On finite element modelling of concrete columns with non-circular cross-sections confined with carbon fibre reinforced polymer

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Abstract

The aim of this study is to evaluate the performance of the damage-plasticity model CDPM2 for modelling CFRP confined columns with non-circular cross-sections subjected to axial compression. Our adopted methodology consists of calibrating CDPM2 using experimental results for columns with circular cross-sections for unconfined and highly confined concrete using a single integration point model. Then, 3D simulations are performed for columns with non-circular cross-sections. For concrete, constant strain tetrahedral elements are used. The CFRP sheets are idealised as truss elements. All the FE modelling is carried out with the program OOFEM modified by the authors. The meshes were generated with T3D. The numerical results are in agreement with those reported in the experiments. Both the effects of circular and non-circular cross-sections on the load-displacement response of the confined columns is well reproduced by the model.

Key words: *CFRP; concrete; confinement; damage-plasticity*

1 Introduction

Carbon Fibre Reinforced Polymer (CFRP) wrapping is a popular technique for strengthening and repairing concrete columns, because the restrained expansion of the concrete results in passive confinement, which increases strength and ductility. In columns with circular cross-sections, it can be assumed that uniform confinement is obtained. Closed form mathematical models for predicting this composite response are well established for columns with circular cross-sections which are centrically loaded [1–5]. However, columns with non-circular cross-sections exhibit a more complicated mechanical response. For non-circular cross-sections, the jacket confining pressure varies around its perimeter, which affects the axial stress distribution in the cross-section [6]. This non-uniformity results in stress concentration and localised failure at the corners of the non-circular cross-sections [4]. Incorporating tensile cracking and confinement variations realistically in closed-form mathematical models is difficult. Therefore, accurate stress–strain models for non-uniformly confined concrete have not yet been fully developed [6]. Existing models are often not capable to predict the column response for varying geometry and material properties with the same model formulation [7]. Therefore, the effect of non-circular cross-sections on confinement requires detailed nonlinear Finite Element (FE) modelling, which can capture the complex stress variations within the cross-section and model cracking and crushing of the concrete [6]. Such a FE approach requires realistic constitutive models for plain concrete [6]. The aim of this study is to evaluate the performance of the concrete damage-plasticity model CDPM2 reported in [8], for modelling CFRP confined columns with non-circular cross-sections subjected to centric compression. Firstly, the capabilities of CDPM2 to predict the response of circular columns is studied by means of a single tetrahedron for concrete confined by three truss elements representing CFRP. Then, full 3D finite element analyses of the columns with non-circular cross-sections are performed.

2 Constitutive model CDPM2

In this section the key equations of the concrete damage plasticity constitutive model CDPM2 are reviewed. A detailed description of the model formulation can be found in Grassl et al. [8]. The stress-strain law is given as

$$\boldsymbol{\sigma} = (1-\omega_t)\,\bar{\boldsymbol{\sigma}}_{\mathrm{t}} + (1-\omega_c)\,\bar{\boldsymbol{\sigma}}_{\mathrm{c}} \tag{1}$$

where $\bar{\boldsymbol{\sigma}}_{\mathrm{t}}$ and $\bar{\boldsymbol{\sigma}}_{\mathrm{c}}$ are the positive and negative parts of the effective stress tensor $\bar{\boldsymbol{\sigma}}$, respectively, and ω_{t} and ω_{c} are two scalar damage variables, ranging from 0 (undamaged) to 1 (fully damaged). The plasticity part is based on the effective stress tensor $\bar{\boldsymbol{\sigma}}$, which is independent of damage.

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_{\mathrm{e}} : \ (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{p}}) \tag{2}$$

Here, $\mathbf{D}_{\mathrm{e}}$ is the material stiffness in the elastic state, $\boldsymbol{\varepsilon}$ is the strain tensor and $\boldsymbol{\varepsilon}_{\mathrm{p}}$ is the plastic strain tensor. The evolution of the damage variables ω_{t} and ω_{c} are controlled by damage history variables based on elastic and plastic strain tensors. They have the form

$$\kappa_{dt} = \max \tilde{\varepsilon}_t \quad \dot{\kappa}_{dt1} = \frac{\|\dot{\varepsilon}_p\|}{\mathrm{x}_s} \quad \dot{\kappa}_{dt2} = \frac{\dot{\kappa}_{\mathrm{dt}}}{\mathrm{x}_s} \tag{3}$$

for tension and

$$\kappa_{dc} = \max \tilde{\varepsilon}_c \quad \dot{\kappa}_{dc1} = \frac{\alpha_c \beta_c \|\dot{\varepsilon}_p\|}{\mathrm{x}_s} \quad \dot{\kappa}_{dc2} = \frac{\dot{\kappa}_{dc}}{\mathrm{x}_s} \tag{4}$$

for compression. Here, $\tilde{\varepsilon}_{\mathrm{t}}$ and $\tilde{\varepsilon}_{\mathrm{c}}$ are equivalent strains for tension and compression, respectively, which depend on the effective stress $\bar{\boldsymbol{\sigma}}$. Parameter β_c provides a smooth transition from pure damage to damage-plasticity softening [8]. Furthermore, x_s is a ductility measure, which takes into account the influence of multiaxial stress states on the damage evolution. It has here the form

$$x_{\mathrm{s}} = 1 + (A_{\mathrm{s}} - 1)\, R_{\mathrm{s}}^{B_{\mathrm{s}}} \tag{5}$$

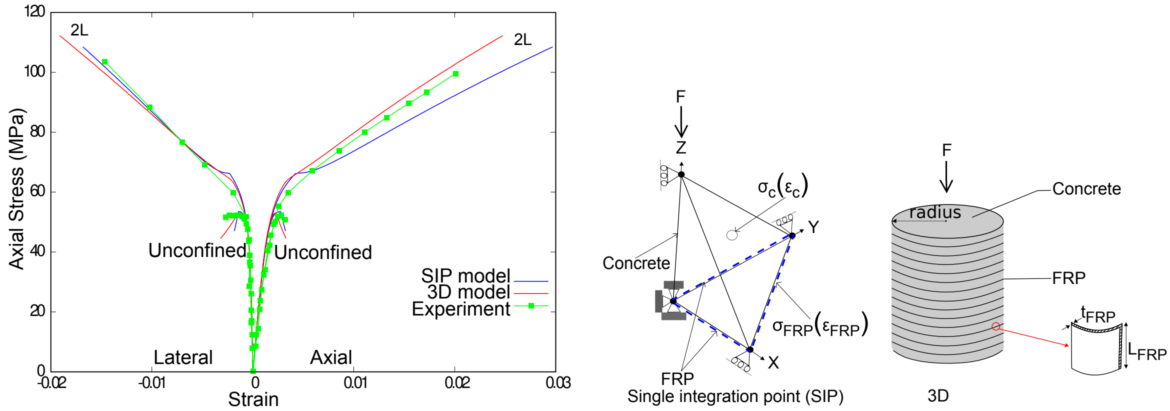
where R_{s} is

$$R_{\mathrm{s}} = \begin{cases} -\dfrac{\sqrt{6}\bar{\sigma}_{\mathrm{V}}}{\bar{\rho}} & \text{if } \bar{\sigma}_{\mathrm{V}} \leq 0 \\ 0 & \text{if } \bar{\sigma}_{\mathrm{V}} > 0 \end{cases} \tag{6}$$

and A_{s} and B_{s} are model parameters. For uniaxial compression $\bar{\sigma}_{\mathrm{V}}/\bar{\rho} = -1/\sqrt{6}$, so that $R_{\mathrm{s}} = 1$ and $x_{\mathrm{s}} = A_{\mathrm{s}}$, which simplifies the calibration of the softening response in this case. This ductility measure differs from Grassl et al. [8] by the additional parameter B_{s}. This addition is important to be able to model passive confinement. The response for unconfined compression does not change.

3 Numerical analysis and results

This section describes the analysis of the experimental study reported in Wang and Wu [9]. A nonlinear finite element analysis of CFRP confined concrete was set up by using an implicit incremental-iterative approach. The load was applied using displacement control applied. Two types of models were used. For the columns with circular cross-sections, a Single Integration Point (SIP) model was used. Concrete was model using single constant strain tetrahedron which was confined in the lateral direction with truss elements representing the CFRP sheets, shown as blue dotted lines in Figure 1b. Here, passive confinement is activated by the deformation of the truss elements. For the 3D model, which is applicable to predict the response of both circular and non-circular cross-sections, concrete was discretised by a prism made of four-node tetrahedral solid elements. The FRP sheets were modelled as truss elements around the circumference of the prism. Each truss element represents a width of the FRP layer wrapped around the concrete prims, as shown in Figure 1b. The nodes of the tetrahedra at the surface of the prism are the same as those of the truss elements. This represents perfect bond between concrete and CFRP. For the boundary conditions, the top nodes were restrained in both x and y directions while displacement was applied in z-direction. The bottom nodes were constrained similar to the experimental test in [9]. The finite element mesh was created with T3D [10]. The open source finite element program OOFEM [11] was used for the nonlinear analyses. The damage-plasticity constitutive model CDPM2 was applied as the material model for concrete while an elastic material model was used for CFRP. The calibration was carried out using the SIP model for the circular cross-section. Material properties were taken from the experiments reported in [9]. The input parameters from the calibration



(a) (b)

Figure 1: Calibration of the model for circular cross-section: (a) Stress-strain response in single integration point (SIP) model and 3D model. (b) Model setup for SIP and 3D.

of the columns with circular cross-section are then used for full 3D FE modelling. The stress-strain response from the SIP and 3D models for the circular cross-section are compared with experimental results in Figure 1a for unconfined compression and two layers of CFRP wrapping. The differences between the SIP and full 3D model are explained by the lateral constraints at the ends of the columns in the 3D model which are not considered in the SIP model. The modelling approach is capable of capturing both the unconfined and confined behaviour of the concrete columns. For presenting the influence of the cross-section shape on the confined response, the modelling results in the form of stress-strain relations and contour plots of the axial stresses for the confined columns are shown in Figure 2 for different corner radii as used in the experiments. Here, blue and red stands for high and low compression, respectively. Figure 2a shows that the strength of the columns reduces with a decrease in the corner radius. This response is in agreement with experimental results. This dependence on the corner radius is well explained by studying the axial stress distribution in the cross-section. From the contour plots of axial stresses at an axial strain of 0.005 mm in Figure 2b, it can be seen that there is a uniform stress distribution in the circular confined column cross-section with highest stresses confined to the centre. This response changes as the corner radius decreases. For a small radius the effective confined area which represents the confinement efficiency reduces with a decrease in the corner radius. Larger areas of low confinement (shown in red) become more visible as the corner radius decreases.

4 Conclusions

Based on the present finite element analyses of the effect of the cross-section shape on CFRP confined columns, with the damage-plasticity model CDPM2, the following conclusions can be drawn. The damage-plasticity model CDPM2 is capable of predicting passive confinement in columns with circular and non-circular cross-sections. The modelling results show that confinement efficiency reduces with a decrease in corner radius, which is in agreement with the experimental results. The axial stress distributions reveals that this reduction is due to the formation of areas of low confinement for non-circular cross-sections.

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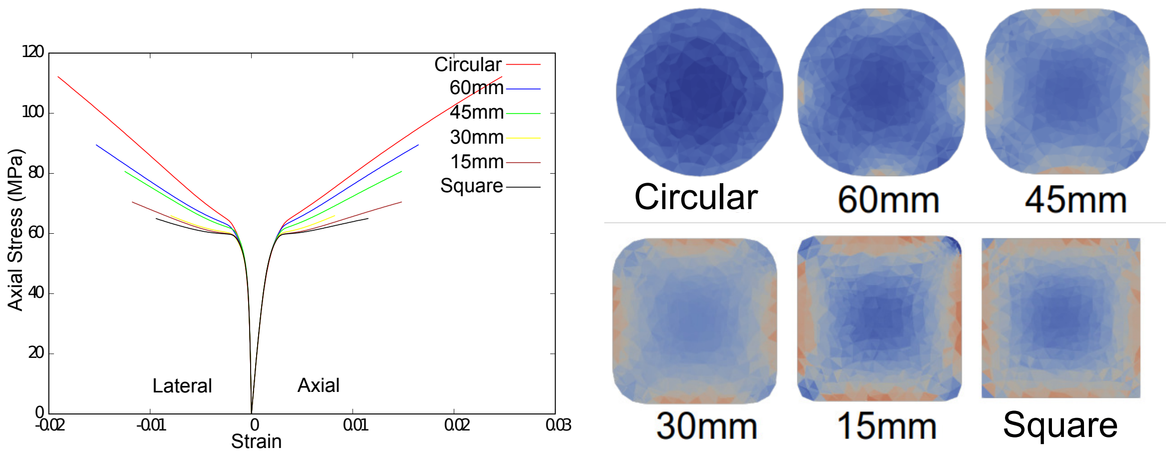

(a) (b)

Figure 2: Effect of cross-section shape: (a) Stress - strain response for all cross-sections (b) Contour plots of axial stresses at axial strain 0.005 for all cross-section shapes considered.

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