MODELING REBAR: THE FORGOTTEN SISTER IN REINFORCED CONCRETE MODELING

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Introduction

Historically reinforcement in concrete was modeled as a layer of 'smeared' material, i.e. a combination of steel and concrete, or via shared (merged) nodes using either truss or beam elements. These methods have been superseded by constraint based methods which allow the user to define the reinforcement mesh independent of the concrete mesh, i.e. the nodes of the two meshes do not need to coincide.

In this manuscript, the various methods of including rebar in reinforced concrete are described. The effectiveness of these methods is assessed through comparative examples of a reinforced concrete slab loaded in axial extension, under self-weight bending and subjected to air blast loading with experimental results comparisons. The manuscript concludes with a mesh refinement study where the reinforcement and concrete mesh densities are changed independently.

Concrete Slab Geometry and Modeling

As part of the "Blind Blast Simulation Contest[1]," organized by the University of Missouri Kansas City, participants were invited to submit predictions of reinforced concrete slabs subjected to air blast loading. There were two classes of concrete: normal strength $f_c' = 5\,\text{ksi}$ (34.5 MPa) and high strength $f_c' = 15$ ksi (103.5 MPa). The normal strength concrete was reinforced with Number 3 Grade 60 steel bars with yield strength of 68 ksi (469 MPa). The high strength concrete was reinforced with Vanadium Number 3 bars with nominal yield strength of about 83 ksi (572 MPa). Each concrete slab design was subjected to two different air blast wave forms with impulses of about 5.38 and 7.04 MPa-ms.

For the purposes of this reinforcement modeling study, the normal strength $f_c' = 5\,\text{ksi}$ (34.5 MPa) concrete reinforced with Number 3 Grade 60 steel bars with yield strength of 68 ksi (469 MPa) will be considered. A description of the reinforced concrete slab and associated modeling is presented next. Interested readers should review the associated web site for additional details.

The overall concrete slab dimensions are 64x33.75x4 inches (1625.6x958.85x101.6 mm) with a single layer of reinforcement, as shown in Figure 1, on the side of the slab away from the blast.

The concrete slab fixture consists of a steel frame with front and back steel cross supports at the long ends of the slab. Figure 2 shows the final assembly of the blast side fixture over the concrete slab. The slab is mounted in the depicted removable end of a large air blast simulator.

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[1] http://sce.umkc.edu/blast-prediction-contest/ (URL last checked 21Oct13)

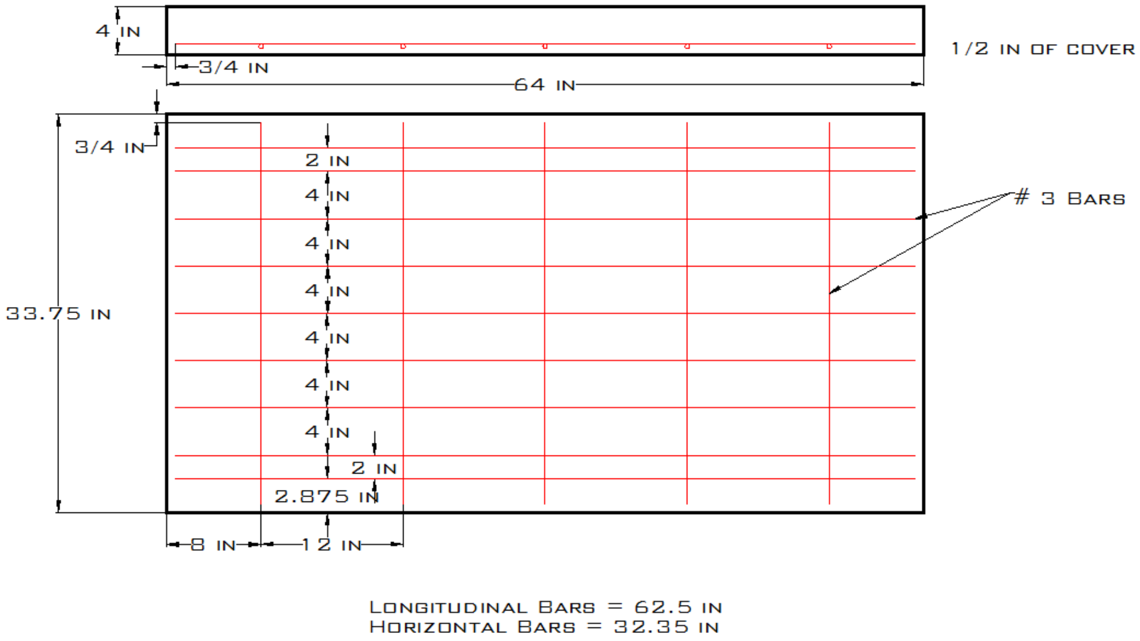

Figure 1 Overall concrete slab dimensions and reinforcement layout.

Figure 2 Photograph of blast side fixture about to be assembled over the previously placed concrete slab (directly behind technician).

Geometric Modeling

The model, see Figure 3, consists of three major components:

1. Concrete slab modeled with solid hexahedra elements,
2. Fixture cross pieces modeled with shell elements,
3. Reinforcement modeled with beam elements.

After some preliminary modeling and assessment, it was decided to use a nominal element size in the concrete slab of 0.5 inches (12.7mm), which provided for one solid element between the reinforcement and the surface of the slab, i.e. concrete cover. This element size corresponds to 8 elements through the thickness of the slab.

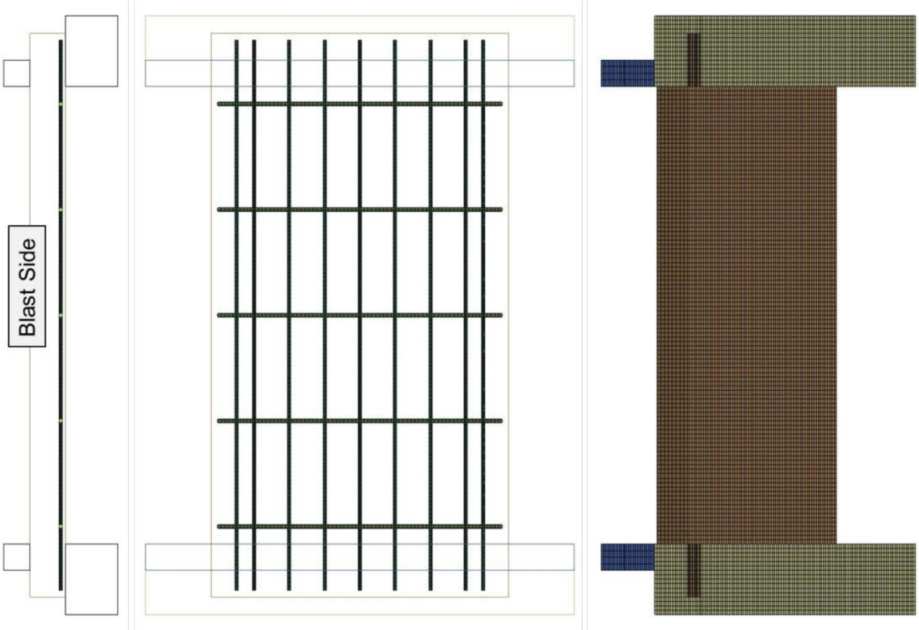

Figure 3 Views of the finite element model.

Having established a nominal element size for the concrete slab model, this same element size (12.7mm) was used to model the steel fixture members. Contact surfaces will be required between the outer surfaces of the concrete slab and the inner surfaces of these fixture members. Using the same mesh size on both sides of the contact surface is a good starting point.

The baseline case for the reinforcement is the mesh required for shared nodes between the concrete solid elements and reinforcement beam elements. All the other reinforcement models will use this mesh, with variations introduced as appropriate. Thus the major mesh lines of the concrete slab are aligned with the reinforcement pattern shown previously in Figure 1. The regions between these mesh lines are then filled with solid elements representing the concrete at a *nominal* dimension of 12.7mm, i.e. not all concrete elements are 12.7mm on a side.

The beam elements comprising the reinforcement grid have the same variable mesh size to accommodate the shared node rebar modeling. The resulting beam element reinforcement mesh had a minimum element size of 0.4 inches (10.16mm) and a maximum element size of 0.483 inches (12.27mm).

Material Modeling

Selection of an appropriate LS-DYNA concrete model was restricted to those with the smeared reinforcement option. Several LS-DYNA concrete models offer the option to include smeared reinforcement:

- *MAT_PSEUDO_TENSOR (MAT016)
- *MAT_CONCRETE_DAMAGE (MAT072)
- *MAT_WINFRITH_CONCRETE (MAT085) used with
- *MAT_WINFRITH_CONCRETE_REINFORCEMENT
- *MAT_BRITTLE_DAMAGE (MAT096)
- *MAT_CONCRETE_EC2 (MAT172)
- *MAT_RC_BEAM (MAT174)
- *MAT_RC_SHEAR_WALL (MAT194)

The Winfrith concrete model (MAT085) was selected based upon the author's previous experience with this model, the minimal user input needed to describe the concrete behavior, the user friendly method of inputting the smeared reinforcement, and the model's popularity in applications dominated by tensile cracking, as is the case for the blast loaded slab.

The Grade 60 reinforcement constitutive model used was *MAT_PIECEWISE_LINEAR_PLASTICITY with a *DEFINE_CURVE to specify the effective plastic strain versus stress behavior. This material data was provided on the *Blind Blast Simulation Contest* web site. For the smeared and truss element reinforcement models, a simplified bi-linear constitutive model was required.

Details of the LS-DYNA input for the concrete and reinforcement material models are provide in the appendices.

Methods of Including Rebar in Reinforced Concrete

Figure 4 is a schematic indicating the methods available in LS-DYNA for including rebar in a reinforced concrete model. Generally, the CPU time required for a given simulation increases as the schematic is traversed from left to right and top to bottom. The meshing effort needed to generate each mesh type is generally greatest for the shared nodes method.

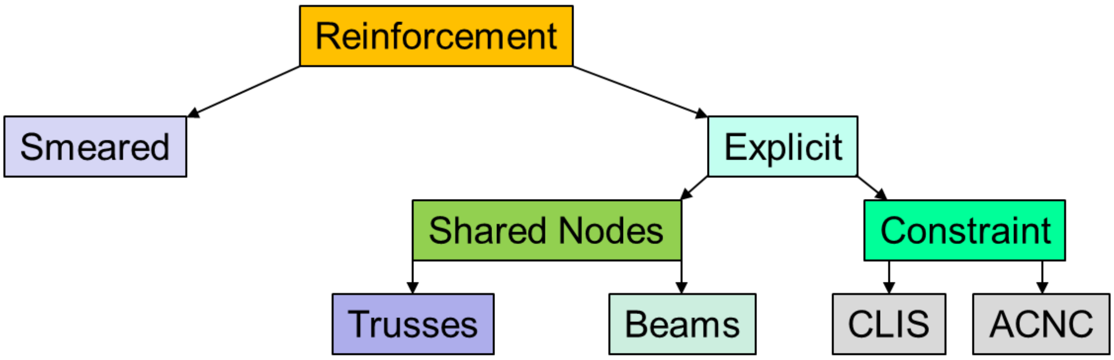

Figure 4 Schematic of various reinforcement modeling methods for rebar in reinforced concrete.

Smeared Reinforcement

Although the smeared rebar approach is a common analytical procedure, its approximation may not be acceptable in some situations. Typically the smeared rebar approach works best for small deformations where the reinforcement remains elastic, or does not stress too far beyond the yield stress.

The amount of reinforcement is specified as a volume fraction f_R that is the volume of steel divided by the volume of concrete. Typically this reinforcement volume fraction is specified per layer of elements in a reinforced concrete structure. For some of the smeared reinforcement concrete models, this may require separate Part IDs for the concrete elements that contain reinforcement and those that are concrete only elements.

The idea behind the smeared reinforcement is to volume-fraction average the material properties, e.g. bulk modulus, shear modulus, and yield strength as for *MAT_PSEUDO_TENSOR (MAT016) via the relation

$$K = \left(1 - f_R\right) K_C + f_R K_R \tag{1}$$

where K is the bulk modulus that has been volume-fraction weighted via combining K_C the concrete bulk modulus and K_R the bulk modulus of the reinforcement. The volume-fraction averaging concept is also used in composite materials to homogenize the material properties; reinforced concrete can be thought of as a composite, i.e. steel and concrete materials. However, just as in composite material analysis, once the matrix or fiber material fails (yields) the accuracy of the homogenization is lost.

A previous drawback of smeared reinforcement was the inability to graphical confirm, with LS-PrePost, the placement of the reinforcement within the concrete model. LS-PREPOST now provides a visualization of the smeared reinforcement planes in the pre-processor mode via **ENTITY DISPLAY > MAT > REBAR.**

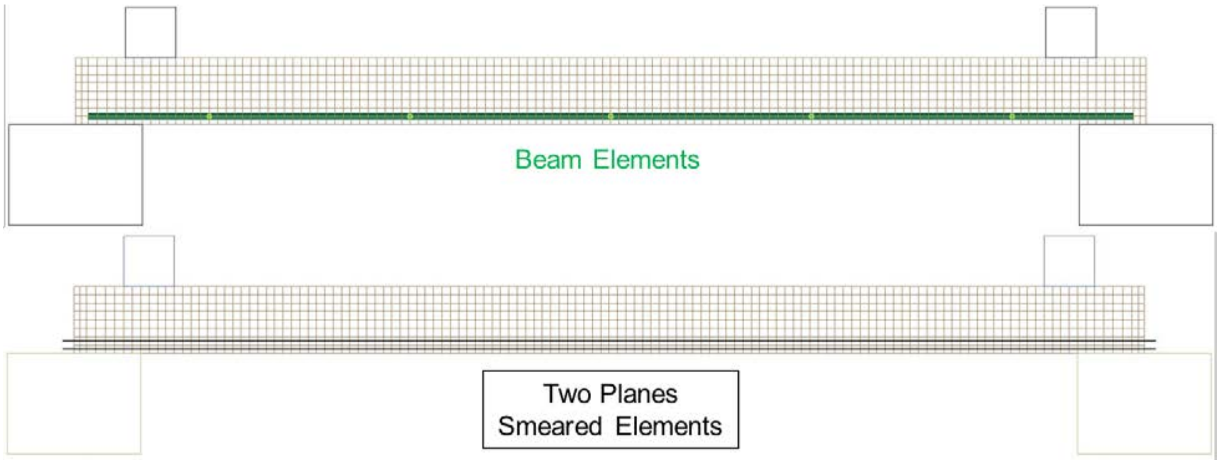

Figure 5 Illustration of two (bottom) rows of concrete elements occupied by the explicit beam element reinforcement or the smeared reinforcement.

Figure 5 shows a side view of the reinforced concrete slab with its supports. The upper portion of this figure shows the explicit beam element reinforcement is aligned with a layer of concrete elements nodes. The lower portion of this figure indicates the two rows of concrete elements that will contain the smeared reinforcement. Two layers of smeared reinforcement are required since for the baseline shared node configuration the reinforcement spans both rows of concrete elements.

If we designate the longest rebar direction as the X-direction, then as per Figure 1, there are nine #3 bars in that direction and five #3 bars in the orthogonal direction, here called the Y-direction. The Winfrith concrete model smeared reinforcement implementation requires the area fraction of steel in each direction, in addition to specifying the normal direction to the plane of the reinforcement, i.e. the Z-direction in this example.

The area of one #3 bar is $\pi\left(\frac{3}{16}\right)^2 = 0.11 \text{ inches}^2 = 71.256 \text{ mm}^2$.

The concrete area in the Y-direction is $(33.75)(0.5) = 16.875 \text{ inches}^2 = 10887.075 \text{ mm}^2$

The concrete area in the X-direction is $(64)(0.5) = 32 \text{ inches}^2 = 20645.12 \text{ mm}^2$

where 0.5 inches=12.7mm is the Z-dimension (thickness) of one row of concrete elements.

The X-direction reinforcement ratio is $0.0589 = 9(71.256)/10887.075$ or about 5.9%.

The Y-direction reinforcement ratio is $0.0173 = 5(71.26)/20645.12$ or about 1.7%.

These ratios are divided by two for two layers of reinforcement, i.e. 0.0295 in the X-direction and 0.00865 in the Y-direction. These reinforcement area ratios, the global normal to the reinforcement plane, and the Z-coordinate of each layer are entered into the additional keyword required for smeared reinforcement when using the Winfrith concrete model:

```
*MAT_WINFRITH_CONCRETE_REINFORCEMENT
$ blank  PID  AXIS  COOR   RQA     RQB
       , 300,   3,  -6.35, 0.0295  0.00865
       , 300,   3, -19.05, 0.0295  0.00865
```

For this example the Part ID for the concrete is 300 and the origin is on the slab face with the concrete cover with the Z-direction positive in the direction of the surface normal, i.e. the two layers of reinforcement are centered -6.25(=-12.7/2)mm and -19.05(=-6.25-12.7/2)mm from the outer surface.

Explicit Reinforcement

As indicated in Figure 4, when the reinforcement is to be included explicitly, the choices are shared (merged) nodes or constraint methods.

Shared Nodes

The shared node approach for including reinforcement requires the nodes of the reinforcement grid and concrete mesh to be identical. The rather trivial single layer of orthogonal reinforcement, considered in this example, does not present too much of a meshing challenge. However, when three dimensional reinforcement grids are required, e.g. multiple layers of reinforcement connected via additional reinforcement or stirrups through the thickness, the shared node meshing becomes a challenge.

The shared node beam or truss element meshes will require a Part ID referencing the previously discussed steel reinforcement constitutive model and Section keywords defining the element type and cross sectional geometry. Both beam and truss elements use the *SECTION_BEAM keyword and the element type ELFORM(=1 or 3) differentiates beam from truss elements, respectively.

```
$   Beam-Rebar (#3 bar = 3/8 = 0.375in = 9.525mm diameter)
*SECTION_BEAM
$ SECID  ELForm   SHRF  QR/IRID  CST SCOOR NSM
   100,     1,      0,      3,     1
$   TS1    TS2    TT3    TT4
   9.525,  9.525,   0.0,   0.0

$   Truss-Rebar (#3 bar = 3/8 = 0.375in => 71.26mm^2 area)
*SECTION_BEAM
$ SECID  ELFORM   SHRF  QR/IRID  CST SCOOR NSM
   100,     3
$   A    RAMPT    STRESS
   71.26
```

Constraint Methods

Unlike the shared node method of including reinforcement, when the constraint method is selected the reinforcement and concrete meshes are constructed independently. The two meshes are then superimposed, in the appropriate relative geometric configuration, and LS-DYNA internally constructs a system of constraints restricting the motion of the two meshes to be consistent.

LS-DYNA offers two keywords for specifying the embedment of reinforcement into the concrete mesh, both keywords use the same algorithm for the internal generation of the constraints. Thus the two keywords differ only by the extent of their user input.

The required inputs are the Part IDs of the reinforcement and the concrete, with the reinforcement considered as the Slave part coupled to the Master concrete part. The coupling type CTYPE=2 is constrained acceleration and velocity at the nodes. As can be seen in the two keyword examples, the *ALE_COUPLING_NODAL_CONSTRAINT (ACNC) input is a simplified subset of the more complex *CONSTRAINED_LAGRANGE_IN_SOLID (CLIS) input. Although to be fair, only values for the first input line of the CLIS keyword are required when coupling reinforcement and concrete.

```
*ALE_COUPLING_NODAL_CONSTRAINT
$    SLAVE    MASTER     SSTYP     MSTYP     CTYPE     MCOUP
      1200       300         0         1         2
$    START       END    FRCMIN
```

```
*CONSTRAINED_LAGRANGE_IN_SOLID
$                                  Couple Rebar to Concrete Target
$    slave    master     sstyp     mstyp     nquad     ctype     direc     mcoup
      1200       300         0         1         0         2         2         0
$    start       end      pfac      fric    frcmin      norm   normtyp      damp
         0         0       0.0       0.0       0.0         0                 0.3
$       cq      hmin      hmax     ileak     pleak   lcidpor     nvent    iblock
                                       0
$   iboxid   ipenchk   intforc   ialesof    lagmul    pfacmm      thkf
                             0
```

Reinforcement Simple Verification & Validation

In this section two simple reinforcement verification examples are presented:

1. Axial extension of a reinforced concrete slab
2. Dynamics self-weight bending of a reinforced concrete slab

The first check the percentage of reinforcement and the second the location of the reinforcement within the concrete slab.

For validation of the various reinforcement methods, the reinforced concrete slab is dynamically loaded and the central deflection compared with the corresponding experimental result.

Reinforced Concrete Slab – Axial Extension

As stated previously, when smeared reinforcement is used, there is a graphical check, with LS-PrePost, that the smeared reinforcement is positioned correctly. However there is no check that the required percentage of steel has been input correctly. To establish the percentage of reinforcement is correct, a reduced thickness version of the above described reinforced concrete slab was loaded in axial extension. Figure 6 shows the reinforced concrete slab with only the lower two rows of concrete elements retained, i.e. the concrete elements on either side of the explicit reinforcement. The reduced thickness is 25.4mm.

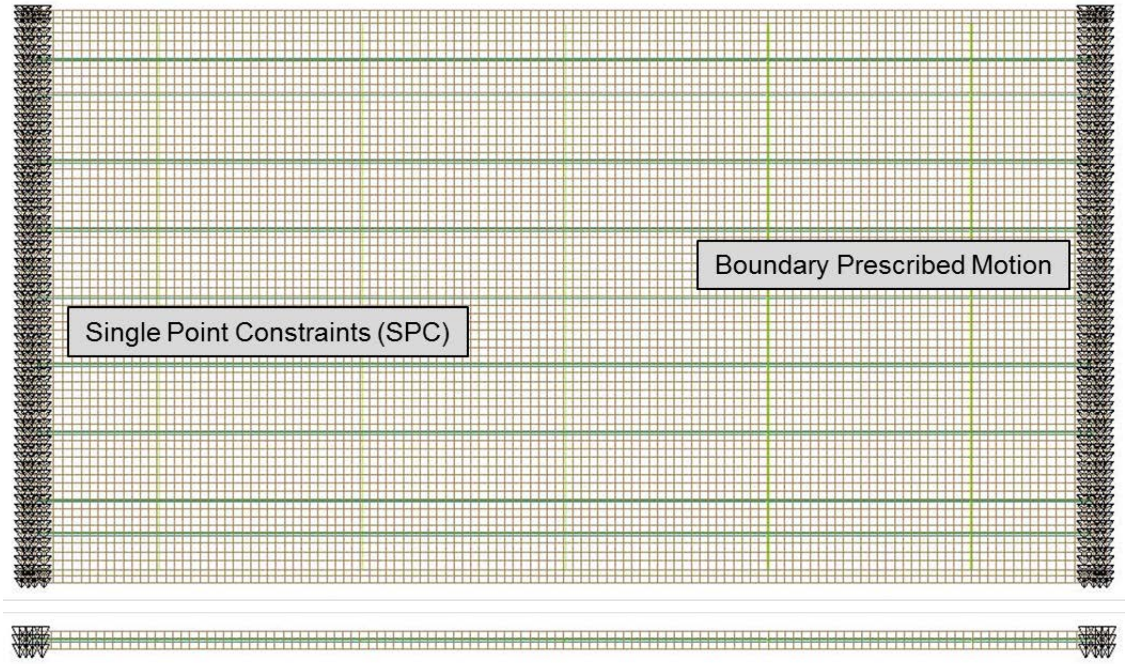

Figure 6 Reduced thickness slab model for axial extension simulations.

The axial extension of the reinforced concrete slab is provided by constraining the motion of a group of *concrete only nodes* at the left end of the slab via Single Point Constraints (*BOUNDARY_SPC) and prescribing an axial velocity of a group of *concrete only nodes* at the right end of the slab via *BOUNDARY_PRESCRIBED_MOTION. Groups of nodes interior to the edges of the slab are selected because under extension the concrete will fail in tension at 4MPa. Since there is no regularization in the Winfrith concrete model, it is most likely the outer column of nodes under prescribed displacement (right side) would fail first and not transmit additional load to the remainder of the slab. Also, by design of the experiment, there is no explicit reinforcement in the outer most concrete elements. Thus concrete failure of these outer elements would not allow for loading of the reinforcement.

The comparisons to be made are of the axial force as a function of displacement. The right end of the slab is displaced 20mm in the longitudinal (X-direction) over 20msec. The X-forces at all the SPC nodes are output via *DATABASE_SPCFORC and summed to provide the total X-force.

The total force for the concrete only (no rebar) case is just the maximum tensile stress, $f_t' = 4$ MPa, multiplied by the area of the concrete on the short edges of the slab $21{,}774\text{mm}^2$ $(= 25.4 \times 857.25)$ or 87.1kN.

When the reinforcement is included, the rebar will continue to carry additional load after the concrete has failed in tension. The total force for this case can be more easily calculated if it is assumed the constitutive response of the reinforcement is elastic-perfectly plastic, see Figure 7. The maximum total force will then be the total area of the rebar 641.3mm^2 $(= 9 \times 71.256)$ times the rebar ultimate stress of 872MPa, or 559kN.

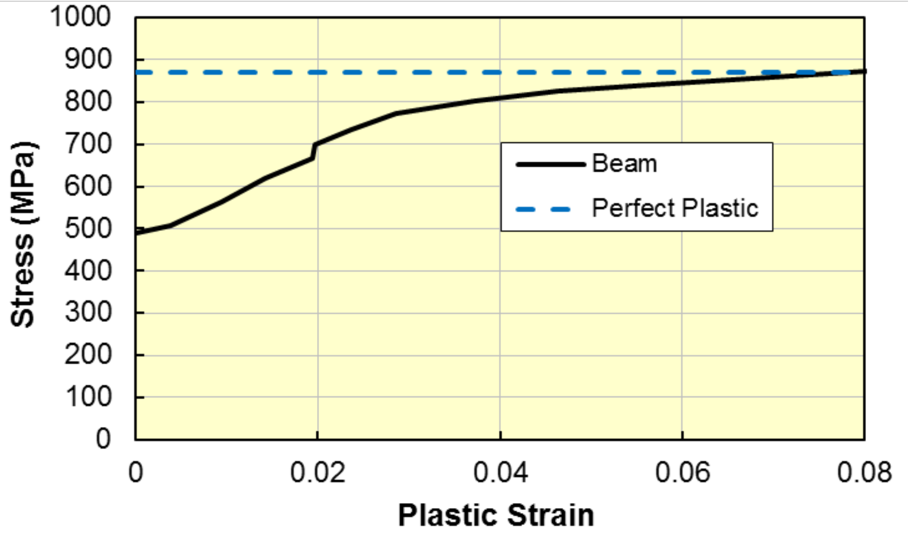

Figure 7 Elastic-perfect plastic assumed constitutive model for reinforcement/

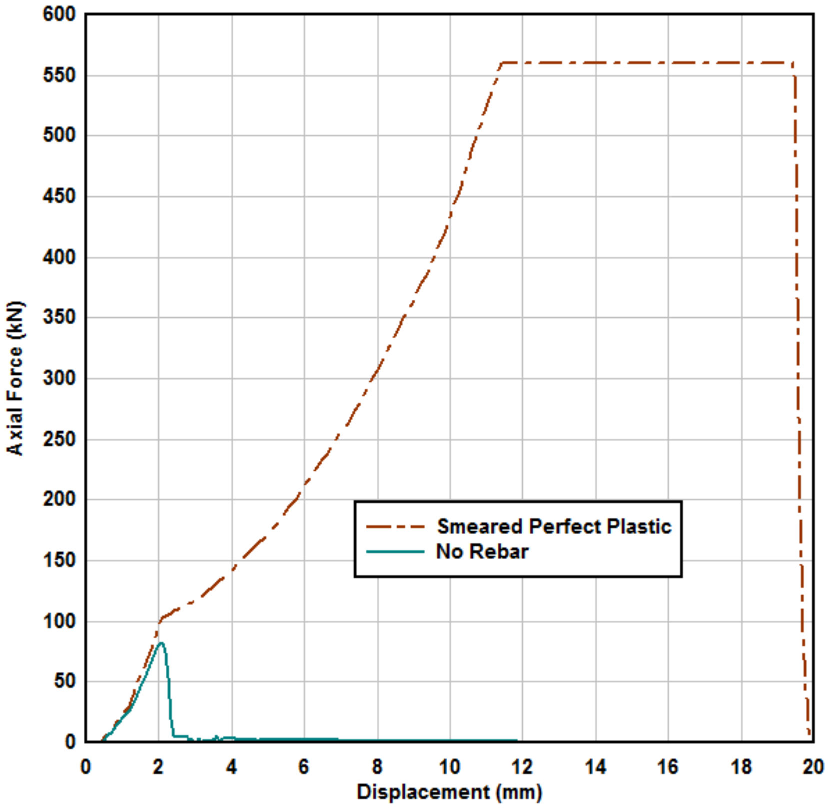

Figure 8 Axial force-displacement for the No Rebar (concrete only) and Perfectly Plastic smeared rebar simulations.

Figure 8 compares the axial force versus displacement for the two check cases:

1. No reinforcement with a maximum force of 82.3kN, a -6% difference.
2. Perfectly plastic smeared reinforcement with maximum force of 568kN, a 2% difference.

Note: a similar extension simulation in the other reinforcement direction (Y-direction) should be performed to check that reinforcement percentage.

Having confirmed the percentage of reinforcement for the smeared Winfrith concrete model, it is informative to check the other reinforcement methods for the axial extension case. Note: the hardening constitutive model for the Winfrith reinforcement, and shared node truss elements, as provided in an appendix, has been reinstated for the remaining simulations, i.e. no longer using the assumed elastic perfectly plastic constitutive model.

Figure 9 shows the axial force versus axial extension of the reinforced concrete slab for the various reinforcement inclusion methods. This figure indicates three grouping of response:

1. Smeared response, for one or two layers of reinforcement,
2. Shared node response, for either beam or truss elements,
3. Constraint method response for either ACNC or CLIS.

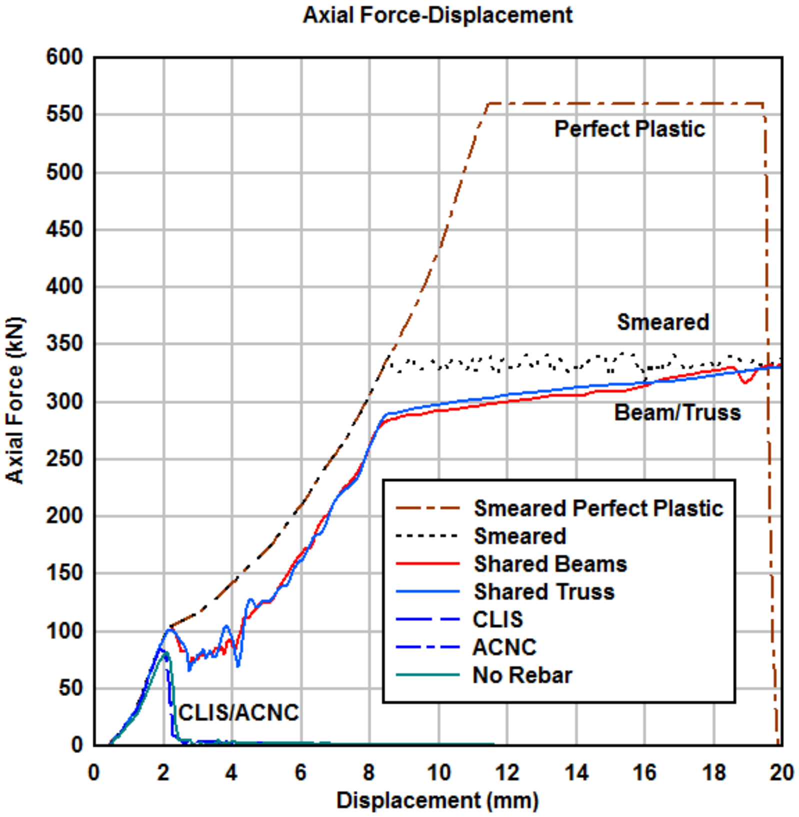

Figure 9 Axial force-displacement for the various methods of including rebar in a reinforced concrete slab.

The smeared and shared node responses have slightly different axial force versus displacement trajectories, but all arrive at about the same maximum force of about 335 to 350kN. This total force agrees with the following check:

- Total axial strain is $\varepsilon = 1.23\times10^{-2} = 20/1625.6$
- Plastic axial strain $\varepsilon_p = \varepsilon - \mathrm{YS/YM} = 1.23\times10^{-2} - 500/2.05\times10^{5} = 9.86\times10^{-3}$
- Rebar Stress $\sigma = \mathrm{YS} + E_h\varepsilon_p = 500 + 4783\left(9.86\times10^{-3}\right) = 547\ \mathrm{MPa}$
- Total Rebar Force $= 9\sigma\ \mathrm{Area} = 9\left(547\right)71.256 = 351\,\mathrm{kN}$

For the smeared models, there is continuous loading since the combination concrete/steel elements can always carry tension. For the shared node models it is similar – as a crack in a concrete element forms, the two element ends move apart, but the beam/truss elements is still connect these nodes and carry load.

The somewhat surprising results are for the constraint methods. Both the CLIS and ACNC do not provide additional axial force beyond that of the concrete failing in tension. This implies that for tensile membrane and cracking dominated reinforced concrete simulations, the constraint methods may under predict the effect of the reinforcement.

The Winfrith concrete model crack patterns and number of cracks for the various reinforcement methods are shown in Figure 10. Each sub-figure includes a view normal to the slab and an edge view to illustrate the through-the-thickness cracks.

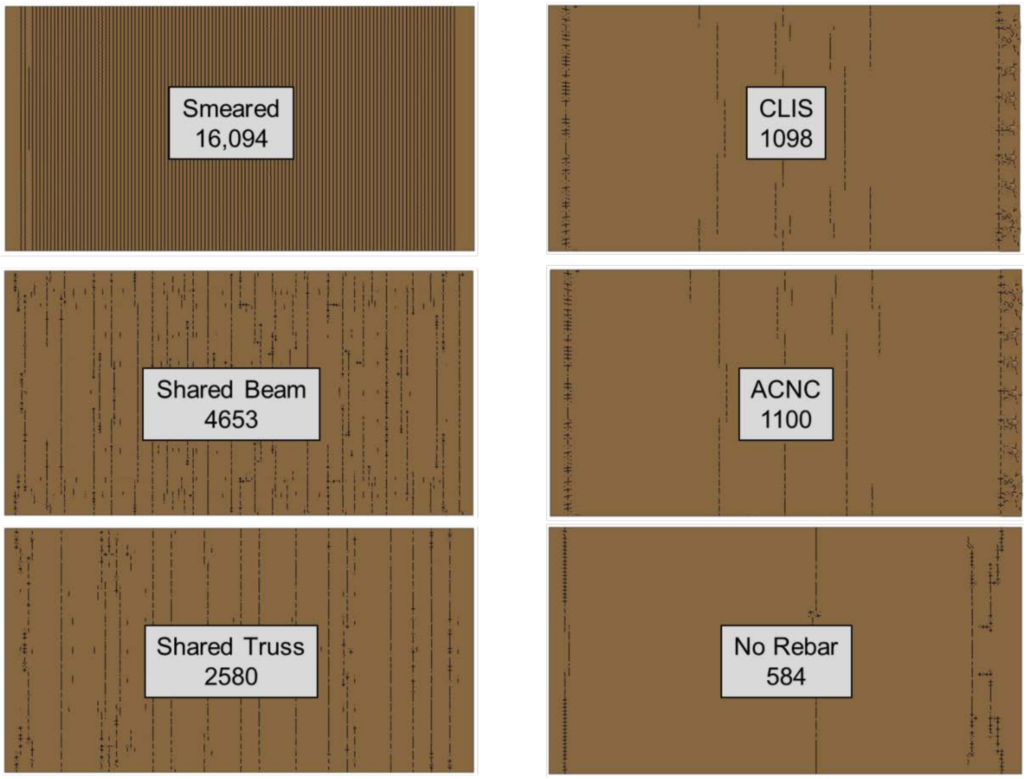

Figure 10 Illustration of crack patterns and number of cracks for the various methods of including rebar in a slab under axial extension.

Reinforced Concrete Slab - Self Weight

As a check on the reinforcement location input for all the methods of including reinforcement in a concrete model, the dynamic central deflection of the reinforced concrete slab acting under self-weight were compared.

The self-weight is imposed via a base acceleration using the LS-DYNA keyword *LOAD_BODY_Z, since in this case the Z-direction is normal to the surface of the slab. The associated acceleration of gravity is applied as a step load, with the maximum central displacement attained in about 6ms. Figure 11 compares the central displacement histories of the slab for the six cases simulated.

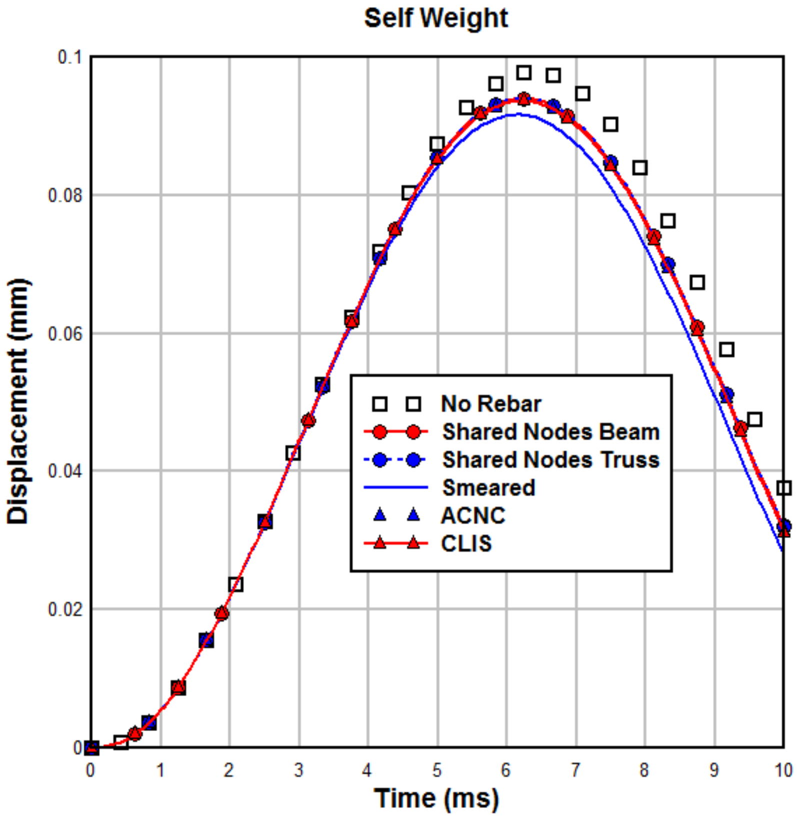

Figure 11 Slab central displacement under dynamic self-weight for six different rebar methods.

A maximum displacement of 0.0978mm was computed for the case of no reinforcement[2] . The shared nodes beam and truss elements had essentially the same maximum displacement of 0.0944mm. The two constraint methods, CLIS and ACNC, had identical maximum displacements of 0.0936mm. The maximum displacement for the smeared reinforcement was 0.091mm; the smeared reinforcement model, and no rebar model, do not include the as of the reinforcement.

The under prediction of the smeared versus shared node models, may also be due to be small differences in cross section moduli. A proper cross section integration, to determine the section modulus, e.g. Izz, might prove this assertion. That is, the two models will have slightly different cross section moduli.

This self-weight example is primarily intended as a check on the reinforcement location input for the various methods, especially the smeared reinforcement. It also serves to illustrate the previously made contention that the smeared reinforcement approach works well for small displacements when the reinforcement remains elastic.

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[2] Using the uniformly loaded elastic beam formula with a span of 1422mm between simple supports provides a maximum static displacement of 0.089mm, which is close to the value reported here for the no-rebar case – albeit with a dynamic load.

Reinforced Concrete Slab – Blast Loading

To provide a more severe test of the available methods for including rebar in reinforced concrete, the slab described above was subjected to a blast loading, i.e. pressure history. Figure 12 shows the measured pressure history, and corresponding impulse, for the blast loading of the reinforced concrete slab. The maximum pressure is 0.34 MPa and the maximum impulse is 7.04 MPa-ms. For simplicity, the pressure history is applied to the entire surface of the concrete on the side opposite the reinforcement.

The resulting slab center deflections are compared to the experimental measurements[3].This *blast* loaded reinforced concrete slab is more typical of the types of reinforced concrete simulations of interest to LS-DYNA users and provides a better basis of assessing the various reinforcement methods.

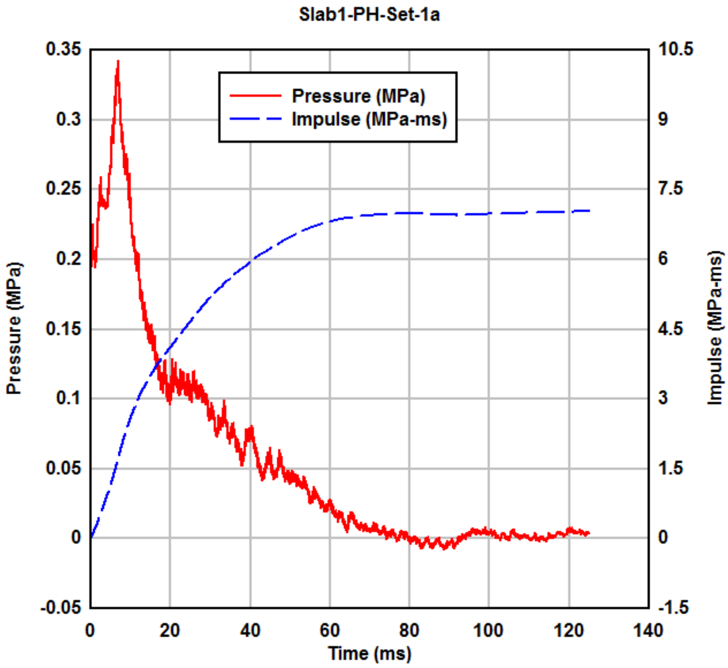

Figure 12 Pressure and corresponding impulse histories applied to surface of slab for blast loading simulation.

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[3] The experimental results were provided after all the "Blind Blast Simulation Contest" entries were submitted.

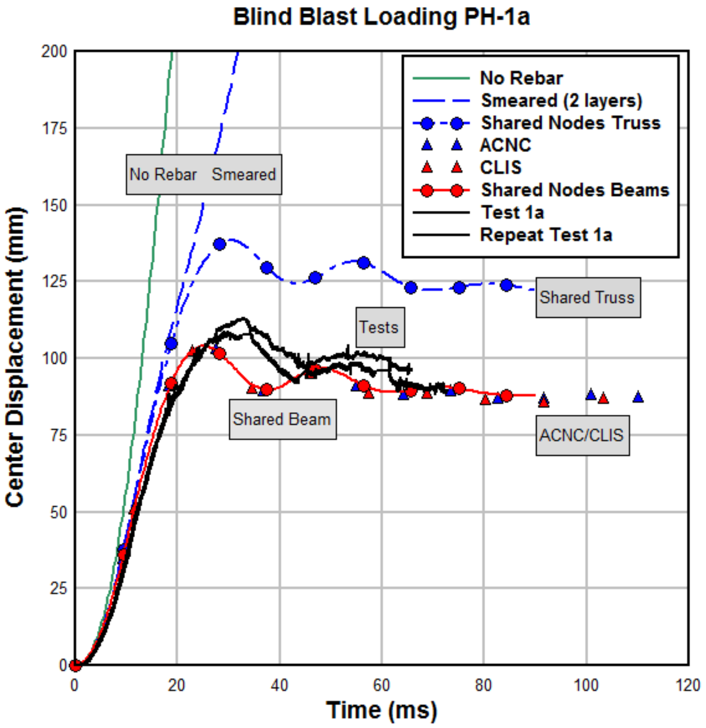

Figure 13 Slab central displacement under blast loading for five different rebar methods compared to the two repeat experimental results.

Figure 13 compares the slab central displacement histories for six reinforcement modeling methods with the results from the two repeat experiments. Of the five reinforcement methods, three, i.e. shared node beams, CLIS and ACNC, provide essentially identical results that agree well with the experimental results. When shared nodes truss elements are used to model the reinforcement, the predicted central displacement is about 30% greater than for the three methods that are in agreement. Since the truss elements do not provide bending resistance, as is the case for beam elements, in this bending dominated deformed configuration, the truss elements do not provide adequate stiffness. The smeared reinforcement does not provide sufficient bending resistance to limit the central deflection of the reinforced concrete slab. Compared to the "No Rebar" case, the smeared reinforcement does provide some bending resistance, just not enough to prevent the slab from failing catastrophically. Note: when a 25% lower impulse blast load[4] was applied to the smeared reinforcement case, the rebar model provided sufficient bending resistant to prevent catastrophic failure.

Figure 14 compares the deformed configuration of the blast loaded reinforced concrete slab with the crack patterns and number of crack indicated for the six reinforcement cases simulated. The deformed slabs shown in the top row include the two shared node configurations, i.e. truss and beam elements, and the two constraint methods, i.e. ACNC and CLIS. It is difficult to discern

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[4] Blind Blast Simulation load Case PH-Set-1b: maximum pressure is 0.28 MPa and a maximum impulse of 5.38 MPa-ms.

much difference in the crack patterns for these four models, and the number of crack is about the same.

The four simulations in the top row of Figure 14 ran to completion at a time of 90 to 110ms. This was not the case for the smeared and no reinforcement configurations shown in the bottom row of this figure. Both simulations terminated early, at 55 and 27ms for the smeared and no rebar configurations, respectively.

In addition to mesh sensitivity, see the next section, the models are sensitive to the Winfrith cracking parameters: f_t' and `FE`, and especially the representation of the rebar stress-strain response, i.e. true stress versus true effective plastic strain.

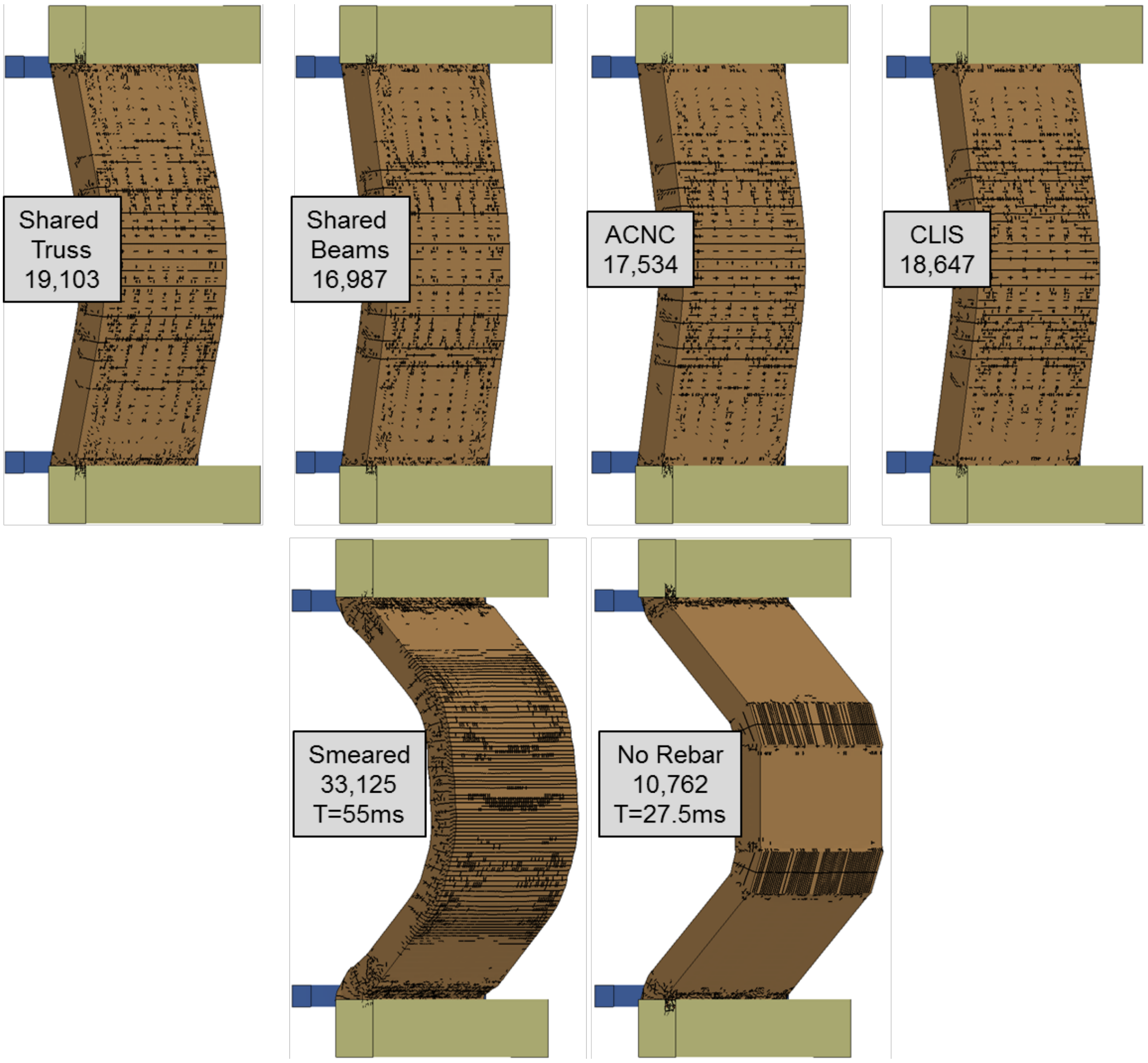

Figure 14 Illustration of the final deformed shapes, crack patterns and number of cracks for the various methods of including the reinforcement.

Rebar and Concrete Mesh Refinement

Mesh refinement is an important step in any analysis, especially when making blind predictions. When the constraint methods are used to model the reinforcement, the two meshes, i.e. reinforcement and concrete, can be constructed independent of each other. This means separate mesh refinement studies for the reinforcement and concrete should be performed.

Uniform Mesh Refinement

The characteristic dimension for the concrete slab is the thickness 4inches (101.6mm) allowing for at least two mesh variants around the nominal 12.7mm mesh spacing:

- 6 elements thru thickness 16.93mm
- 8 elements thru thickness 12.70mm
- 10 elements thru thickness 10.16mm

The characteristic dimension for the reinforcement is 2inch (50.4mm) horizontal bar spacing, again allowing for at least two mesh variants around the nominal 12.7mm mesh spacing:

- 25.4mm (1inch)
- 12.7mm (0.5inch)
- 6.35mm (0.25inch)

This selection of mesh sizes provides for *nine* possible uniform mesh refinement combinations.

Figure 15 shows the slab center displacement results for all nine uniform mesh refinement combinations, all using the constraint CLIS method. The baseline result used a 12.7mm nominal mesh size for both the reinforcement and concrete; this was the baseline mesh in the previous studies; this nominal discretization element size was required for the shared node reinforcement models. As noted previously in the Geometric Modeling section, the major mesh lines for the concrete slab are aligned with the X and Y direction reinforcement. When the concrete mesh density is changed, only the concrete nodes in between these major mesh lines are changed. This means the concrete and reinforcement nodes at the intersection of the X and Y reinforcement are always collocated, but never merged when using the CLIS method.

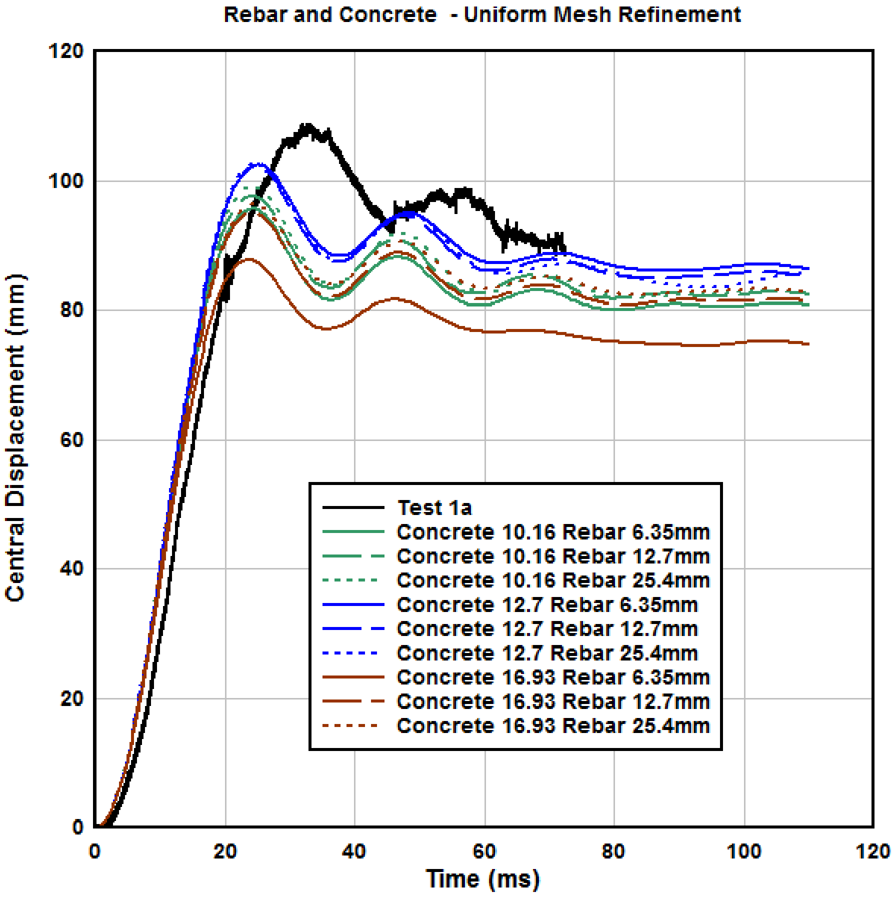

Figure 15 Comparison of slab central displacement for combinations of concrete and reinforcement mesh refinements.

Figure 16 shows the slab central displacement for the rebar and concrete nominal mesh size of 12.7mm, and the concrete mesh refined to 10.16mm, or 10 elements through the slab thickness, and made coarser concrete mesh with a nominal element size of 16.93mm or 6 elements through the slab thickness; the latter two cases are the dashed green and brown lines. The 10.16 and 16.93mm results are quite similar to the baseline 12.7mm result, but all three models slightly under predict the experimental result (Test 1a).

When the nominal concrete mesh size is held constant at 12.7mm, or 8 elements through the thickness of the slab, changing the reinforcement discretization produces a minor change in the central displacement; these results are shown Figure 17. The more coarse 25.4mm rebar discretization and the refined 6.35mm reinforcement results are essentially identical to the baseline 12.7mm reinforcement discretization. However, a further refinement of the reinforcement to 5.08mm shows a small decrease in the slab central displacement.

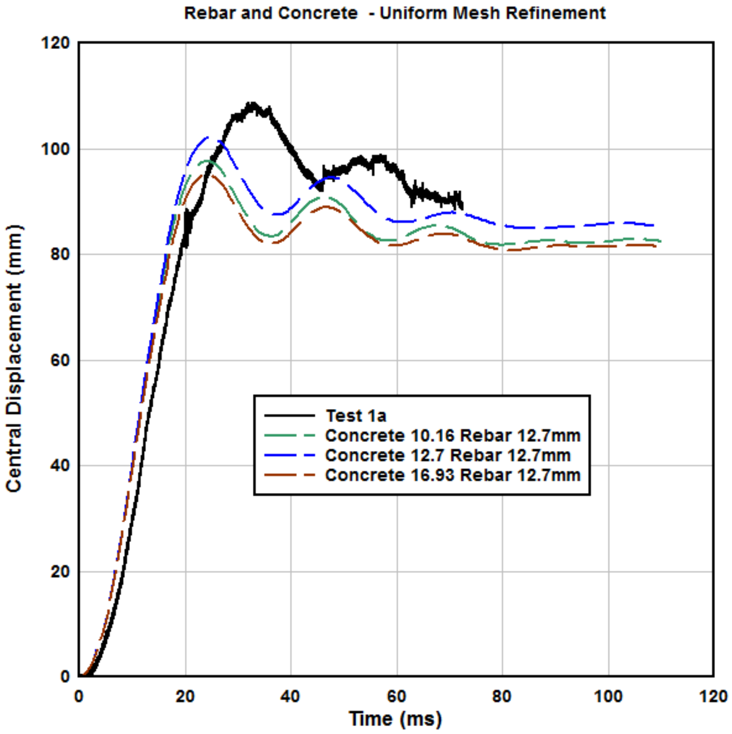

Figure 16 Comparison of central displacement for 12.7mm rebar mesh with three concrete mesh refinements.

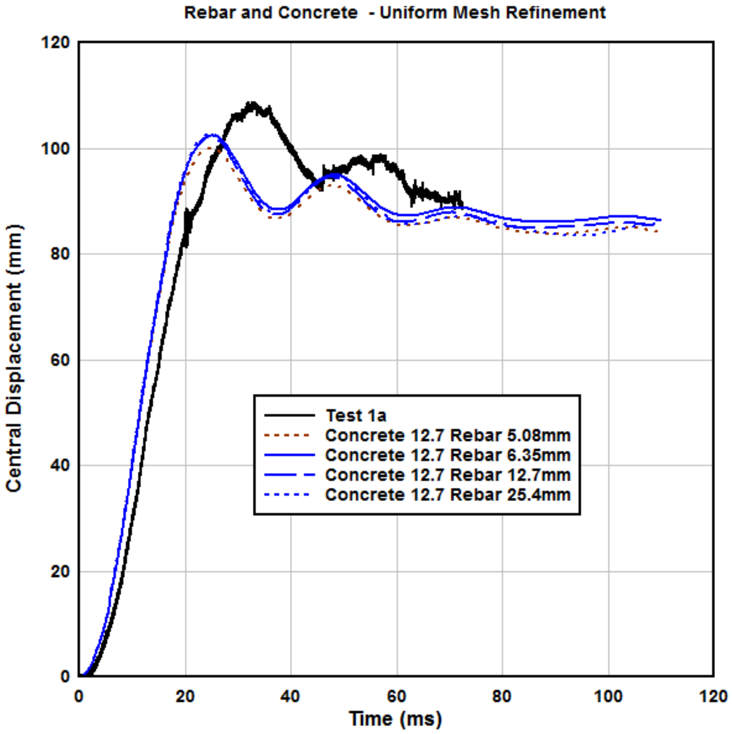

Figure 17 Comparison of central displacement for 12.7mm concrete mesh with four rebar mesh refinements.

Perhaps there are limits to CLIS effectiveness as the ratio of the concrete to rebar element characteristic length increases. In the above example the largest ratio was 2.5 (=12.7/5.08). The concrete to rebar ratios for all nine of the models shown previously in Figure 15 are listed in Table 1.

Table 1 Ratio of concrete (green) to rebar (blue) characteristic element sizes for nine mesh refinements.

	6.35	12.70	25.40
10.16	1.60	0.80	0.40
12.70	2.00	1.00	0.50
16.93	2.67	1.33	0.67

The largest concrete to rebar ratio of 2.67, occurs for the 16.93mm concrete and 6.35mm rebar characteristic lengths. This also corresponds to the smallest computed concrete slab central deflection. Figure 18 reproduces that result along with the other central displacements for the 16.93mm concrete and the three rebar mesh refinements considered.

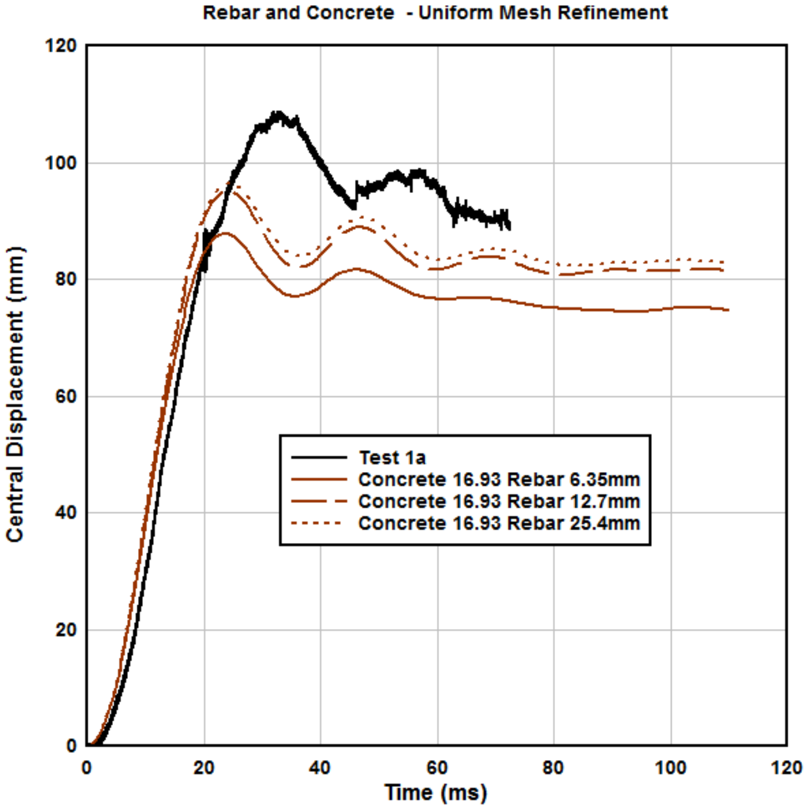

Figure 18 Comparison of central displacement for 16.93mm concrete mesh with three rebar mesh refinements.

Collocation Mesh Refinement

Another mesh refinement strategy is to mesh both parts identically, i.e. as if they were to share nodes, but use CLIS coupling. This will be referred to as *collocation meshing* to differentiate from shared node models discussed previously.

Since the rebar are 12.7mm (0.5inch) from the slab surface, the characteristic mesh lengths are again: 10.16, 12.7 and 16.93mm, but with the one row always of characteristic length 12.7mm to allow alignment of the concrete and rebar nodes.

Figure 19 shows the concrete slab central displacement for three collocated mesh refinements of the concrete and reinforcement. First note that 10.16 and 12.7mm collocation refinements provide a close match to the maximum slab displacement – although the computed maximum displacement occurs about 6ms sooner than in the test. These maximum displacements are larger than those computed using the uniform mesh refinements shown previously; Figure 20 compares the 12.7mm concrete and rebar mesh uniform and collocated refinement results.

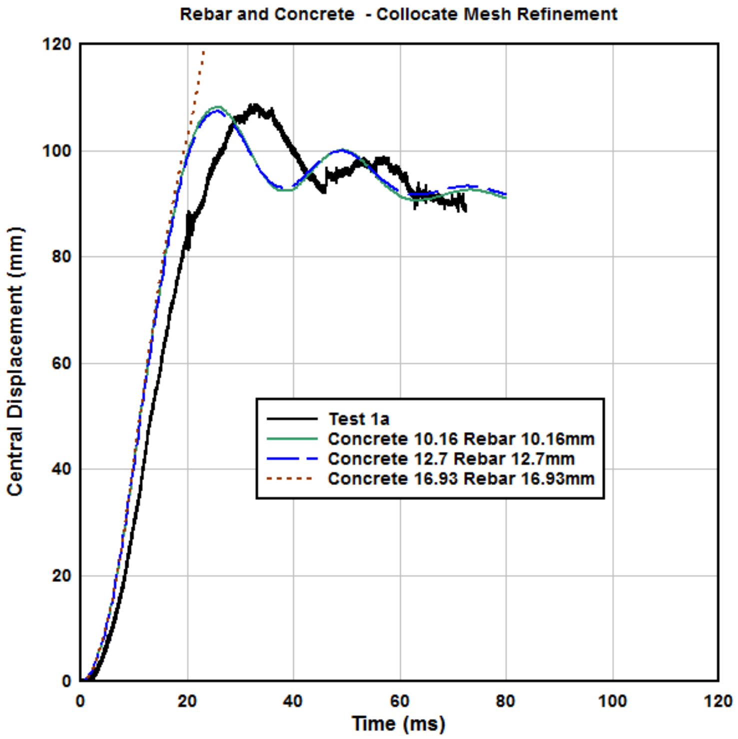

Figure 19 Comparison of central displacement for three collocated concrete and rebar mesh refinements.

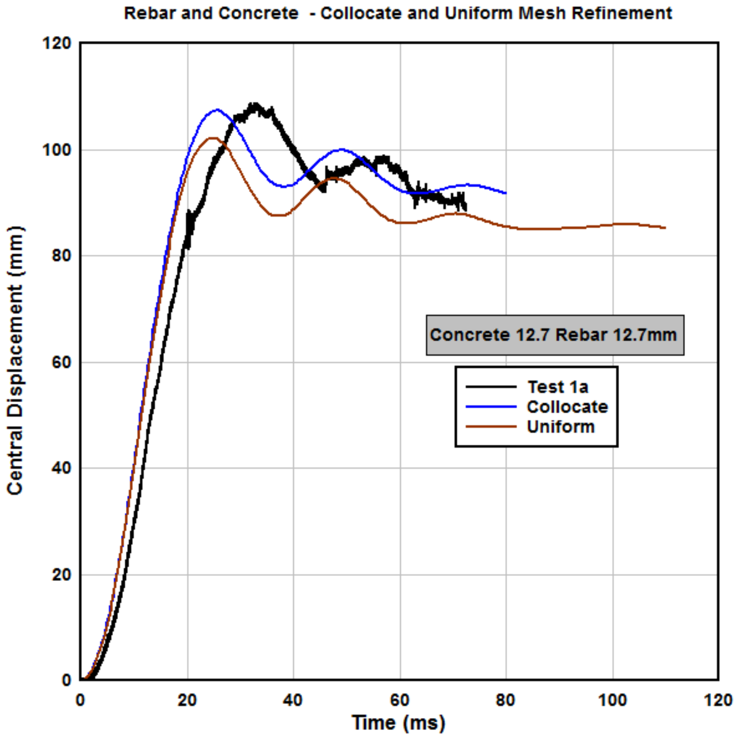

Figure 20 Comparison of central displacement for collocated and uniform concrete and rebar mesh refinements.

The obvious question about the collocated displacement results, shown previously in Figure 19, is what happens with the 16.93mm meshing model. It turns out, in this model *only*, the rebar fails near the center of the slab, see Figure 21, and due to concrete cracking (MAT085) the slab loses all bending resistance.

This potential rebar failure is a ‘hidden’ design flaw in the Blind Blast experiments – if the simulation results are to be believed. Figure 22 shows the prescribed true axial stress versus true effective plastic strain as prescribed for the Blind Blast Simulation (MAT024) and two collocated mesh refinements. The maximum axial stress is prescribed as 882MPA with a maximum effective plastic strain of about 7%. The collocated 12.7mm mesh reaches a maximum axial stress of 870MPa while the 16.93mm mesh reaches a maximum axial stress of 879MPa – this from the rather coarse time sampling of the beam element output – is sufficient to fail the rebar.

It is assumed that in all the blast loading simulations, both presented here and in the Blind Blast Simulation Contest, the reinforcement were near failure and thus the results were sensitive to how the reinforcement stress versus strain was modeled; not all participant bothered to convert the provided engineering stress versus strain to true stress versus true (natural) strain.

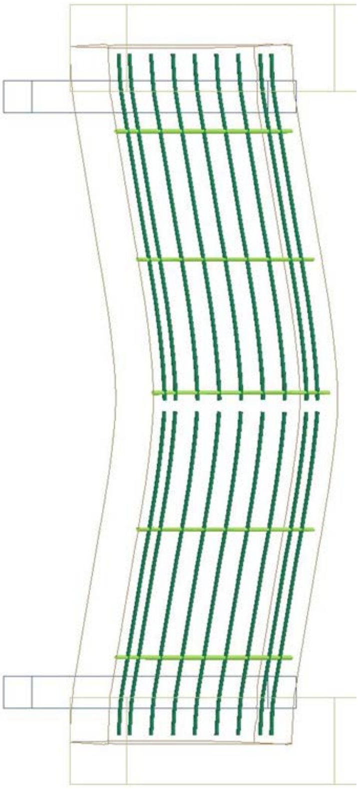

Figure 21 Failure of reinforcement after about 20ms.

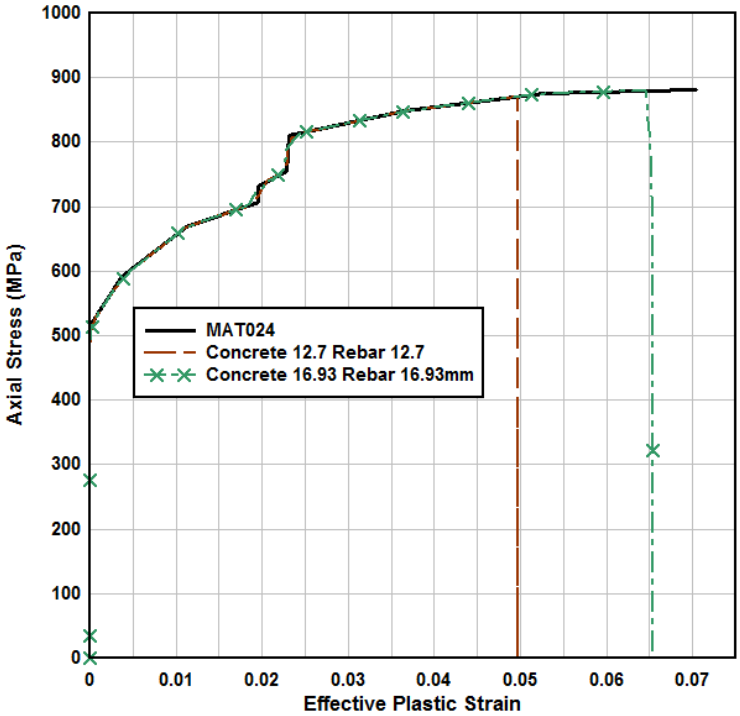

Figure 22 Rebar axial stress versus effective plastic strain as prescribe (MAT024) and for two collocated mesh refinements (12.7 and 16.93mm).

It is important to note that these are not to be construed as general mesh refinement results; the intent here is to motivate mesh refinement studies for both the concrete and reinforcement.

Summary

Five methods of including rebar in a reinforced concrete slab have been demonstrated for reinforced concrete slabs subjected to loading via axial extension, self-weight and blast loading.

While it is generally recommended to use the constraint methods, i.e. ACNC and CLIS, in the axial extension case these methods did not provide the appropriate axial resistance of the reinforcement. All five reinforcement inclusion methods provided acceptable results for the self-weight load case – this was small displacement and the results for the reinforced slab did not differ much from the result for an unreinforced (no rebar) case. For the blast loading case, the smeared reinforcement provided only a small amount of bending resistance relative to the unreinforced simulation result. The relative weakness of using truss elements versus beam elements for the reinforcement was also illustrated for the blast loaded slab simulations.

Finally, motivation was provided for mesh refinement of both the concrete and reinforcement meshes. Only the constraint CLIS method was used in the mesh refinement demonstration, but mesh refinement needs to be performed for the other methods of including rebar in reinforced concrete.

Acknowledgement

The author is most grateful to Richard Stuart of ARUP UK for the careful read and important suggestions concerning this manuscript.

Appendix – Input Parameters for the Winfrith Concrete Model

The Winfrith concrete model input is for the MAT085 no strain rate version (RATE=1.0) and the bi-linear material properties for the smeared reinforcement are included on the second input line.

```
*MAT_WINFRITH_CONCRETE
$#     MID        RO        TM        PR       UCS       UTS        FE     ASIZE
      2185   2.30e-3   29053.0       0.2     34.48      4.00     0.041     4.763
$#       E        YS        EH    UELONG      RATE      CONM      CONL      CONT
   205.0e3     500.0   4.783E3      0.14       1.0      -3.0     0.000     0.000
$#    EPS1      EPS2      EPS3      EPS4      EPS5      EPS6      EPS7      EPS8
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
$#      P1        P2        P3        P4        P5        P6        P7        P8
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
$
```

Appendix – Input Parameters for the Reinforcement Material Models

Figure 23 illustrates the stress versus effective plastic strain models used for the Grade 60 reinforcement when beam or truss elements were used. The smeared reinforcement uses the same bi-linear representation as the truss elements.

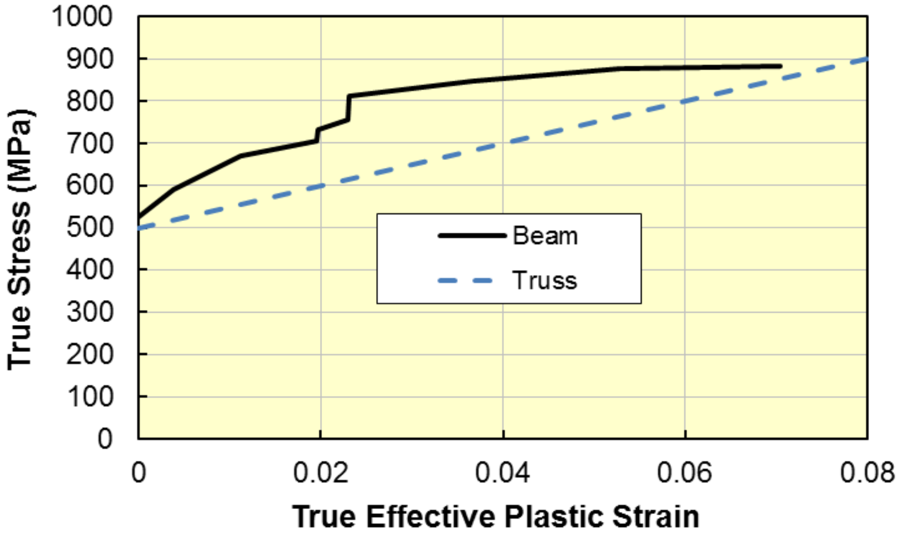

Figure 23 Beam and truss element stress-effective plastic strain models for Grade 60 reinforcement.

Beam Elements (MAT024)

```
$              60 ksi Tension Test (414 MPa)
*MAT_PIECEWISE_LINEAR_PLASTICITY
$ Material Type 24 (units: Newtons-millimeter-millisecond-MPa)
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$      MID         RO           E          PR        SIGY        ETAN        FAIL        TDEL
        24    7.85E-3     205.0e3        0.29                                0.14
$        C          P        LCSS        LCSR          VP         LCF
                               24                     1.0
$     EPS1       EPS2        EPS3        EPS4        EPS5        EPS6        EPS7        EPS8

$      ES1        ES2         ES3         ES4         ES5         ES6         ES7         ES8

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$         |         |         |         |         |         |         |         |
$========1=========2=========3=========4=========5=========6=========7=========8
$
*Define_Curve
$ LCID SIDR SFA SFO OFFA OFFO DATTYP
    24,   0,  1.0, 1.0
$ T-EPS     T-Stress
0.0000,  514.88
0.0038,  591.11
```

```
0.0112,  669.00
0.0195,  706.40
0.0196,  732.62
0.0229,  755.02
0.0231,  810.19
0.0366,  847.93
0.0527,  875.52
0.0704,  882.11$
```

Truss Elements (MAT003)

```
$
*MAT_PLASTIC_KINEMATIC
$  MID    RO        E      PR      SIGY  ETAN       BETA
    24, 7.85E-3, 205.0e3, 0.29, 500.0,  5000.0
$  SRC    SRP  FS   VP
   0.0, 0.0, 0.14
$
$
```