Modelling the failure of steel reinforced concrete frame structures

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Abstract

Numerical modelling of the failure processes of steel reinforced concrete frame structures requires numerically efficient finite element models. The most suitable approach to model the interplay of possible failure modes in reinforced concrete structures are frame elements in which the constitutive model is formulated in the form of internal forces and generalised strains. In this work, a constitutive model for frame elements for the failure process of reinforced concrete, which links the six internal frame forces to the corresponding generalised strain components, is proposed. The constitutive model developed in this study is based on a combination of elasto-plasticity and damage mechanics. The performance of the proposed constitutive model is compared with experiments reported in the literature. The model is implemented in a linear Timoshenko frame element. Mesh independent description of failure processes is demonstrated by means of a simply supported beam analysis subjected to bending and shear.

Key words: *Reinforced concrete; Frame elements; Plasticity; Damage*

1 Introduction

The finite element modelling approaches for reinforced concrete are divided in this study into continuum element, fibre beam and frame models. In continuum element models, concrete is modelled using solid finite elements, whereby the reinforcement is often modelled using beam or truss elements. These approaches are limited to simulating the response of individual members but are not suitable for entire structures because of the computational demands of discretising the entire concrete volume using solid finite elements. In fibre beam approaches, concrete members are modelled as structural elements (beam or truss elements) and the constitutive response of reinforcement and concrete is modelled by fibres within the cross-section of these structural elements. Fibre beam models are numerically efficient but are not capable of modelling localised shear failure [1]. Finally, frame element models are suitable for modelling the interplay of all failure modes in reinforced concrete frames [8, 9]. These models are numerically efficient and well suited for localised failure. The challenge is to propose constitutive models for these approaches which predict strength as well as ductility correctly and mesh-independently. The aim of this study is to propose a plasticity-damage constitutive model for frame elements for the failure process of steel reinforced concrete.

2 Method

The proposed constitutive model for frame element for modelling the failure process of reinforced concrete links the six internal frame forces to the corresponding generalised strain components. The constitutive model is based on a combination of elasto-plasticity and damage mechanics. The plasticity part, which uses a generalised ellipsoidal strength envelope, is used to model the strength limit. The damage part of the model predicts the limit on ductility. The graphical representation of the generalised ellipsoidal strength envelope depends on the internal forces to be depicted. For instance, Figure 1 shows the four possible cases of the strength envelope for a shear-moment interaction. The constitutive model was used in combination with a linear Timoshenko frame element with the option to shift integration points as proposed previously in [9] for modelling the failure of frame structures made of steel (Figure 2). This frame element is very efficient, because it has only one integration point and all the information about the nonlinearity and cross-section is included in the form of force-generalised strain relationship. The main force-strain law, which is used in this model, has the form

$$\mathbf{s} = (1-\omega)\,\mathbf{D}\,(\mathbf{e}-\mathbf{e}_{\mathrm{p}}) \tag{1}$$

where $\mathbf{s}$ are the internal forces, $\mathbf{e}$ are the generalised strains, $\mathbf{e}_{\mathrm{p}}$ is the plastc strain, ω is the damage variable varying from 0 (undamaged) to 1 (fully damaged) and $\mathbf{D}$ is the elastic stiffness matrix which is based on the elastic modulus E and the shear modulus G of the material.

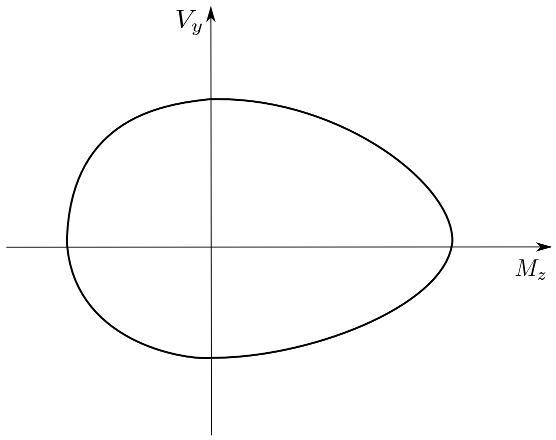

Figure 1: The ellipsoidal strength envelope for four quarter-ellipsoidal shear-moment interaction curves with different limits for shear and moment.

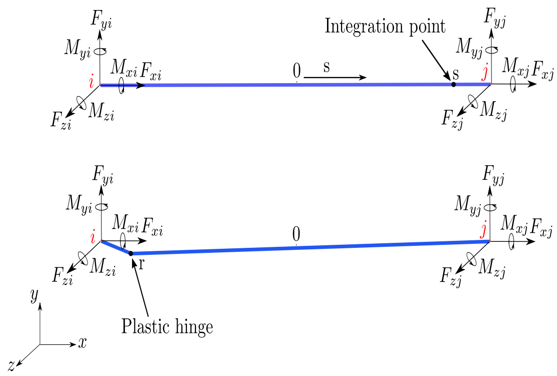

Figure 2: Linear Timoshenko beam element with the option to shift integration points [9].

The plasticity model consists of the yield function, flow rule and loading-unloading conditions. The yield function is

$$\begin{aligned} f = & \left(\frac{\langle N_x\rangle_+}{N_{x0}^+}\right)^2 + \left(\frac{\langle V_y\rangle_+}{V_{y0}^+}\right)^2 + \left(\frac{\langle V_z\rangle_+}{V_{Z0}^+}\right)^2 + \left(\frac{\langle M_x\rangle_+}{M_{x0}^+}\right)^2 + \left(\frac{\langle My\rangle_+}{M_{y0}^+}\right)^2 + \left(\frac{\langle M_z\rangle_+}{M_{z0}^+}\right)^2 \\ & + \left(\frac{\langle N_x\rangle_-}{N_{x0}^-}\right)^2 + \left(\frac{\langle V_y\rangle_-}{V_{y0}^-}\right)^2 + \left(\frac{\langle V_z\rangle_-}{V_{Z0}^-}\right)^2 + \left(\frac{\langle M_x\rangle_-}{M_{x0}^-}\right)^2 + \left(\frac{\langle M_y\rangle_-}{M_{y0}^-}\right)^2 + \left(\frac{\langle M_z\rangle_-}{M_{z0}^-}\right)^2 - 1 \end{aligned} \tag{2}$$

where N_x, V_y, V_z, M_x, M_y and M_z are the axial force, shear in y-direction, shear in z-direction, torsional moment about the x-axis, the bending moment about the y-axis and the bending moment about the z-axis respectively. The subscript $+_0$ and $-_0$ denote a totally plastic value when each component of resultant forces acts independently on a cross-section. In the yield function, $\langle\rangle_+$ and $\langle\rangle_-$ are positive and negative Macaulay brackets, respectively.
The flow rule for the plastic strain rate is

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{3}$$

where $\dot{\lambda}$ is the rate of plastic multiplier and $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\sigma}}$ is the first derivative of the yield function.
Loading-unloading conditions have the form

$$f(\boldsymbol{\sigma}) \leq 0, \qquad \dot{\lambda} \geq 0, \qquad \dot{\lambda} f(\boldsymbol{\sigma}) = 0 \tag{4}$$

In the damage part, the damage variable is a function of the damage history variable κ_d, which is determined as

$$\kappa_d = \underset{\tau<t}{max}\tilde{e} \tag{5}$$

where t is the maximum step size, τ is a variable from zero to maximum step size. Furthermore, $\tilde{e}$ is the equivalent strain which is

$$\tilde{e} = \|\mathbf{e}_p\| \tag{6}$$

Damage is introduced in the form of a threshold

$$\omega = \begin{cases} 1, & \text{if } \kappa_d h_e \geq w_u \\ 0, & \text{if } \kappa_d h_e < w_u \end{cases} \tag{7}$$

where h_e is the element length.

3 Analyses and results

In the first part of the analyses, the plasticity-damage constitutive model is used to analyse a simply supported beam subjected to monotonically increasing vertical displacement at its mid-span to investigate the capability of the plasticity-damage constitutive model to provide mesh-independent results. Input parameters for this analyses are the second moment of area around the y-axis $I_y = 7e7$ mm^4, the second moment of area around z-axis $I_z = 1.9e8$ mm^4, shear coefficient $= 0.833$, Young's modulus $E = 2000$ MPa, Poisson's ratio $v = 0.2$, the length of the beam $L = 3$ m, fully-plastic axial force $N^+_{x0} = 200$ kN, fully-plastic shear in $y-$direction $V^+_{y0} = 91.7$ kN, and fully-plastic moment about z-axis $M^+_{z0} = 16.8$ kNm. The ability of the plasticity-damage constitutive model to provide mesh-independent results is investigated by using 2, 4, 8, 16, and 32 element to discretise the beam. The results in the form of load-displacement curves are shown in Figure 3. It can be seen that upon mesh refinement the response converges.

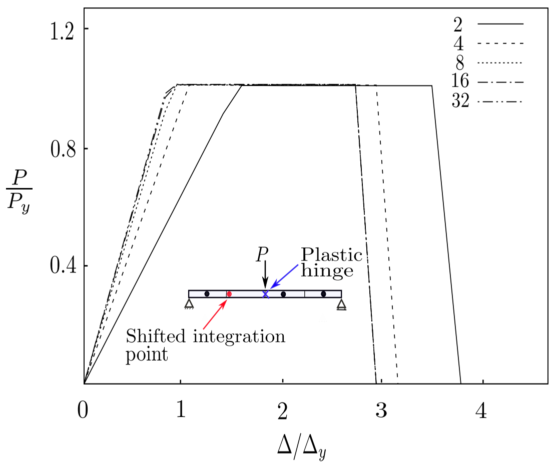

Figure 3: Effect of mesh-refinement on the load-displacement curve.

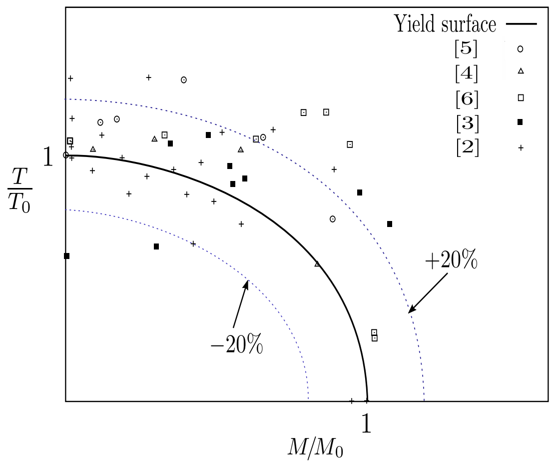

Figure 4: Comparison of torsion-moment interaction of the model and experiments reported in the literature.

In the second part of the analysis, the ellipsoidal strength envelope, which is used to model the strength limit for the plasticity part, is investigated by comparing it with the experimental results reported in the literature. Figure 4 shows the comparison of the interaction equation between torsion-moment with the experimental results reported in the literature [4, 3, 2, 6, 5]. The interaction ellipsoidal measured the experimental results almost within ±20% and indicates the torsion-moment's ellipsoidal is in good agreement with the experimental results. Here, T and M are the torsion and bending moment applied on the structure member, respectively. Furthermore, T_0 and M_0 are fully plastic value of torsion and moment, respectively, when each component of the resultant forces acts independently on a cross-section.

4 Conclusions

A new plasticity-damage constitutive model for frame elements for the failure process of reinforced concrete has been proposed. The constitutive model is numerically efficient, capable of modelling localised failure including shear failure and provides mesh-independent results for load-displacement relationship for fine meshes. The ellipsoidal strength envelope, which is used in the plasticity part of the model is in a good agreement with the experimental results.

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