

1.6.10 CDPM2

This model is an extension of the ConcreteDPM presented in 1.6.9. CDPM2 has been developed by Grassl, Xenos, Nyström, Rempling and Gylltoft for modelling the failure of concrete for both static and dynamic loading. It is described in detail in [9]. The main differences between CDPM2 and ConcreteDPM are that in CDPM2 the plasticity part exhibits hardening once damage is active. Furthermore, two independent damage parameters describing tensile and compressive damage are introduced. The parameters of CDPM2 are summarised in Tab. 41

The stress for the anisotropic damage plasticity model (ISOFLAG=0) is defined as

$$\boldsymbol{\sigma} = (1 - \omega_t) \bar{\boldsymbol{\sigma}}_t + (1 - \omega_c) \bar{\boldsymbol{\sigma}}_c \quad (163)$$

where $\bar{\boldsymbol{\sigma}}_t$ and $\bar{\boldsymbol{\sigma}}_c$ are the positive and negative parts of the effective stress tensor $\bar{\boldsymbol{\sigma}}$, respectively, and ω_t and ω_c are two scalar damage variables, ranging from 0 (undamaged) to 1 (fully damaged).

The stress for the isotropic damage plasticity model (ISOFLAG=1) is defined as

$$\boldsymbol{\sigma} = (1 - \omega_t) \bar{\boldsymbol{\sigma}} \quad (164)$$

The effective stress $\bar{\boldsymbol{\sigma}}$ is defined according to the damage mechanics convention as

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_e : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p) \quad (165)$$

Plasticity:

The yield surface is described by the Haigh-Westergaard coordinates: the volumetric effective stress $\bar{\sigma}_V$, the norm of the deviatoric effective stress $\bar{\rho}$ and the Lode angle $\bar{\theta}$. The yield surface is

$$f_p(\bar{\sigma}_V, \bar{\rho}, \bar{\theta}; \kappa_p) = \left\{ [1 - q_{h1}(\kappa_p)] \left(\frac{\bar{\rho}}{\sqrt{6}f_c} + \frac{\bar{\sigma}_V}{f_c} \right)^2 + \sqrt{\frac{3}{2}} \frac{\bar{\rho}}{f_c} \right\}^2 + m_0 q_{h1}^2(\kappa_p) q_{h2}(\kappa_p) \left[\frac{\bar{\rho}}{\sqrt{6}f_c} r(\cos \bar{\theta}) + \frac{\bar{\sigma}_V}{f_c} \right] - q_{h1}^2(\kappa_p) q_{h2}^2(\kappa_p) \quad (166)$$

It depends also on the hardening variable κ_p (which enters through the dimensionless variables q_{h1} and q_{h2}). Parameter f_c is the uniaxial compressive strength. For $q_{h2} = 1$, the yield function is identical to the one of CDPM. The shape of the deviatoric section is controlled by the Willam-Warnke function

$$r(\cos \bar{\theta}) = \frac{4(1 - e^2) \cos^2 \bar{\theta} + (2e - 1)^2}{2(1 - e^2) \cos \bar{\theta} + (2e - 1) \sqrt{4(1 - e^2) \cos^2 \bar{\theta} + 5e^2 - 4e}} \quad (167)$$

Here, e is the eccentricity parameter. The friction parameter m_0 is given by

$$m_0 = \frac{3(f_c^2 - f_t^2)}{f_c f_t} \frac{e}{e + 1} \quad (168)$$

where f_t is the tensile strength.

The flow rule (26) is split into a volumetric and a deviatoric part, i.e., the gradient of the plastic potential is decomposed as

$$\mathbf{m} = \frac{\partial g}{\partial \bar{\boldsymbol{\sigma}}} = \frac{\partial g}{\partial \bar{\sigma}_V} \frac{\partial \bar{\sigma}_V}{\partial \bar{\boldsymbol{\sigma}}} + \frac{\partial g}{\partial \bar{\rho}} \frac{\partial \bar{\rho}}{\partial \bar{\boldsymbol{\sigma}}} \quad (169)$$

Description	CDPM2
Record Format	con2dpm $d_{(rn)}$ # $E_{(rn)}$ # $n_{(rn)}$ # $tAlpha_{(rn)}$ # $ft_{(rn)}$ # $fc_{(rn)}$ # $wf_{(rn)}$ # $[stype_{(in)}]$ # $[ft1_{(rn)}]$ # $[wf1_{(rn)}]$ # $[efc_{(rn)}]$ # $[ecc_{(rn)}]$ # $[kinit_{(rn)}]$ # $[Ahard_{(rn)}]$ # $[Bhard_{(rn)}]$ # $[Chard_{(rn)}]$ # $[Dhard_{(rn)}]$ # $[Asoft_{(rn)}]$ # $[helem_{(rn)}]$ # $[dilation_{(rn)}]$ # $[hp_{(rn)}]$ # $[isoflag_{(in)}]$ # $[rateflag_{(in)}]$ # $fcZero_{(in)}$ # $[yieldtol_{(rn)}]$ # $[newtoniter_{(in)}]$
Parameters	<ul style="list-style-type: none"> - d material density - E Young modulus - n Poisson ratio - $tAlpha$ thermal dilatation coefficient - ft uniaxial tensile strength - fc uniaxial compressive strength - wf parameter that controls the slope of the softening branch to be used - $stype$ allows to choose different types of softening laws, default value 1: <ul style="list-style-type: none"> 0 - linear softening 1 - bilinear softening 2 - exponential softening - $ft1$ parameter for the bilinear softening law ($stype = 1$) defining the ratio between intermediate stress and tensile strength, optional, default value 0.3 - $wf1$ parameter for the bilinear softening law ($stype = 1$) defining the intermediate crack opening, optional, default value 0.15 - efc parameter for exponential softening law in compression, optional, default value 100×10^{-6} - ecc eccentricity parameter, optional, default value 0.525 - $kinit$ initial value of hardening law, optional, default value 0.3 - $Ahard$ hardening parameter, optional, default value 0.08 - $Bhard$ hardening parameter, optional, default value 0.003 - $Chard$ hardening parameter, optional, default value 2 - $Dhard$ hardening parameter, optional, default value 1×10^{-6} - $Asoft$ softening parameter, optional, default value 15 - $helem$ element size h, optional (if not specified, the actual element size is used) - $dilation$ dilation factor, optional, default value 0.85 - hp hardening modulus, optional, default value 0.01. - $isoflag$ flag which allows for the use of only one damage parameter if set to 1, default value 0 - $rateflag$ flag which allows for the consideration of the effect of strain rates if set to 1, default value 0. If $rateflag = 1$, then hp should be chosen as $hp=0.5$. - $fcZero$ Input parameter for modelling the effect of strain value. Recommended value 10 MPa. Only if $rateflag=1$, $fcZero$ has to be specified. - $yieldtol$ tolerance for the implicit stress return algorithm, optional, default value 1×10^{-6} - $newtoniter$ maximum number of iterations in the implicit stress return algorithm, optional, default value 100
Supported modes	3dMat, PlaneStrain

Table 41: CDPM2 – summary.

Taking into account that $\partial\bar{\sigma}_V/\partial\bar{\sigma} = \delta/3$ and $\partial\bar{\rho}/\partial\bar{\sigma} = \bar{s}/\bar{\rho}$, restricting attention to the post-peak regime (in which $q_{h1} = 1$) and differentiating the plastic potential (147), we rewrite equation (169) as

$$\mathbf{m} = \frac{\partial g}{\partial \bar{\sigma}} = \frac{\partial m_g}{\partial \bar{\sigma}_V} \frac{\delta}{3f_c} + \left(\frac{3}{f_c} + \frac{m_0}{\sqrt{6}\bar{\rho}} \right) \frac{\bar{s}}{f_c} \quad (170)$$

The flow rule is non-associative which means that the direction of the plastic flow is not normal to the yield surface. This is important for concrete since an associative flow rule would give an overestimated maximum stress for passive confinement.

The dimensionless variables q_{h1} and q_{h2} that appear in (143), (147) and (148) are functions of the hardening variable κ_p . They control the evolution of the size and shape of the yield surface and plastic potential. The first hardening law q_{h1} is

$$q_{h1}(\kappa_p) = \begin{cases} q_{h0} + (1 - q_{h0}) (\kappa_p^3 - 3\kappa_p^2 + 3\kappa_p) - H_p (\kappa_p^3 - 3\kappa_p^2 + 2\kappa_p) & \text{if } \kappa_p < 1 \\ 1 & \text{if } \kappa_p \geq 1 \end{cases} \quad (171)$$

The second hardening law q_{h2} is given by

$$q_{h2}(\kappa_p) = \begin{cases} 1 & \text{if } \kappa_p < 1 \\ 1 + H_p(\kappa_p - 1) & \text{if } \kappa_p \geq 1 \end{cases} \quad (172)$$

The evolution law for the hardening variable,

$$\dot{\kappa}_p = \frac{\|\dot{\boldsymbol{\varepsilon}}_p\|}{x_h(\bar{\sigma}_V)} (2 \cos \bar{\theta})^2 = \frac{\dot{\lambda} \|\mathbf{m}\|}{x_h(\bar{\sigma}_V)} (2 \cos \bar{\theta})^2 \quad (173)$$

sets the rate of the hardening variable equal to the norm of the plastic strain rate scaled by a hardening ductility measure, which is identical to the one used for the CDPM.

Damage:

Damage is initiated when the maximum equivalent strain in the history of the material reaches the threshold $\varepsilon_0 = f_t/E$. This expression is determined from the yield surface ($f_p = 0$) by setting $q_{h1} = 1$ and $q_{h2} = \tilde{\varepsilon}/\varepsilon_0$. From this quadratic equation for $\tilde{\varepsilon}$, the equivalent strain is determined as

$$\tilde{\varepsilon} = \frac{\varepsilon_0 m_0}{2} \left(\frac{\bar{\rho}}{\sqrt{6}f_c} r(\cos \theta) + \frac{\bar{\sigma}_V}{f_c} \right) + \sqrt{\frac{\varepsilon_0^2 m_0^2}{4} \left(\frac{\bar{\rho}}{\sqrt{6}f_c} r(\cos \theta) + \frac{\bar{\sigma}_V}{f_c} \right)^2 + \frac{3\varepsilon_0^2 \bar{\rho}^2}{2f_c^2}} \quad (174)$$

Tensile damage is described by a stress-inelastic displacement law. For linear and exponential damage type the stress value f_t and the displacement value w_f must be defined. For the bi-linear type two additional parameters f_{t1} and w_{f1} are required.

For the compressive damage variable, an evolution based on an exponential stress-inelastic strain law is used. The stress versus inelastic strain in the softening regime in compression is

$$\sigma = f_t \exp\left(-\frac{\varepsilon_i}{\varepsilon_{fc}}\right) \quad \text{if } 0 < \varepsilon_i \quad (175)$$

where ε_{fc} is an inelastic strain threshold which controls the initial inclination of the softening curve. The use of different damage evolution for tension and compression is one important improvement over CDPM.

The history variables κ_{dt1} , κ_{dt2} , κ_{dc1} and κ_{dc2} depend on a ductility measure x_s , which takes into account the influence of multiaxial stress states on the damage evolution. This ductility measure is given by

$$x_s = 1 + (A_s - 1) R_s \quad (176)$$

where R_s is

$$R_s = \begin{cases} -\frac{\sqrt{6}\bar{\sigma}_V}{\bar{\rho}} & \text{if } \bar{\sigma}_V \leq 0 \\ 0 & \text{if } \bar{\sigma}_V > 0 \end{cases} \quad (177)$$

and A_s is a model parameter.

Strain rate:

Concrete is strongly rate dependent. If the loading rate is increased, the tensile and compressive strength increase and are more prominent in tension than in compression. The dependency is taken into account by an additional variable α_r . The rate dependency is included by scaling both the equivalent strain rate and the inelastic strain. The rate parameter is defined by

$$\alpha_r = (1 - X)\alpha_{rt} + X\alpha_{rc} \quad (178)$$

where X is the continuous compression measure ($= 1$ means only compression, $= 0$ means only tension).

The functions α_{rt} and α_{rc} depend on the input parameter f_{c0} . A recommended value for f_{c0} is 10 MPa.