ON A GEOMETRIC NONLINEAR TIMOSHENKO FRAME ELEMENT WITH MATERIAL NONLINEARITY

Gumaa Abdelrhim*, **Peter Grassl**

University of Glasgow, Glasgow, UK.
g.abdelrhim.1@research.gla.ac.uk, peter.grassl@glasgow.ac.uk

Abstract. Collapse of reinforced concrete frame structures is a highly nonlinear process governed by material nonlinearities in the form of cracking and crushing of concrete, as well as yielding of steel reinforcement. These processes are accompanied by large rotations and deflections, which strongly affect the load transfer. In this study, a Timoshenko frame element is developed for modelling the collapse of reinforced concrete frame structures, considering both geometric and material nonlinearities. The frame element is formulated using the rigid-body-spring concept. For the material nonlinearity, a plastic-damage constitutive model is used, which was developed by the authors in previous work. For geometric nonlinearities, an incremental approach is used. In this approach, the internal forces are expressed in the local coordinate system, which follows the displaced geometry of the structure. The model response is compared to benchmarks for buckling and bending reported in the literature.

Key words: *Reinforced concrete; Frame elements; Geometric nonlinearity; Large rotations*

1 Introduction

Progressive collapse of a reinforced concrete frame structure is a failure mode in which initial local failure of a limited number of structural elements caused by, e.g., accidental loading results in partial or entire collapse of the structure. Collapse of reinforced concrete frame structures is a highly nonlinear process governed by material nonlinearity in the form of cracking and crushing of concrete, as well as yielding of steel reinforcement. These processes are accompanied by large rotations and deflections, which strongly affect the load transfer in the structure. For predicting the progressive collapse of reinforced concrete structures, a finite element approach is required which is capable of reproducing both material and geometric nonlinearities in an efficient manner so that the analysis of entire structural systems is feasible. Detailed approaches in which the steel reinforced concrete is modelled using a continuum approach have the advantage of being capable of predicting the effect of reinforcement arrangements on failure processes, but are normally limited to the analysis of individual structural components. Numerically more efficient approaches are based on frame elements in which the composite material response of reinforced concrete is modelled by relating the internal forces to generalised strain components. In this study, a Timoshenko frame element model is proposed for modelling the collapse of reinforced concrete frame structures, considering both geometric and material nonlinearities. The frame element is formulated using the rigid-body-spring concept [1]. The geometric nonlinearities are introduced in this formulation following the work in [3, 4]. Furthermore, the material nonlinearities are considered by a damage-plasticity approach presented previously by the authors in [2]. In the next sections, the two-dimensional model formulation is described followed by a comparison of the model response with benchmarks for geometric and material nonlinearities.

2 Method

In this section, the proposed frame element model for modelling the collapse of reinforced concrete frame structures considering both geometric and material nonlinearities is described. The frame element is formulated using the rigid-body-spring concept [1]. For geometric nonlinearities, an incremental approach based on the work in [3, 4] is used. In this approach, the internal forces of the frame element are expressed in the local coordinate system, which follows the displaced geometry of the structure (Figure 1a). The nodal deformations at nodes i and j result in displacement jumps at the contact point of two rigid bodies $i^{'}$-$j^{'}$ (Figure 1b). The displacement jump is transformed into generalised strains which are used to calculate forces at the contact point by means of a constitutive model. For each rigid body, the forces at the contact point are in equilibrium with forces at the nodes as shown in Figure 1c.

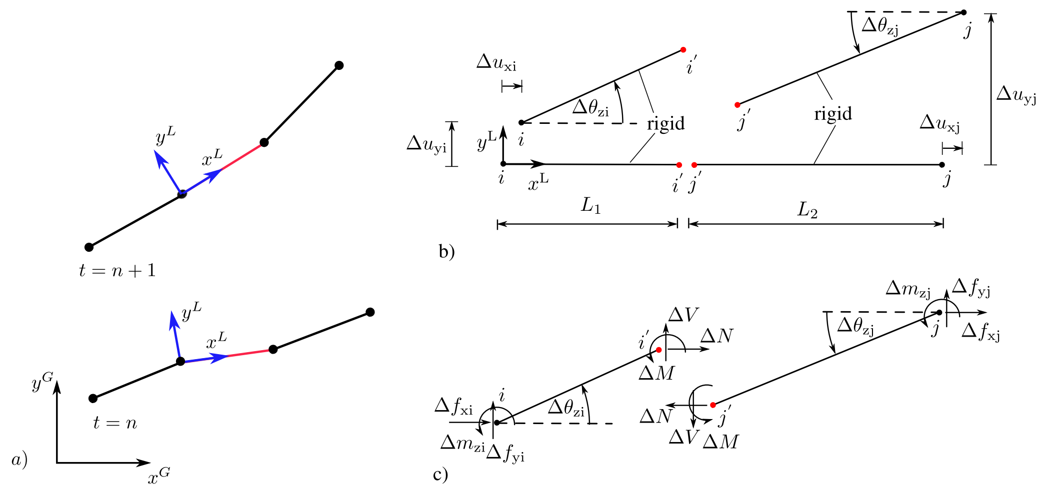


Figure 1: Frame element with geometric nonlinearities: a) update of local coordinate system, b) compatibility condition, c) equilibrium in deformed configuration.

Mathematically, the generalised strain is related to the nodel displacements as

$$\Delta \mathbf{e} = \mathbf{B}_{\rm d} \Delta \mathbf{u} \tag{1}$$

where $\Delta \mathbf{e} = \{\Delta\varepsilon, \Delta\tau, \Delta\kappa\}^T$ are the generalised strains and $\Delta \mathbf{u} = \left\{\Delta u_{\rm xi}, \Delta u_{\rm yi}, \Delta\theta_{\rm zi}, \Delta u_{\rm xj}, \Delta u_{\rm yj}, \Delta\theta_{\rm zj}\right\}^T$ are the nodal displacements. Furthermore, $\mathbf{B}_{\rm d}$ is a compatibility matrix, which has the form

$$\mathbf{B}_{\rm d} = \frac{1}{L}\begin{pmatrix} -1 & 0 & \frac{L_1(1-\cos\Delta\theta_{\rm zi})}{\Delta\theta_{\rm zi}} & 1 & 0 & \frac{L_2(1-\cos\Delta\theta_{\rm zj})}{\Delta\theta_{\rm zj}} \\ 0 & -1 & -\frac{L_1 \sin\Delta\theta_{\rm zi}}{\Delta\theta_{\rm zi}} & 0 & 1 & -\frac{L_2 \sin\theta_{\rm zj}}{\Delta\theta_{\rm zj}} \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where L_1 and L_2 are the two lengths of the rigid bodies in Figure 1b, and $L = L_1 + L_2$. It should be noted that only for $L_1 = L_2 = L/2$ the rigid body spring model is identical to a Timoshenko beam with a single integration point [1].

The internal forces $\Delta \mathbf{s} = \{\Delta N, \Delta V, \Delta M\}^T$ are then determined by

$$\Delta \mathbf{s} = \mathbf{F}_{\rm dp}\left(\Delta \mathbf{e}\right) \tag{3}$$

where $\mathbf{F}_{\text{dp}}$ is the constitutive model based on a damage plasticity approach presented in [2]. The forces at the nodes are then determined from equilibrium conditions for each of the rigid bodies, which are expressed in matrix form as

$$\mathbf{f} = \mathbf{B}_{\text{f}}\mathbf{s} \tag{4}$$

where $\Delta\mathbf{f} = \left\{\Delta f_{\text{xi}}, \Delta f_{\text{yi}}, \Delta m_{\text{zi}}, \Delta f_{\text{xj}}, \Delta f_{\text{yj}}, \Delta m_{\text{zj}}\right\}^T$ are the nodal forces and $\mathbf{B}_{\text{f}}$ is the equilibrium matrix of the form

$$\mathbf{B}_{\text{f}} = \begin{pmatrix} -1 & 0 & L_1 \sin\Delta\theta_{\text{zi}} & 1 & 0 & L_2 \sin\Delta\theta_{\text{zj}} \\ 0 & -1 & -L_1 \cos\Delta\theta_{\text{zi}} & 0 & 1 & -L_2 \cos\Delta\theta_{\text{zj}} \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}^T \tag{5}$$

It should be noted that only rotations enter the compatibility and equilibrium matrices, because the nodal forces are determined for the deformed geometry, but in the direction of the local coordinate system chosen at the start of the step.

3 Numerical results

The model has been implemented in the finite element programme OOFEM. In this section, two benchmarks were modelled to illustrate the proposed frame element's performance. The first benchmark is the elastic William's toggle [6], for which the geometry and loading setup, and load versus displacement is shown in Figure 2a and b, respectively. The second benchmark is a braced portal frame made of elasto-

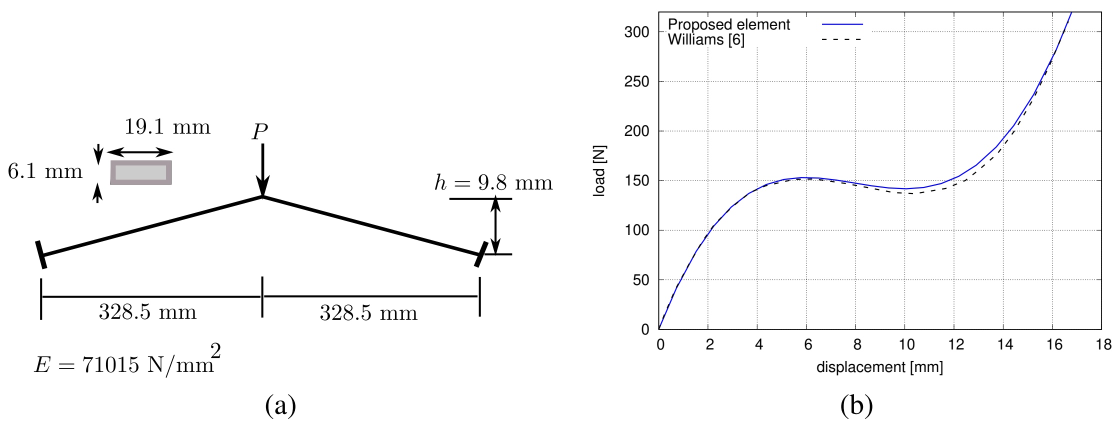


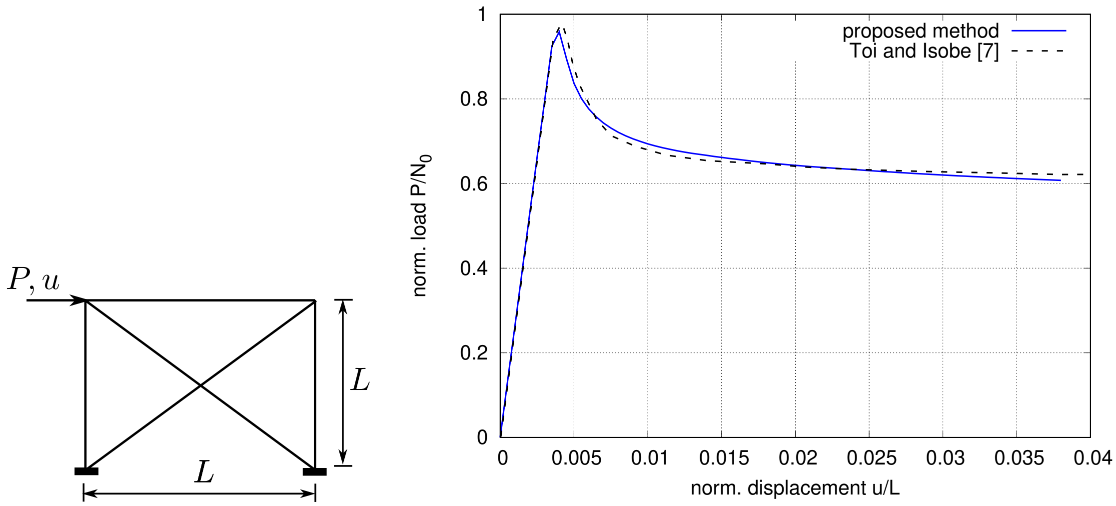
Figure 2: (a) Williams' toggle geometry and setup [6], (b) comparison of results of proposed method and analytical solution in [6].

plastic material for which the geometry and loading setup, and the load-displacement curve is shown in 3a and b. For details on the material properties and cross-sectional geometry, see [7].

The results obtained with the proposed method are in good agreement with those reported in the literature.

4 Conclusions

In this work we have developed an approach for the geometric nonlinear analysis of 2D frame structures based on rigid body kinematics. The model has been applied to two benchmarks for which the results agreed well with those in the literature. In future research, the model will be extended to 3D.



(a) (b)

Figure 3: (a) Geometry and loading setup of the braced portal frame in [7], (b) comparison of the proposed element with the numerical results in [7].

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