

1 On the dynamic response of reinforced concrete beams subjected to  
2 drop weight impact

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9 **Abstract**

10 To improve the impact resistance of reinforced concrete structures, a detailed understanding of  
11 the dynamic response is required. This study investigates this impact resistance using  
12 experiments in combination with 3D non-linear finite element (FE) simulations. The  
13 experiments made use of high-speed photography and digital image correlation (DIC), while a  
14 damage-plasticity constitutive model for concrete was used in the FE simulations. Drop weight  
15 impact tests of simply supported reinforced beams made of plain concrete and fibre reinforced  
16 concrete were made, and it was shown that the addition of fibres reduced crack spacing, crack  
17 widths and mid-point deflections. For the FE approach, tetrahedral elements were shown to be  
18 well suited for capturing inclined shear cracks and the structural response obtained in  
19 experiments and analyses agreed very well. The FE analyses showed that the reinforcement  
20 strains were more localised for concrete with fibres, and hence predicted an increased risk of  
21 reinforcement rupture.

22 *Keywords:* concrete, fibres, drop weight impact, digital image correlation (DIC), CDPM2

23 **1. Introduction**

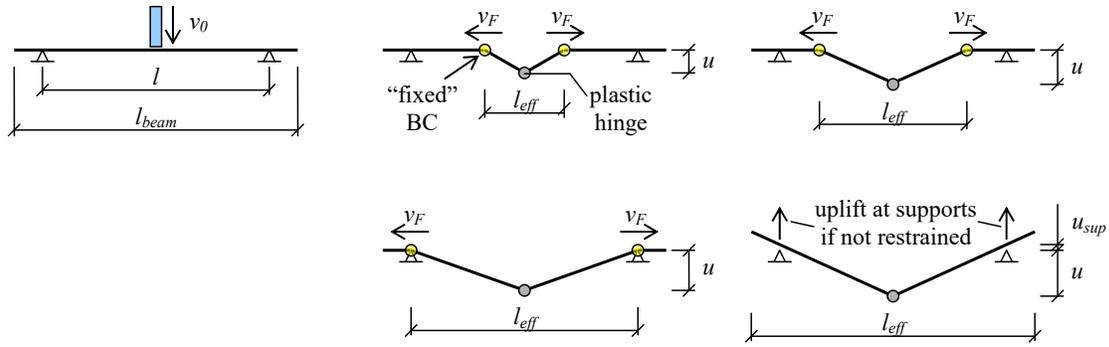
24 Understanding the response of reinforced concrete structures subjected to impulse loading (i.e.,  
25 blast and impact) is essential for designing critical infrastructure with confidence. Concrete  
26 cracking in tension, concrete crushing in compression, yielding of reinforcement, and interplay  
27 between concrete and reinforcement interact in a very short period, often resulting in a complex  
28 structural response. This response is difficult to predict without detailed experimental and  
29 numerical analyses.

30 The structural response of an impulse loaded structure may be very different compared to when  
31 subjected to a quasi-static load with the same geometric configuration. Due to the intense  
32 energy released in a short time, different phenomena will occur that affect both the material and  
33 structural response of the loaded structure. High strain rates cause the material in the structure  
34 to become stiffer, stronger, and more brittle [1–3], while wave propagation effects at the same  
35 time may greatly influence the structural response. Several studies [4–7] have noted that the  
36 load capacity increases with increased load rate. The load capacity increase is partly due to  
37 increased material strength as well as inertia effects causing a change in the structural response.

38 When a structure is subjected to a load, it takes time for this information to transfer within the  
39 structure. This time lapse is the case in all structures and for all types of loading. However, the  
40 information travels very fast within the structure (the shear wave velocity in concrete is  
41 approximately 2300 m/s); consequently, it is only when rapid loading is applied that such so  
42 called wave propagation effects will be important. In impact loading the impact force may be  
43 very high but with a short duration – in the order of just a few milliseconds or less. This means  
44 that all, or a large part, of the load may have been transferred to the structure before information  
45 of the load application has reached all parts of the structure. For example, in an impact loaded  
46 beam, there will be a notable lag in the time between impact and when the supports experience  
47 the applied load. This lag also means that the initial deformed shape of the beam will be very

48 different compared to the same beam subjected to a static point load. Since it takes some time  
49 for the information of the applied load to reach all parts of the structure and it takes time for the  
50 structure to react, parts of the beam will initially be ‘unaware’ of both the support conditions  
51 and the applied load.

52 This initial response is denoted as the local response [8] and the deformed shape will vary as  
53 schematically illustrated in Fig. 1 [9]. Here, a plastic hinge is assumed to form in the section of  
54 impact and the affected part of the beam responds as if it was a fixed beam with an effective  
55 span length  $l_{eff}$ . Consequently, tensile stresses that may cause cracks will form at the top of the  
56 beam. The effective span length increases with time and, therefore the stiffness and load  
57 capacity of the beam will vary during the local response. The time it takes for the initial response  
58 to develop depends on the beam length and average force propagation velocity  $v_F$ . This velocity  
59 is considerably smaller than the shear wave velocity and decreases approximately  
60 proportionally with increased beam slenderness (approximately 250 to 1000 m/s for  
61 length/depth of the beam varying between 10 and 4) [10,11]. Eventually, the effective length  
62 reaches the supports. If the beam is not vertically restrained, it will briefly lift up due to the  
63 inertia forces gathered within the effective length [12]. Once the whole beam is affected (i.e.,  
64  $l_{eff} = l_{beam}$ ), the overall response [8] is initiated in which a deformed shape approximately  
65 corresponding to that of static loading is obtained. However, this takes some time and until then  
66 the beam’s boundary conditions can be regarded as time-dependent fixed supports, where the  
67 locations of these supports gradually move toward the free ends of the beam.



68

69 **Fig. 1.** Schematic illustration of how the deformed shape of an impact loaded beam  
 70 develops during the local response. Based on [9] and [12].

71 The capacity of a structure to withstand the effect of impulse loading mainly depends on its  
 72 ability to absorb the external energy applied by the load. Using a single degree of freedom  
 73 model and assuming plastic impact, the external energy acting on the beam can be  
 74 approximately determined as  $W_e = m_w/(m_w + m_b) \cdot E_{k,0}$ , where  $m_w$  is the mass of the drop weight,  
 75  $m_b$  is the effective mass of the beam,  $E_{k,0} = m_w \cdot v_0^2/2$  is the kinetic energy of the drop weight at  
 76 impact, and  $v_0$  is the impact velocity [13]. Hence, the external energy applied to the beam will  
 77 decrease with an increased beam mass. To withstand the effect of impact, the external energy  
 78 should be balanced by the internal energy –  $W_i = \int R(u) du$  – provided by the beam, where  $R(u)$   
 79 is the load capacity as a function of the deflection  $u$ . Consequently, in an impulse-loaded  
 80 structure, the internal energy  $W_i$  (not the maximum load capacity) is essential to withstand the  
 81 load. In such structures it is usually preferred that the energy consumption of the structure is  
 82 provided by large deformations rather than large internal forces [14–17]. This arrangement is  
 83 ensured by designing the structure with regard to bending and providing it with a large plastic  
 84 deformation capacity. To fulfil the latter criterion, the structure should be designed so it does  
 85 not experience premature brittle failure, for example, due to shear. Hence, avoiding shear failure  
 86 in an impulse-loaded structure is essential and has been the focus of several studies [18–20]. A  
 87 potential problem here is that the mechanics of shear failure in impulse loaded structures are

88 not as well understood as failure due to bending [18] and several studies have noted that a  
89 statically-loaded structure that fails due to bending may instead fail in shear when subjected to  
90 impulse loading [18–20].

91 In the equation for the external energy, the mass of the impacting body is important. However,  
92 the impacting body's geometric properties are also important and as such have been the subject  
93 of several studies [21–25]. These studies have concluded that an increased radius on the drop  
94 weight head results in increased contact stiffness, that this stiffness significantly influences the  
95 impact force (force increase with increased stiffness), and that a decreased head radius leads to  
96 more severe local concrete damage at the impact zone. However, the contact stiffness has only  
97 minor effect on the resulting impulse, impact energy, and resulting mid-span displacement of  
98 the loaded structure. Furthermore, a flat head is sensitive to the inclination angle of the drop  
99 weight [24], suggesting that it is preferable to use a head with a radius that is not too large.

100 If the beam's internal energy capacity is high enough (i.e.,  $W_i \geq W_e$ ), the structure can withstand  
101 the effect of impulse loading and reach its maximum deformation when  $W_i = W_e$ . However, due  
102 to elastic energy gathered in the structure, it will rebound in the opposite direction of the load,  
103 and the beam will lose contact with its supports if it is not restrained. Consequently, restrained  
104 boundary conditions may greatly affect the response of the rebounded beam, although it will  
105 have negligible effect on the response up to maximum deformation in the direction of the  
106 applied load.

107 Simplified calculation tools, based on spring-mass models of one or several degrees of freedom,  
108 are commonly used in practical design of impulse-loaded structures [15–17,26]. Such tools are  
109 advantageous in the sense that they provide an improved understanding to the fundamental  
110 physics behind complex events such as impact-loaded structures. Furthermore, they are fast and  
111 therefore may be very powerful since, once defined, it is easy to make many calculations

112 quickly. The improvement of such simplified methods is also the subject of several studies  
113 [8,9,23]. However, to understand in detail what happens during impact loading, more detailed  
114 models – e.g., FE models using 3D continuum elements – may be needed that correctly describe  
115 more complex phenomena such as concrete cracking and crushing, reinforcement yielding, and  
116 loss of bond. Although such tools are potentially powerful, experiments are still needed to check  
117 whether the FE results correspond to experimental results. Therefore, FE analyses should be  
118 regarded as a complement to experiments; that is, even if both methods have their advantages  
119 when used alone, they are even more powerful when used together. In the present study,  
120 reinforced beams of plain and fibre reinforced concrete subjected to impact loading were tested.  
121 The test set-up was designed for the specimens to obtain a bending mode response and to avoid  
122 shear failure with the purpose to use these results as a benchmark for nonlinear FE analyses.

123 Drop weight impact tests have been widely used to experimentally investigate the response of  
124 concrete beams [8,18,19,27–34] recorded in the form of global quantities such as impact force,  
125 support reaction force, midspan deflections, and final crack patterns. Recently, digital image  
126 correlation techniques (DIC) [35] have been used to investigate the evolution of crack patterns  
127 in impact tests of concrete beams reinforced with traditional steel bars (RC) [10,36]. This  
128 technique makes it possible to reproduce the strain field on the concrete surface and therefore  
129 provide improved understanding of the dynamic response of the loaded structure. Here, DIC  
130 was used to monitor the crack evolution of RC and steel fibre reinforced concrete beams  
131 reinforced with traditional steel bars (FRC). This experimental study provides in depth  
132 information about the response of concrete subjected to impact loading and a benchmark for  
133 evaluating the performance of numerical modelling.

134 Numerical modelling approaches for evaluating the detailed nonlinear response of concrete  
135 structures subjected to impact loading are commonly based on the finite element method, which  
136 is used to model concrete and steel reinforcement as interacting phases. For concrete, models

137 for cracking and crushing are commonly based on plasticity [37,38], damage mechanics  
138 [39,40], or combinations of these two [41–45]. Alternative approaches include micro-plane  
139 based models [46,47] and discrete methods [48,49].

140 FE analyses use an implicit or explicit solution method [50]. In the implicit method, equilibrium  
141 is based on information in both the current and previous increments, so that an iteration  
142 procedure is needed to find the equilibrium. In the explicit method, the equilibrium in the  
143 current increment is based solely on information in previous load increments, and therefore, an  
144 iteration procedure is not required. Hence, using an explicit method may be an effective way to  
145 avoid numerical convergence problems encountered when using an implicit method to solve  
146 nonlinear problems. However, with the explicit method very small time steps have to be used  
147 to obtain a stable solution. When modelling impact loading, small time steps are anyway needed  
148 to correctly capture the response. Consequently, an explicit solution method is suitable to model  
149 impact loading.

150 Here, in the numerical analyses, the damage-plasticity model CDPM2 proposed by the third  
151 author [51,52], and implemented in LS-DYNA as \*MAT\_CDPM (\*MAT\_273), was used as  
152 the concrete constitutive model. This model reproduces many of the key characteristics of  
153 concrete – e.g., softening in tension and low confined compression with reduction of stiffness  
154 and permanent strains, and pressure sensitive strength and deformation capacity for triaxial  
155 compression [51]. This model also reproduces the strain rate dependence of concrete, which is  
156 important for modeling the dynamic response of structures [52].

157 In LS-DYNA, several different types of damping are available; e.g. Rayleigh damping and  
158 artificial bulk viscosity. The former method is a mass and stiffness proportional viscous  
159 damping that presents a common approach to simulate the energy dissipation in structural  
160 dynamics, while the latter is a method used to handle discontinuous shock waves in materials.

161 In an impulse loaded structure, it is usually the initial response that is of interest; i.e. the peak

162 deformation. Hence, the effect of mass weighted damping is small and can often be neglected  
163 [14]. However, for structures subjected to very high strain rates ( $>10 \text{ s}^{-1}$ ), e.g. high-speed  
164 projectiles impact on concrete slabs, it has in [53] been shown that the bulk viscosity can have  
165 a significant influence on the resulting projectile penetration depth. There, it is also argued that  
166 bulk viscosity, due to both physical aspects and numerical reasons, must be included in the  
167 analysis. This statement, though, is contradicted by [54], where it is instead argued that bulk  
168 viscosity should not be included but an enhanced constitutive model should be used. For  
169 impulse loading of RC and FRC structures, it is common to use hexahedral elements in the  
170 numerical model in the FE program LS-DYNA [24,55–58]. However, it is well known that the  
171 element type affects the crack patterns [59,60]. It was shown that the use of tetrahedral elements  
172 made it easier to follow diagonal shear cracks. In addition, CDPM2 has been shown to perform  
173 well for quasi-static loading with tetrahedral elements [61]. Studies using tetrahedral elements  
174 for dynamically loaded concrete structures have been carried out in LS-DYNA and other FE  
175 programs [62–64]. Response of steel fibre reinforced concrete to impact and blast loading can  
176 be predicted accurately using tetrahedral elements in three-dimensional numerical studies [62].  
177 A model based on continuum mechanics, rate dependent microplane, and standard tetrahedral  
178 finite elements has accurately predicted the complex dynamic fracture process of concrete [63].  
179 Similarly, using tetrahedral elements can predict tension-shear damage and mixed-mode  
180 fracture in solids subjected to dynamic loading [64]. Merging the material model CDPM2 with  
181 tetrahedral elements has satisfactorily reproduced impact loading with respect to the bond  
182 between steel bars and concrete, merged and coincident with slip [13]. Compared to hexahedral  
183 elements, tetrahedral elements in LS-DYNA with CDPM2 seem to capture better diagonal shear  
184 cracks due to impact loading [65]. However, these studies neglect the strain rate effects both  
185 for the concrete and the steel bars [13,65].

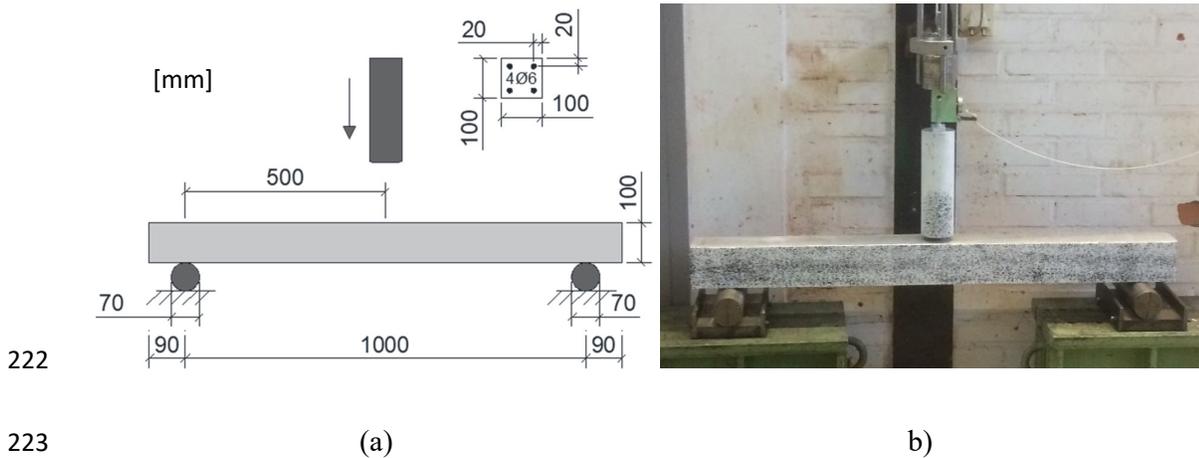
186 This paper investigates the difference in results when using hexahedral and tetrahedral elements  
187 for RC and FRC structures subjected to impact loading when using the previously developed  
188 material model CDPM2, including strain rate effects (for concrete and steel rebar). Drop weight  
189 (made of steel) impact tests were carried out on plain concrete and fibre reinforced concrete  
190 beams with ribbed steel bars. A combination of high-speed photography and DIC made it  
191 possible to compare these drop weight test results of crack patterns and structural response with  
192 FE analyses. The aim of this comparison was to find whether CDPM2 can sufficiently model  
193 impact loaded RC and FRC structures and to use CDPM2 to study phenomena that are difficult  
194 to extract from experiments only, such as reinforcement strains. Specifically, CDPM2 was used  
195 to study the effect of strain localisation and development of plastic hinge in an impact-loaded  
196 structure made of fibre-reinforced concrete and plain concrete.

## 197 **2. Experimental test set-up**

198 In the present study, a 10.1 kg drop weight, released from a height of 5.5 m, impacted simply  
199 supported beams placed on rollers (Fig. 2). The test specimens were small: the length of the  
200 beams was 1.18 m, the length of the span was 1.0 m, and the beam cross section was  
201 0.1 x 0.1 m. The beam height corresponds to a height of a slab in, e.g., a Swedish civil defence  
202 shelter with a scale factor of 1:3.5. The beams were reinforced with 2+2 ribbed steel bars with  
203 a nominal diameter of 6 mm resulting in a reinforcement ratio of about 0.7 %. The roller  
204 supports had a diameter of 70 mm and were free to roll against their supports, which were made  
205 of thick steel plates. The beams were not restrained, so during the test the beam could obtain a  
206 rigid body movement upwards once rebounding occurred – i.e., the beam’s boundary conditions  
207 were not the same for downward and upward deformation. The main reason for this test set-up  
208 was to avoid restraint moments. This was deemed to be an acceptable solution since the purpose  
209 here was to study the beam’s structural response when deforming in the direction of the applied  
210 load (i.e., downward). This configuration also made it easy to correctly position the beams in

211 the test set-up, providing for an effective testing procedure with good repeatability. No impact  
212 or reaction forces were measured.

213 A beam is normally restrained at the connections, and in impact tests typically an upper restraint  
214 is added to the supports to prevent rebounding [8,18,19,28,31]. However, in impact tests  
215 simplify support conditions are also often used, by using only simply supported beams without  
216 an upper restraint [30,32–34], and some studies have used both upper restrained and  
217 unrestrained beams [27,29]. Simply supported boundary conditions cannot capture the rebound  
218 effect. Typically, the initial deflection causes the most damage and therefore is the most  
219 dangerous. Furthermore, small scale experiments are commonly used for impact tests – small  
220 cross sections, e.g., 0.1 m x 0.1 m [27,32,33] or 0.15 x 0.15 m [29,34] and span lengths up to  
221 1.0 m.



224 **Fig. 2.** Experimental test set-up: (a) geometry, and (b) photo of beam and drop weight  
225 with speckled pattern applied.

226 The experiments analysed in this paper were part of a larger experimental series [12,36]. From  
227 this series of experiments, five RC and four FRC beams were selected for the present  
228 investigation.

229 A high-speed camera was used during the drop weight impact tests. Images were taken every  
230 0.2 ms (i.e., 5000 frames per second, fps). This frame rate is not high enough to capture what  
231 happens locally in the impact zone, but it is accurate enough to obtain information about the  
232 global response of the beam and its crack propagation. Only half of the beam was recorded to  
233 obtain good resolution. DIC with the software GOM Correlate ARAMIS [66] was used to  
234 analyse the high-speed camera images. This technique is commonly used to measure  
235 deformations and strain fields of a surface. The basic idea is to analyse the change of a surface  
236 speckle pattern in a series of digital images taken during the test. This analysis is done by  
237 tracking the position of discrete pixel subsets of the speckle pattern. If the time between each  
238 image taken during the test is known, the deformations, strains, velocities, and accelerations  
239 can be measured. To create the surface component in GOM Correlate, a facet size of 15 pixels  
240 and a point distance of 10 pixels were used for the RC beam. For the FRC beam, a facet size of  
241 15 pixels and a point distance of 5 pixels were used. Each pixel had a point distance of  
242 approximately 0.6 mm in GOM Correlate. DIC was used for all beams tested to determine crack  
243 patterns, deformed shape and midpoint-deflection. In addition, DIC was used to determine that  
244 the average impact velocity of the drop weight was 10.35 m/s.

245 The specimens were cast with self-compacting concrete (Table A.1). The fibre reinforced  
246 concrete,  $40 \text{ kg/m}^3$  ( $V_f = 0.5 \%$ ) of Dramix steel fibres, was 35 mm long and 0.55 mm in  
247 diameter. The compressive strength  $f_c$  and tensile strength  $f_t$  and fracture energy  $G_F$  were tested  
248 according to standard procedures [67–70].

249 For the plain concrete, wedge splitting tests [69] were used to determine the fracture energy.  
250 For the fibre reinforced concrete, RILEM beams were tested according to [70], and the force  
251 and crack mouth opening displacement (CMOD) were recorded (Fig. A.1).

252 Material testing on the day of the impact experiments (45 days after casting) gave the following  
253 mean values for the three test specimens for each test:  $f_c = 45.5$  (49.7) MPa,  
254  $f_t = 3.28$  (3.55) MPa; and  $G_F = 113$  (1676) Nm/m<sup>2</sup>. The values given refer to RC (FRC).

255 Material tests for the reinforcement were also conducted. The reinforcement was coil  
256 reinforcement of class C [71] with a characteristic yield strength of 500 MPa. The stress-strain  
257 curve for the tested reinforcement bars are shown in Fig. A.2. The data for the drop weight used  
258 in the experiments are shown in Table A.2.

### 259 **3. FE analyses**

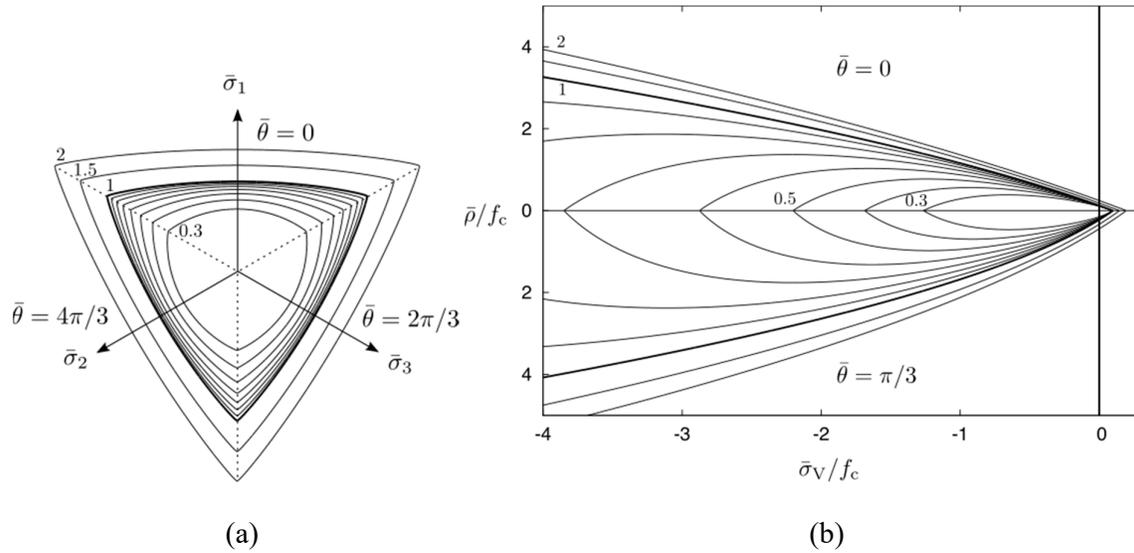
#### 260 **3.1 Concrete constitutive model**

261 The previously developed concrete damage-plasticity model 2 (CDPM2) [42,51,52] was used.  
262 This constitutive model can reproduce many features of the failure process of concrete, which  
263 are important for modelling structures subjected to impact loading. The model has been shown  
264 to accurately reproduce the effect of confinement on strength and strain capacity in  
265 compression, which is important for capturing the effect of lateral confinement generated below  
266 the impact point. The model can describe the stiffness change for a transition from tension to  
267 compression and vice-versa, which occurs often in dynamic simulations due to the effect of  
268 wave propagation and reflection. In addition, the model describes the dependence of tensile and  
269 compressive strength on strain rate [52].

270 The techniques in CDPM2, which can capture these features, are outlined briefly. In CDPM2,  
271 the stress evaluation is based on a combination of damage mechanics and elasto-plasticity. With  
272 respect to plasticity, the effective stress in the undamaged material is determined. The damage  
273 is then used to evaluate the nominal stress by applying tensile and compressive damage  
274 variables to positive and negative parts of the principal effective stress, respectively.

275 The plasticity part of the model is formulated in the effective stress space by means of the  
276 Haigh-Westergaard stress coordinates, which are the volumetric effective stress  $\bar{\sigma}_v$ , the length

277 of the deviatoric effective stress  $\bar{\rho}$  and the Lode angle  $\bar{\theta}$  [72]. The yield surface is based on an  
 278 extension of the static strength envelope [73], which was shown to reproduce experimental  
 279 results well. This strength envelope is characterised by curved meridians and deviatoric sections  
 280 varying from almost triangular in tension to almost circular in highly confined compression  
 281 (Fig. 3).

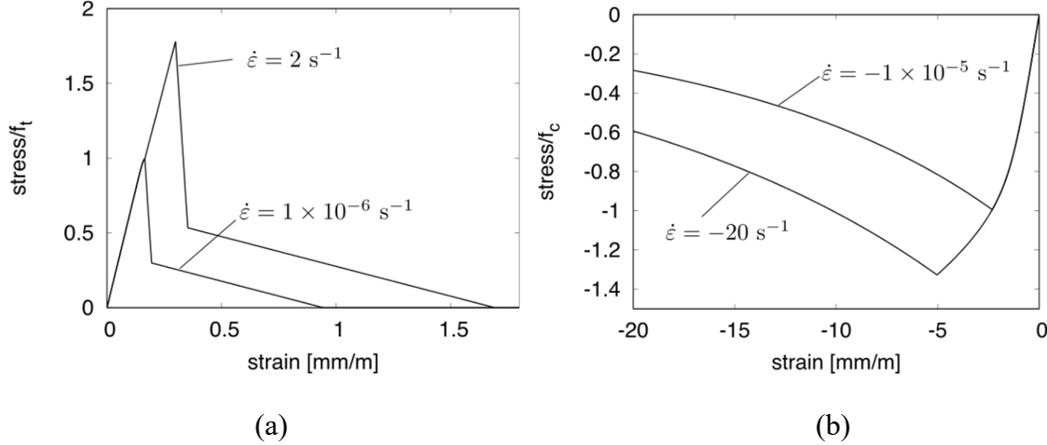


282  
 283  
 284 **Fig. 3.** Evolution of the yield surface for varying values of the hardening variable  $q_h$   
 285 (from 0.3 to 2), which is smaller than 1 in the pre-peak and larger than 1 in the post-peak regime:  
 286 (a) deviatoric section for a constant volumetric stress of  $\bar{\sigma}_v = -f_c/3$  and (b) meridians at  $\bar{\theta} = \pi/3$   
 287 (compression) and  $\bar{\theta} = 0$  (tension). A thicker line indicates static strength surface where  $q_h =$   
 288 1.  
 289 Based on this static strength envelope, static pre- and post-peak regimes were defined. In the  
 290 pre-peak regime, the yield surface is capped both in hydrostatic tension and compression. At  
 291 peak, the static strength envelope previously proposed in [73] is reached, which is open in  
 292 hydrostatic compression. In the post-peak regime, the yield surface further extends with the  
 293 shape being similar to the strength envelope. The hardening in the post-peak regime is  
 294 controlled by the hardening modulus  $H_p$ . The greater the value of  $H_p$ , the smaller the  
 295 contribution of plasticity in the post-peak regime.

296 In quasi-static simulations, damage is initiated once the strength envelope is reached. For  
297 dynamic simulations in which the strain rate dependence of concrete is considered, damage  
298 initiation is made dependent on the strain rate: the greater the strain rate, the greater the delay  
299 of the damage initiation [52]. This is achieved by dividing the rates of the equivalent strains by  
300 a strain rate dependent factor which is equal to one for quasi-static loading and increases with  
301 increasing strain rate. This way to model the rate dependence of strength is a simplified version  
302 of the often used damage delay approach [74–76]. A visco-elastic model to describe the rate  
303 dependence of the elastic response as used, for instance, in [75,77] was not considered in this  
304 study. Consequently, there is no damping in the analyses due to sudden changes in strain rate  
305 as they occur during cracking. Consequently, the strength at which damage (and softening) is  
306 initiated exceeds the static strength of the envelope (Fig. 4). This means that for dynamic  
307 loading, the pre and post-peak regimes differ from those defined by the static strength envelope.  
308 For this technique to produce reasonable results in the pre-peak in compression,  $H_p$  must be set  
309 to a large value so that the plastic strain before the onset of damage remains small. Therefore,  
310  $H_p = 0.5$  was used. The effect of strain rate on compressive and tensile strengths were modelled  
311 using equations proposed in Model Code 1990 [78] and [2]. All the equations used in CDPM2  
312 for modelling the strain rate effect are shown in [52] in detail.

313 Once damage is initiated, the response is a combination of the theory of plasticity and damage  
314 mechanics. Evolution laws for tensile and compressive damage are formulated as functions of  
315 positive and negative parts of the principal effective stress so that tensile and compressive  
316 softening responses can be described independently. Mesh-independent results are obtained  
317 using the crack-band approach [79]. In this approach, the function for the tensile damage  
318 variable is derived from a bilinear stress-crack opening ( $\sigma - w_c$ ) curve, so the results of analyses  
319 of tensile failure in which strains localise in mesh-dependent regions are independent of the  
320 finite element mesh [79–81]. Consequently, the finer the mesh, the larger the strain at constant

321 crack opening. If the strain rate in cracked elements were used, a strong mesh-dependence  
 322 would be obtained, because, in the crack band approach, for a constant crack opening the strain  
 323 is a function of the element size. Therefore, once damage is initiated in a material point, the  
 324 strain rate effect on strength is set constant to the value reached in that time step.



325  
 326

327 **Fig. 4** Effect of strain rate on softening responses used to determine damage variables:  
 328 (a) bilinear stress-crack opening curve used to determine tensile damage variable and (b)  
 329 exponential stress-inelastic strain curve used to evaluate the compressive damage variable.

330 The tensile fracture energy of concrete  $G_F$  is defined as the area under the stress-crack opening  
 331 curve for quasi-static loading (strain rate factor equal to one). For the present bilinear curve,  
 332 this results in  $G_F = (f_t w_{f1} + f_{t1} w_f)/2$ . For the default in CDPM2,  $f_{t1} = 0.3 f_t$  and  $w_{f1} = 0.15 w_f$ , so  
 333  $G_F = 0.225 f_t w_f$ . Thus, the crack opening threshold is related to the area under the stress-crack  
 334 opening curve (fracture energy) as  $w_f = 4.444 G_F/f_t$ . Within the crack band approach in CDPM2,  
 335 the crack opening  $w_c$  is converted into an inelastic strain  $\epsilon_c = w_c/h_b$ , where  $h_b$  is the assumed  
 336 width of the band in which strains localise. For linear hexahedral elements with reduced  
 337 integration used in this study, the band width is set to  $h_b = \sqrt[3]{V_e}$ , where  $V_e$  is the element  
 338 volume. For linear tetrahedral elements, the band width is  $h_b = 1.79 \sqrt[3]{V_e}$ . This practical  
 339 approach produces similar global energy dissipations for tetrahedral and hexahedral meshes  
 340 [61], which was acceptable for the present comparison. More refined approaches in which the

341 band width is determined based on principal directions of stresses and/or strains have also been  
342 presented [82].

343 The compressive damage variable is linked to a stress-inelastic strain curve, since the  
344 deformation patterns in the compressive zones of applications dominated by bending are often  
345 mesh-size independent [51].

346 CDPM2 requires many input parameters, which can be divided into groups related to the elastic,  
347 plastic and damage parts of the model. In the present work, most of these parameters are set to  
348 their default values as described, where it was shown that these parameters provide a good  
349 match with experimental results for quasi-static loading [51]. Some of the parameters are  
350 directly linked to experimental results, such as the  $f_t$  and  $f_c$  (Section 2). Young's modulus was  
351 calculated according to EN 1992-1-1 [71] as  $E_{cm} = 22 \cdot (f_{cm}/10)^{0.3}$ , and the density  $\rho$  was set to  
352  $2400 \text{ kg/m}^3$ . The damage threshold  $w_f$  was adjusted to match material data available for the  
353 different groups of analyses. For plain concrete,  $w_f$  was calculated using the fracture energy  
354 stated in Section 2. For fibre reinforced concrete, three point bending tests were performed as  
355 described [70]. Then, inverse analysis was used to determine the input parameters for the  
356 bilinear stress-crack opening law in CDPM2 (Fig. A.1). In CDPM2, the default value of the  
357 inelastic strain threshold  $\varepsilon_{fc}$  results in a very brittle response in compression. To avoid  
358 numerical difficulties, it is sometimes required to choose a more ductile compressive response  
359 to avoid premature failure in regions close to supports or applied loads. Therefore, for  
360 tetrahedral meshes, this parameter was set to  $\varepsilon_{fc} = 0.001$ , whereas for hexahedral meshes the  
361 default  $\varepsilon_{fc} = 0.0001$  was used.

### 362 **3.2 Reinforcement constitutive model**

363 The material model Johnson-Cook [83] was used to describe the stress-strain response of the  
364 reinforcement steel. The yield stress is expressed as follows:

$$365 \quad \sigma = (A + B \cdot \varepsilon_p^n)(1 + C \cdot \ln \dot{\varepsilon}^*)(1 - T^{*m}),$$

366 with five material constants:  $A$ ,  $B$ ,  $n$ ,  $C$  and  $m$ . Here,  $\varepsilon_p$  is the equivalent plastic strain,  $\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}$   
367 is the dimensionless plastic strain rate and  $T^*$  is the dimensionless temperature.

368 The constants  $A$ ,  $B$ , and  $n$  (expression in the first set of brackets) give the stress as a function of  
369 strain for  $\dot{\varepsilon}^* = 1$  and  $T^* = 0$ , which correspond to the static material tests. The input material  
370 parameters of the model ( $A = 400$  MPa,  $B = 450$  MPa and  $n = 0.21$ ) were chosen by fitting  
371 material tests of the reinforcement bars (Fig. A.2). The strain rate was modelled per previous  
372 recommendations [84], with quasi-static threshold strain rate  $\dot{\varepsilon}_0 = 10^{-4}$  and the strain rate  
373 constant  $C = 0.017$ . Temperature effects were not considered in the FE analyses.

374 There was no need to model rupture of the reinforcement as the effective plastic strains were  
375 expected to be smaller than the ultimate strain. The analyses used the following values:  
376  $E = 200$  GPa, Poisson's ratio  $\nu = 0.3$  and  $\rho = 7850$  kg/m<sup>3</sup>.

### 377 **3.3 Numerical model**

378 Three-dimensional numerical analyses of the drop weight impact test were conducted using the  
379 software LS-DYNA [85]. An explicit method based on central difference scheme [85] with an  
380 automatic time step control was used in the numerical analyses. The time step was in the order  
381 of  $3.5 \cdot 10^{-4}$  ms and  $2.3 \cdot 10^{-4}$  ms for hexahedral and tetrahedral elements, respectively. Default  
382 values in LS-DYNA [85] were used for both Rayleigh damping (no damping) and artificial bulk  
383 viscosity (1.5 for quadratic and 0.06 for linear viscosity coefficients). For the impact tests  
384 studied, the maximum strain rate was in the order of about  $1-10$  s<sup>-1</sup>, and therefore bulk viscosity  
385 was not expected to influence the results from the FE analyses. A sensitivity analysis was  
386 performed, which confirmed this expectation.

387 The model consisted of four parts: the roller supports, the concrete beam, reinforcement  
388 embedded in the concrete beam, and the drop weight (including the radius of the head), see  
389 Fig. 5. For the beams and the supports, solid hexahedral and tetrahedral elements were used.

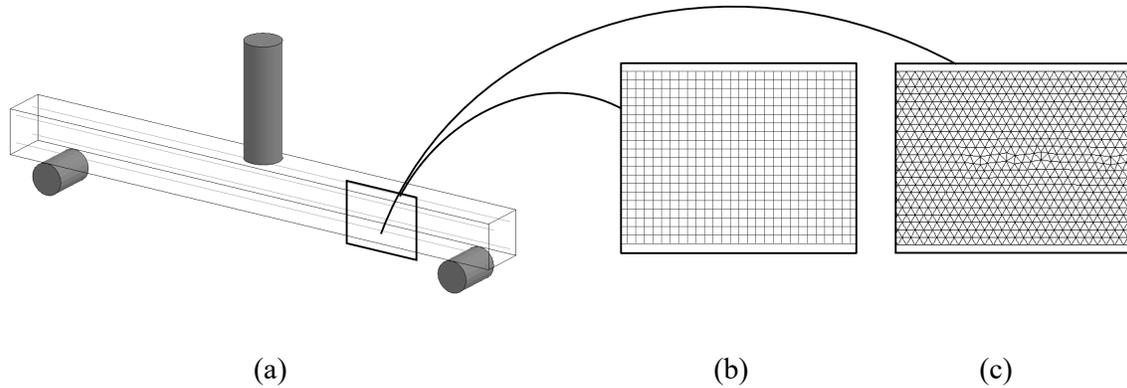
390 The hexahedral and tetrahedral elements were 8-node and 4 node solid elements, respectively,  
391 with one integration point. The drop weight was modelled with hexahedral mesh. The  
392 reinforcement was modelled with beam elements using Hughes-Liu formulation [85] with  
393 circular cross section. Hourglass control was needed to ensure stability of hexahedral elements,  
394 and the Belytschko-Bindeman formulation [85] was used with default input parameters.  
395 Convergence of the element size was previously studied for the hexahedral and tetrahedral  
396 meshes, with element edge lengths of one side of 2.5 mm (only for hexahedral), 5 mm, 10 mm,  
397 and 20 mm [65]. In this study, the same geometries of the beam and drop weight were used; the  
398 drop heights, though were different (2.5 m and 5.0 m). It was concluded that an element edge  
399 length of 5 mm for both hexahedral and tetrahedral mesh were adequate to accurately represent  
400 the deflections and crack patterns. Thus, in this study an element length of 5 mm for both  
401 element types was chosen, which resulted in about 100000 elements and 900000 elements for  
402 the beam with hexahedral elements and tetrahedral elements, respectively.

403 To simulate the simply supported boundary conditions, rollers were fixed at the bottom and an  
404 automatic surface to surface contact, based on Taylor and Flanagan [85], without friction was  
405 used between the drop weight and the concrete beam. The same contact definition was used  
406 between the beam and the rollers.

407 The reinforcement was modelled with 5 mm beam elements assuming perfect bond between  
408 reinforcement and concrete, which has been shown to provide acceptable results [13]. The same  
409 nodes were used for concrete and reinforcement elements for both FE models.

410 The drop weight and the roller supports were modelled to be elastic:  $E = 200$  GPa and  $\nu = 0.3$ .  
411 A small hole was drilled in the drop weight where an accelerometer (not used in the tests) was  
412 attached. This hole was not included in the FE model. Hence, to account for this, the steel

413 density of the drop weight was adjusted so that the mass corresponded to the mass in the  
414 experiments ( $\rho = 7753 \text{ kg/m}^3$ ).



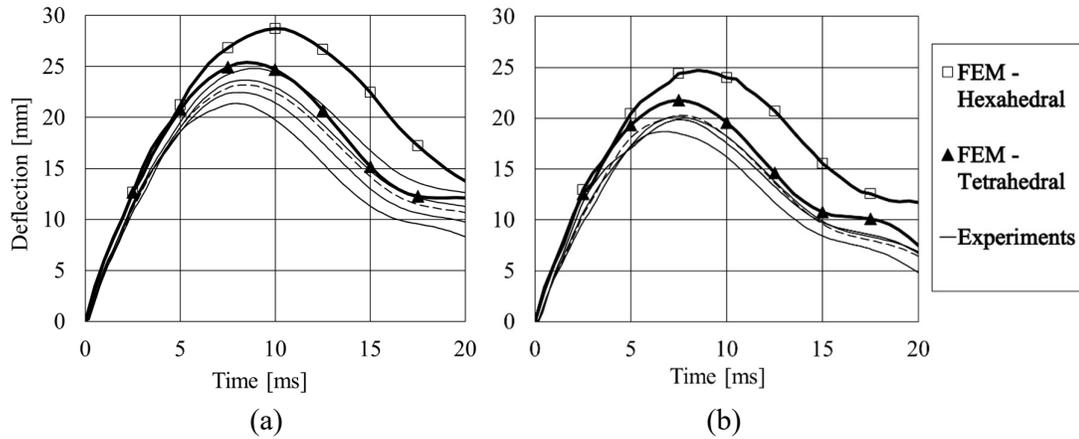
417 **Fig. 5.** Numerical model used in the FE analyses: (a) 3D model, (b) hexahedral mesh,  
418 and (c) tetrahedral mesh.

#### 419 4. Results

420 The midpoint deflections obtained from DIC and FE analyses for RC and FRC beams are  
421 compared with experimental results (5 RC and 4 FRC) in Fig. 6; dashed lines in the figure  
422 indicate the beams used to compare crack patterns in Fig. 7 and Fig. 8 and the initial relative  
423 deformation in Fig. 9. This comparison shows that in general the beam deflections can be  
424 modelled effectively using CDPM2. However, the maximum midpoint deflection is  
425 overestimated using hexahedral elements. For the drop weight tests with RC beams, the  
426 maximum midpoint deflections were between 21 and 25 mm, with an average value of about  
427 23 mm and in the FE analysis the maximum midpoint deflection was 29 mm using hexahedral  
428 elements. However, when using tetrahedral elements, the maximum midpoint deflection was  
429 25 mm, which is very close to the experimental results.

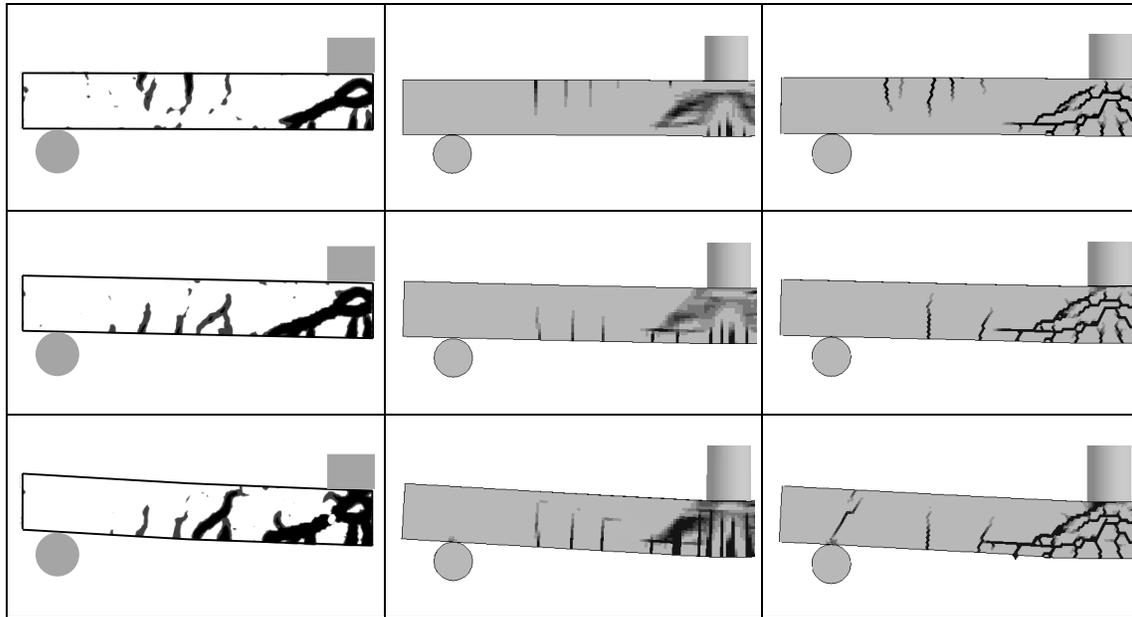
430 For the FRC beams, it was found that the addition of fibres decreased the maximum midpoint  
431 deflection; and in the tests the maximum midpoint deflections were between 18 and 20 mm,

432 with an average value of 19 mm. In the FE analysis, the maximum midpoint deflection was  
 433 25 mm and 22 mm using hexahedral and tetrahedral elements, respectively.



434 **Fig. 6.** Deflection in the midpoint of the beams, comparison with experiments (5 + 4  
 435 beams) and FE analyses using hexahedral and tetrahedral elements, (a) RC beams,  
 436 and (b) FRC beams. Dashed lines correspond to the experimental beams used to  
 437 compare crack patterns in Fig. 7 and Fig. 8 and initial relative deformation in  
 438 Fig. 9.

439 DIC made it possible to study the crack propagation as a function of time in the experiments.  
 440 Here, just two beams from the experiments, one for RC and one for FRC, are presented. In  
 441 Fig. 7, DIC and FE show the crack patterns at an initial stage and the stage at maximum  
 442 deflection. In the DIC analyses, black indicates a crack visible to the eye (approximately  
 443 0.1 mm). In the numerical analyses the maximum principal strains were plotted and black  
 444 corresponds to the cracking strain:  $\epsilon_{\text{crack}} = 0.017$  (for tetrahedral elements).



445

446

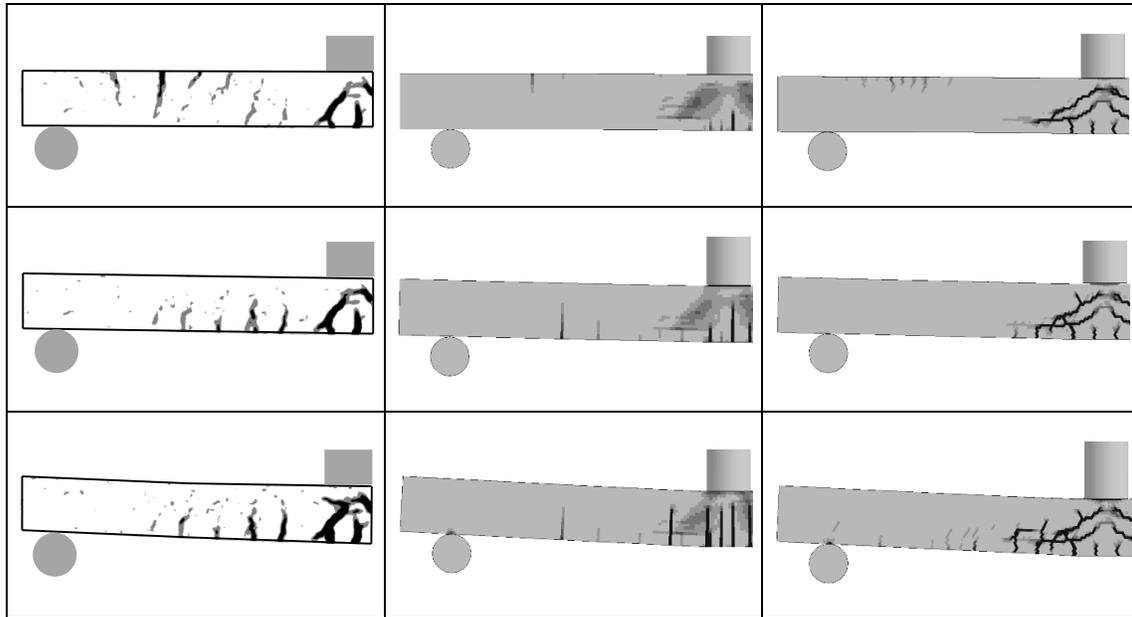
(a)

(b)

(c)

447 **Fig. 7.** Comparison of DIC (dashed line in Fig. 6a) and FE-analyses for RC beam. (a)  
 448 DIC, (b) hexahedral elements, and (c) tetrahedral elements. Time from impact:  
 449 first row after 0.6 ms; second row after 2 ms; and third row after 9 ms.

450 Similar analyses were carried out for the FRC beams (Fig. 8). The response was very similar to  
 451 the responses of the RC beams. However, the crack widths obtained using FRC are smaller than  
 452 in the RC beam, both in the experiments and in the FE analysis. In the FE analysis, the early  
 453 stage cracking is accurately captured (same concept as for RC shown in Fig. 7).



454

455

(a)

(b)

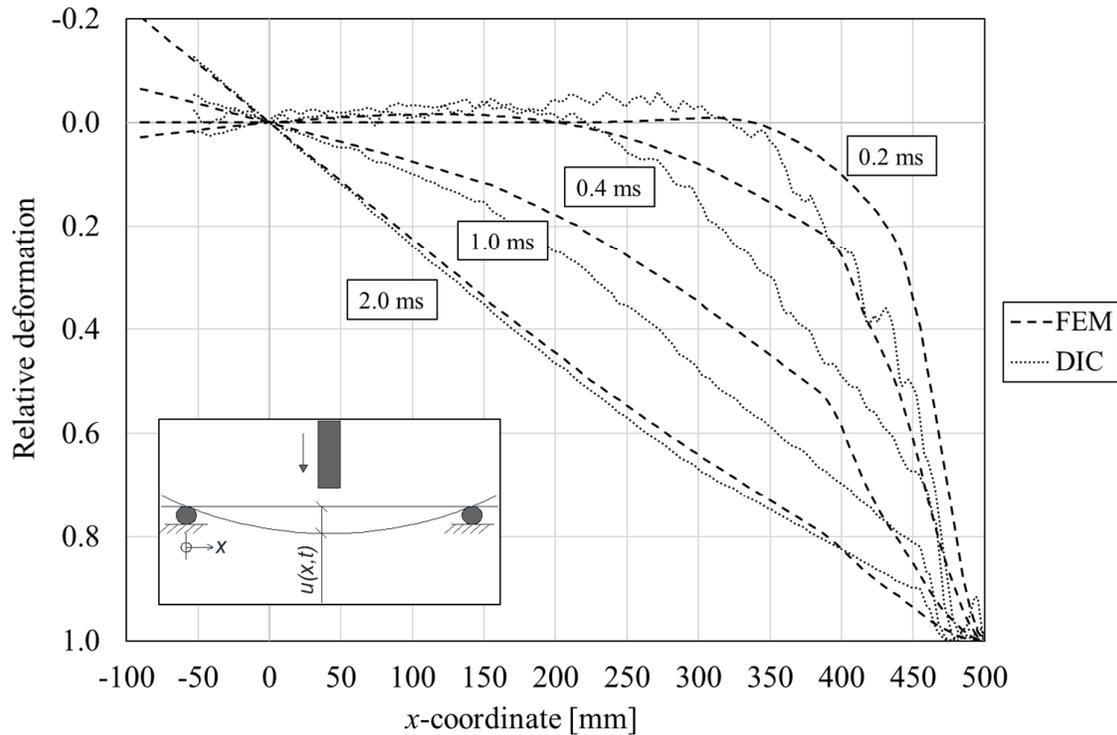
(c)

456 **Fig. 8.** Comparison of DIC (dashed line in Fig. 6b) and FE-analyses for FRC beam. (a)  
 457 DIC, (b) hexahedral elements, and (c) tetrahedral elements. Time from impact:  
 458 first row after 0.6 ms; second row after 2.0 ms; and third row after 8.0 ms.

459 The numerical analyses of the hexahedral elements show that the crack tend to follow the mesh  
 460 and that the diagonal shear cracks cannot be captured as well as with tetrahedral elements.

461 Initially, bending cracks form at the bottom of the beam and shear cracks form close to the  
 462 impact point. Then, bending cracks form at the top of the beam at a distance of roughly one-  
 463 fourth of the span length from the support. The appearance of bending cracks at the top of the  
 464 beam is due to wave propagation effects as schematically shown in Fig. 1. In Fig. 9, the relative  
 465 deformation,  $u_{rel}(x,t) = u_{mod}(x,t)/u_{max}$ , from DIC and FE analyses using hexahedral elements in  
 466 the RC beam is compared for the first 2 ms of loading. In the initial stage, the beam lifts at the  
 467 supports. This effect, however, has been negated in Fig. 9 by setting  
 468  $u_{mod}(x,t) = u(x,t) - u_{support}(t)$ . The relative deformation along the beam shows that the beam has  
 469 very high positive curvature beneath the drop weight and a negative curvature at the top soon

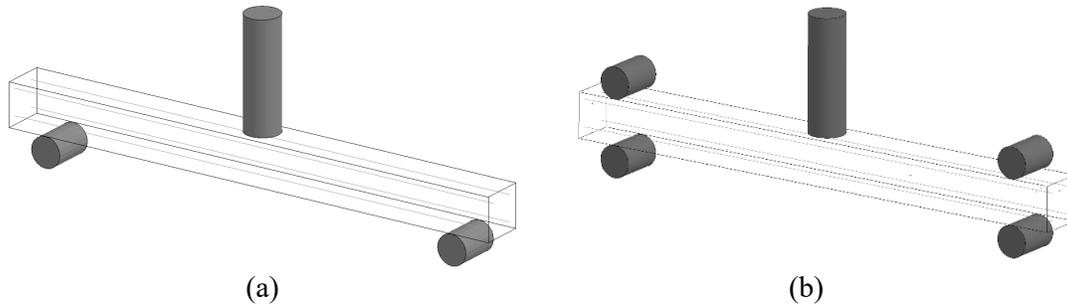
470 after impact. After the first two milliseconds, the deformed shape of the beam approaches that  
 471 of a simply supported beam subjected to a static point load, with a plastic hinge in the middle  
 472 of the beam. The relative deformation can be captured very effectively both in the DIC and the  
 473 FE analyses and corresponds well with that shown in Fig. 1.



474  
 475 **Fig. 9.** Relative deformation for RC. Comparison with DIC (dashed line in Fig. 6a) and  
 476 FE analyses (hexahedral elements) for the first 2 ms after impact.

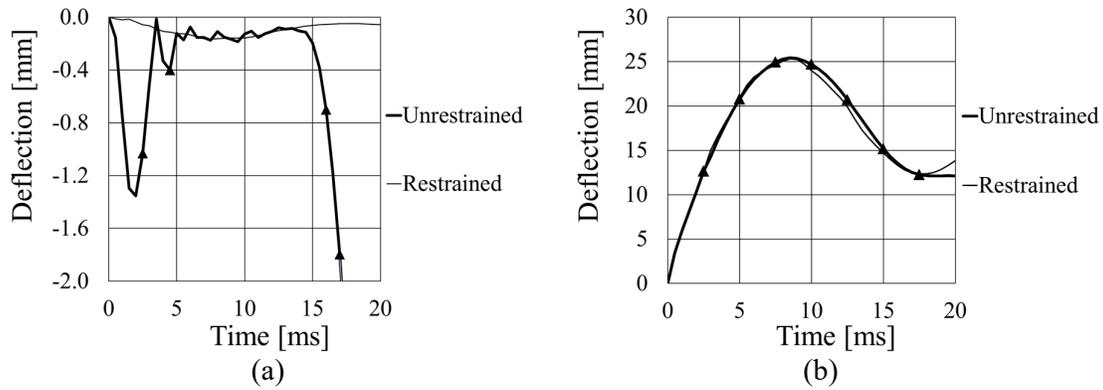
477 Based on the observations made above, it can be concluded that the FE model using tetrahedral  
 478 elements effectively reproduces the results obtained in the experiments both for RC and FRC  
 479 beams. Accordingly, it is also possible to use the FE analyses to further complement the  
 480 experiments; that is, they can be used to study different settings or phenomena that otherwise  
 481 may be difficult to study using experiments alone. The influence of different boundary  
 482 conditions, the impact force, the support reactions, and the strain localisation in the  
 483 reinforcement were studied using only FE analysis.

484 The effect of different boundary conditions was studied with FEM for the RC beam using  
485 tetrahedral elements. The beams were supported on rollers – one beam was unrestrained and  
486 the other beam had an upper restraint to prevent upward movements at the support (Fig. 10).



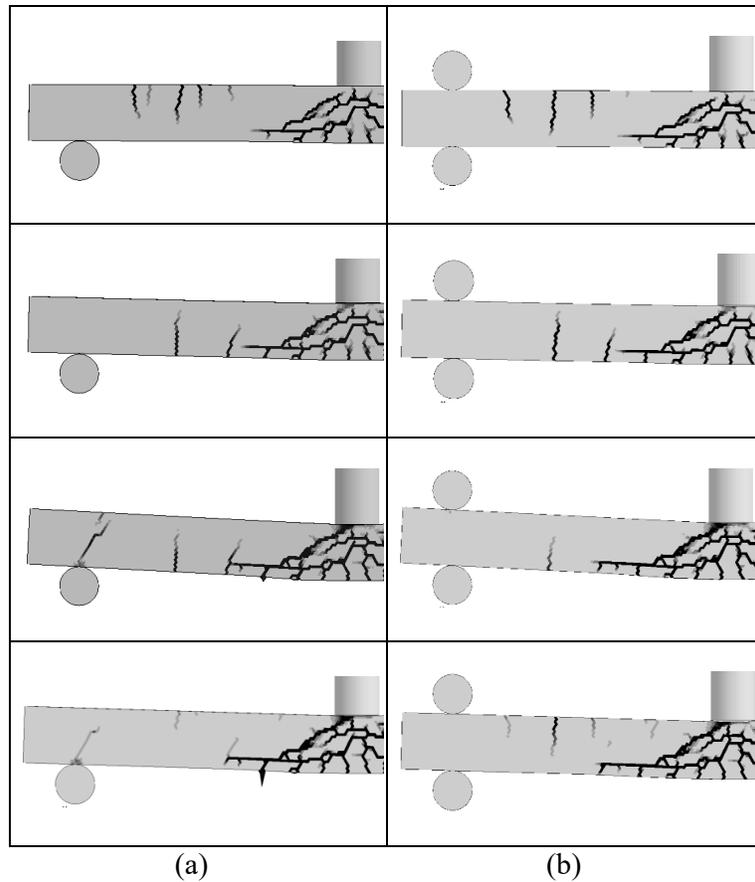
487 **Fig. 10.** Numerical models used in FE analyses for study boundary conditions (a)  
488 unrestrained beam (same model as in Fig. 5), and (b) restrained beam.

489 The deflections from the FE analyses at the support and the midpoint deflection for the  
490 boundary conditions with and without upper restrain are compared in Fig. 11. In Fig. 11 (a), the  
491 beam deflections, in the initial contact point between bottom roller and the beam, at the bottom  
492 support are compared; in Fig. 11 (b), the midpoint deflections at the top of the beam for the two  
493 models are compared. The results show that the behaviour of the beam is hardly affected by the  
494 upper restraint for the first 15 ms, which is the time when the beams start to lift from their  
495 supports and the midpoint deflections start to deviate, after approximately 17 ms. These results  
496 show that the effect of the upper restrain has negligible effect on the midpoint deflection before  
497 rebounding occurs even if the unrestrained beam initially lifted from the supports for a few  
498 milliseconds. In addition, the restrained beam seems to have a small deflection at the support,  
499 but this deflection is due to the rotation of the beam at the support – i.e., the beam is in contact  
500 with the support at all times.



501 **Fig. 11.** Comparison of deflections with FE analyses with different boundary conditions  
 502 (a) at the support, and (b) midpoint deflections. Positive direction is downwards.

503 The effect on crack propagation due to different boundary conditions is compared in Fig. 12.  
 504 As the crack propagation is very similar and independent of boundary conditions, the upper  
 505 restraint had very little effect on the crack propagation for the first 15 ms – i.e., the time when  
 506 the unrestrained beam started to lift from its supports. The boundary condition may affect the  
 507 response of the rebounded beam. However, the boundary condition has negligible effect on the  
 508 response up to the maximum midpoint deflection in the direction of the applied load, which  
 509 often causes the most damage for the beam.



510

511 **Fig. 12.** Comparison of FE analyses for RC beam: (a) unrestrained beam, and (b)

512 restrained beam. Time from impact: first row after 0.6 ms; second row after

513 2.0 ms; third row after 9 ms; and fourth row after 15 ms.

514 The force and impulse from the impact and support reaction, taken from the FE analyses with

515 tetrahedral meshes, are shown in Fig. 13. At 0.1 ms after the impact, the magnitude of the

516 impact force was very similar for the RC and FRC beams with peak values of 261 kN and

517 263 kN, respectively. The support reactions were also very similar: peak values of 224 kN (RC)

518 and 222 kN (FRC). The delayed appearances of the support reactions, 3.4 ms (RC) and 3.0 ms

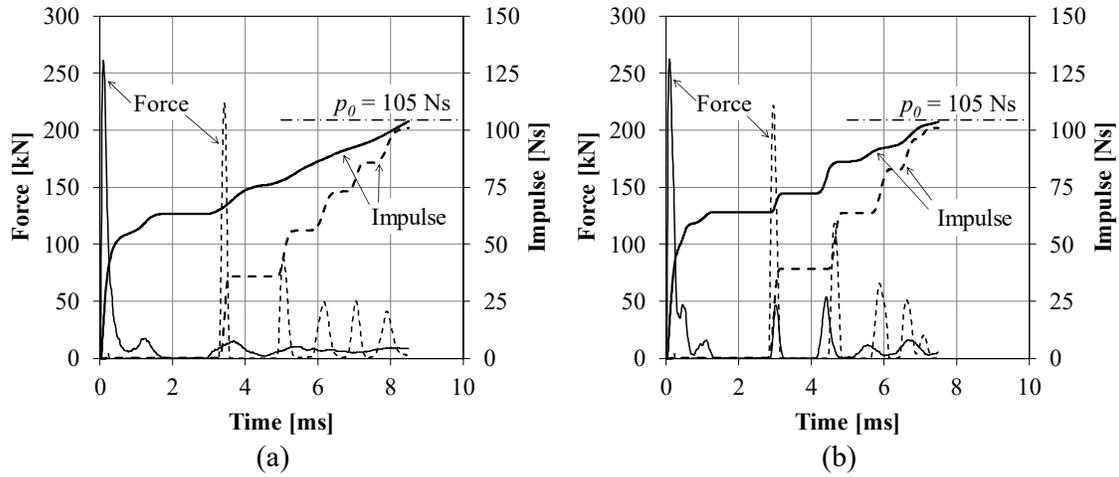
519 (FRC), were due to a combination of wave propagation effects and the initial uplift of the beams

520 at the supports; compare with Fig. 11 (a). The development of the impulses of the impact and

521 support reaction is presented until a time corresponding to maximum deflection, reaching

522 104 Ns for the impact force and 101 Ns for the support reaction. The same values were obtained

523 for RC and FRC beams and can be compared to the momentum of the dead-weight just prior to  
 524 impact:  $p_0 = m \cdot v = 10.1 \cdot 10.35 = 105 \text{ Ns}$ .

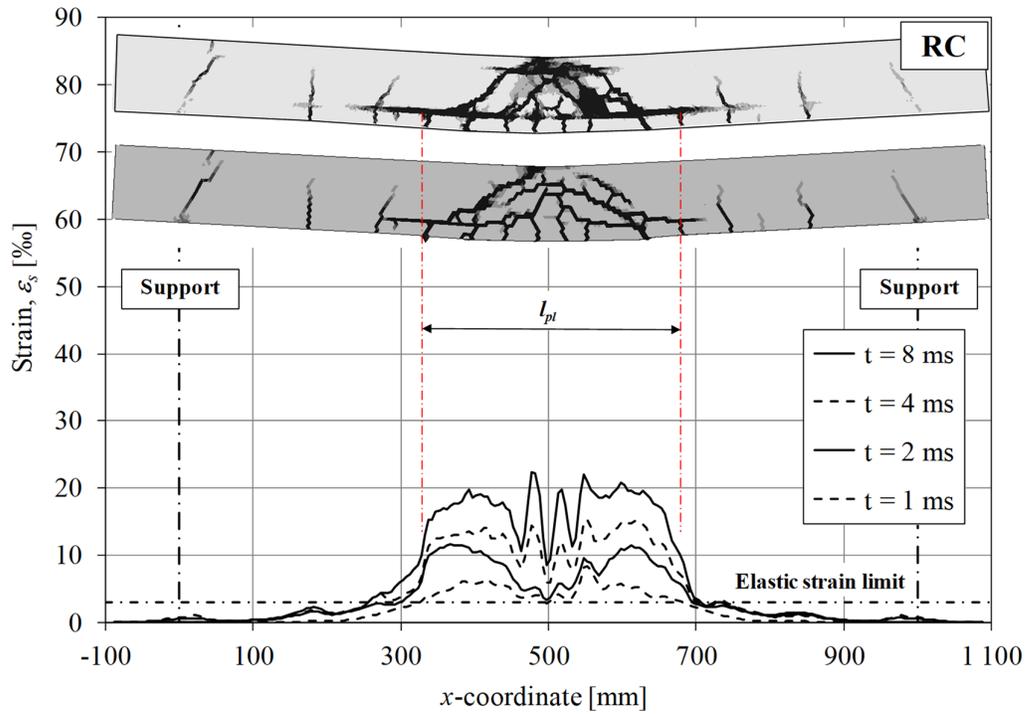


525 **Fig. 13.** Force and impulse from impact force (solid lines) and total support reaction  
 526 (dashed lines) from FE analyses using tetrahedral elements of (a) RC beam and (b) FRC beam.  
 527 The momentum  $p_0$  of the drop weight just prior to impact is marked as comparison. Impact or  
 528 support reaction forces were not measured in the experiments.

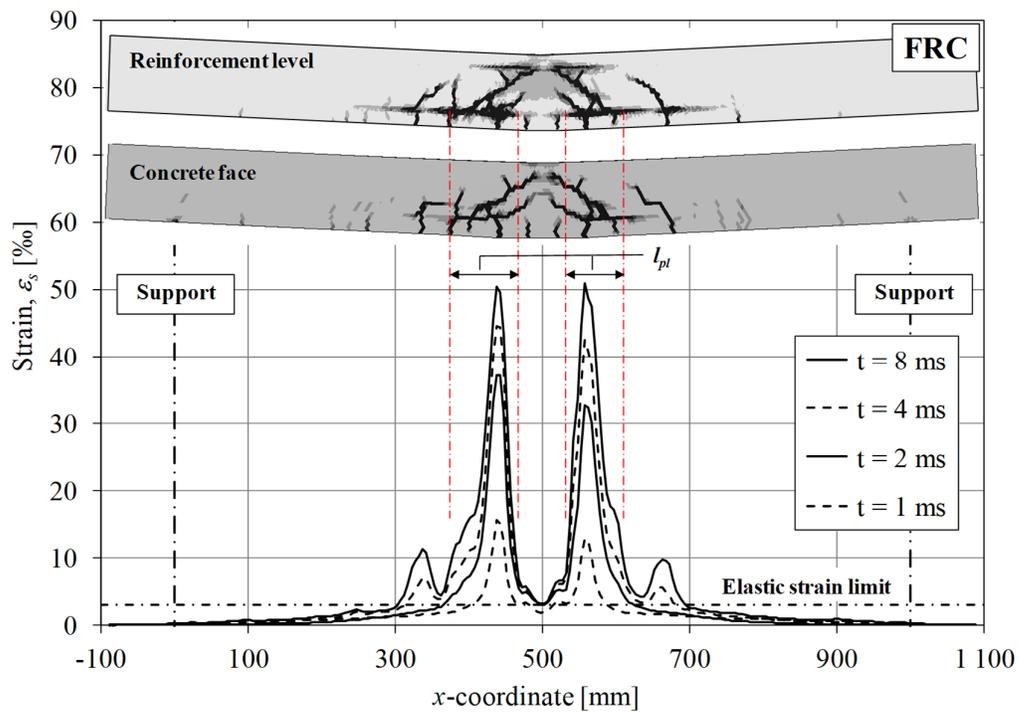
529 Fig. 14 shows strains in the reinforcement along the length of the RC and FRC beams. Although  
 530 the total deflection in the RC beam was larger than in the FRC beam, the maximum  
 531 reinforcement strain in the FRC beam was more than twice as high as in the RC beam. In the  
 532 RC beam, the maximum reinforcement strain did not exceed 25 %, and the strains were rather  
 533 evenly distributed in the middle of the beam, giving rise to a well-defined plastic hinge with a  
 534 length  $l_{pl}$  of approximately 0.35 m ( $l_{pl}$  is here defined as the total distance where  $\varepsilon_s \geq 10 \%$ ).  
 535 However, for the FRC beam, the maximum reinforcement strain reached 50 %, and the length  
 536 of its plastic hinge was considerably smaller, about 0.20 m.

537 The difference in response observed is due to different interaction between reinforcement and  
 538 concrete in the two models. In the RC beam, the strain in the reinforcement damages  
 539 surrounding concrete; i.e., spalling cracks appear that reduce the interaction stiffness between

540 reinforcement and concrete. This phenomenon enables the reinforcement strain to be more  
541 evenly distributed over a longer distance, resulting in a more uniform strain distribution over a  
542 larger length. In the FRC beam, the fibres make the concrete much more ductile; its fracture  
543 energy is almost 15 times higher than that of the RC beam. Because the fibres prevent spalling  
544 cracks from forming, the effective bond between reinforcement and concrete remains more or  
545 less intact. A stiffer bond interaction leads to an increased amount of strain localisation, and  
546 therefore the maximum reinforcement strain becomes higher. This increase is also evident in  
547 Fig. 14 and has previously been observed for statically-loaded beams subjected to bending  
548 failure [86,87].



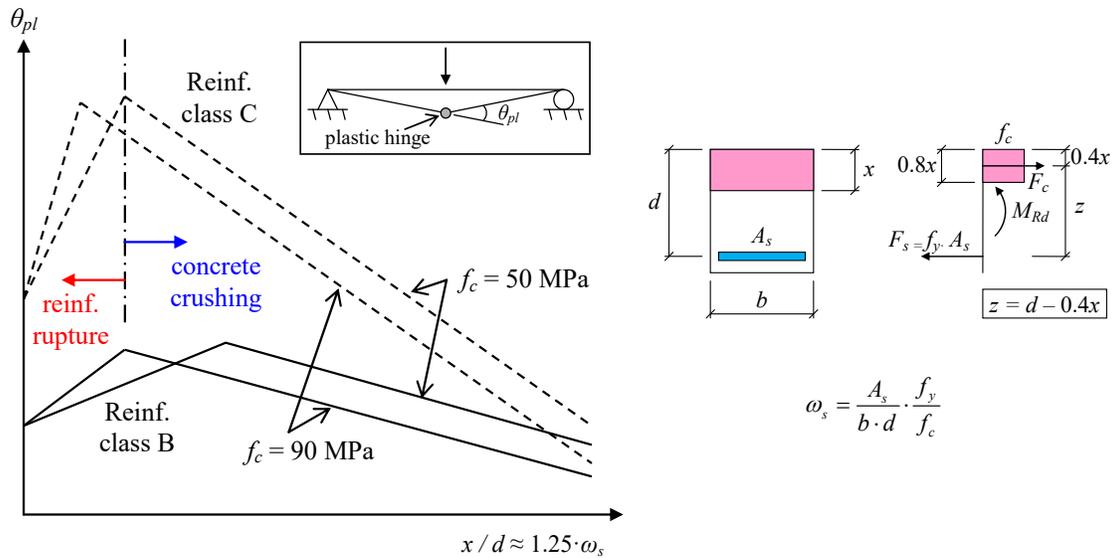
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550

551 **Fig. 14.** Strain in bottom reinforcement in FE analyses of RC (top) and FRC (bottom)  
 552 beams using tetrahedral elements. As comparison, the crack patterns obtained at the  
 553 reinforcement level and at the concrete face are shown.

554 In Section 1, it is concluded that the design of concrete structures subjected to impulse loading  
555 needs to make sure that the structure can manage large plastic deformations prior to failure.  
556 There are many parameters that affect the plastic deformation capacity of a reinforced concrete  
557 structure (e.g., reinforcement ductility (strength ratio  $f_u/f_y$  and ultimate strain), bond between  
558 reinforcement and concrete, mechanical reinforcement ratio, and structure slenderness); how to  
559 predict plastic deformation capacity is its own research field [88–90]. However, one important  
560 criterion for obtaining this response is to obtain a large plastic hinge in which the reinforcement  
561 can develop plastic strains. Whether the reinforcement or the concrete is critical for the final  
562 plastic deformation capacity depends on the reinforcement's mechanical properties and the  
563 mechanical reinforcement ratio. Low reinforcement ductility, high concrete strength and low  
564 reinforcement amount increase the risk of reinforcement rupture (Fig. 15). Hence, the  
565 observation made above is important, since it indicates that the maximum reinforcement strain  
566 in reinforced concrete structures may increase due to the presence of fibres – i.e., the risk of  
567 reinforcement rupture may increase. Depending on the configuration of the structure, this may  
568 have considerable negative effects on the structure's deformation capacity. In this study, the  
569 deformation capacities proved to be adequate for both the RC and the FRC beams. However,  
570 in a structure with smaller mechanical reinforcement amount, there is increased risk for ruptured  
571 reinforcement leading to collapse.



572 **Fig. 15.** Schematic illustration of plastic deformation as function of the mechanical  
 573 reinforcement ratio  $\omega_s$  for different classes of reinforcement (class C is more ductile than  
 574 class B) and concrete strength. For each line, the ascending and descending branch indicates  
 575 reinforcement rupture and concrete crushing, respectively. Based on [71].

576 In the FE models used here, perfect bond between reinforcement and concrete was assumed.  
 577 Hence, the bond-slip relation between reinforcement and surrounding concrete was not  
 578 explicitly incorporated in the model. Nevertheless, the effect of reduced bond is still included  
 579 in an approximate way since the concrete in the vicinity of the reinforcement bars are affected  
 580 (cracked) by the force in the reinforcement. In Fig. 14, the effect of reduced bond is indicated  
 581 by the horizontal cracks at the level of the bottom reinforcement - i.e., spalling cracks. The  
 582 extension of this zone of reduced bond also agrees well with the location of large reinforcement  
 583 strains in both the RC and the FRC beam.

## 584 5. Conclusions

585 Based on experiments and numerical analyses of drop weight impact tests of RC and FRC  
 586 beams, the following conclusions can be drawn.

587 Digital image correlation (DIC) analysis of high-speed camera images is a very powerful tool  
588 for analysing the response of drop weight impact tests. The deformation and crack propagation  
589 of the beams tested could be followed in detail.

590 FE analyses with the material model CDPM2, using tetrahedral elements, effectively captures  
591 the response of the impact loaded beams. The use of hexahedral elements is not adequate for  
592 representing correct response when diagonal cracks are formed.

593 The FE analyses showed that the strains in the reinforcement were more evenly distributed and  
594 that the length of the plastic hinge was larger in the RC beams than for FRC beams. In the latter,  
595 larger maximum strains were also obtained, even though both deflection and crack widths were  
596 smaller in the experiments. This finding indicates that there may be an increased risk of  
597 reinforcement rupture in FRC structures, a finding that calls for further study.

598 The CDPM2 material model shows promising results for use in the evaluation of impulse loaded  
599 structures; however, there is need for further investigating into how to model the influence of  
600 strain rates on concrete properties in general and, in particular on fracture energy.

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819 **Appendix**

820 **Table A.1**

821 Data summary for the self-compacting concrete mixtures.

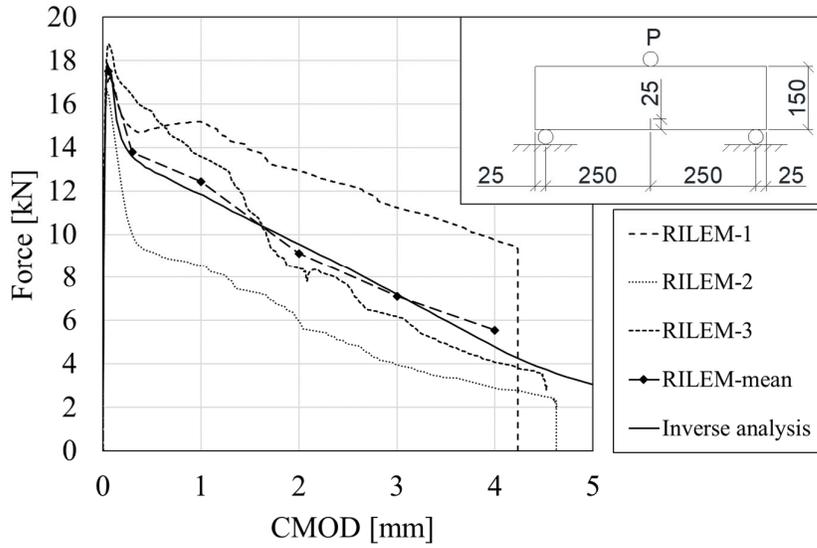
	Plain	Fibre reinforced
Constituents	Amount [kg/m <sup>3</sup> ]	Amount [kg/m <sup>3</sup> ]
Cement	335	335
Limestone filler	160	160
Sand	747.3	741.1
Aggregates		
4 – 8 mm	268.9	266.8
8 – 16 mm	717.1	711.4
Superplasticiser	5.36	5.36
Fibre	-	40
Water	184.3	184.3
w-c ratio	0.55	0.55

822

823 **Table A.2**

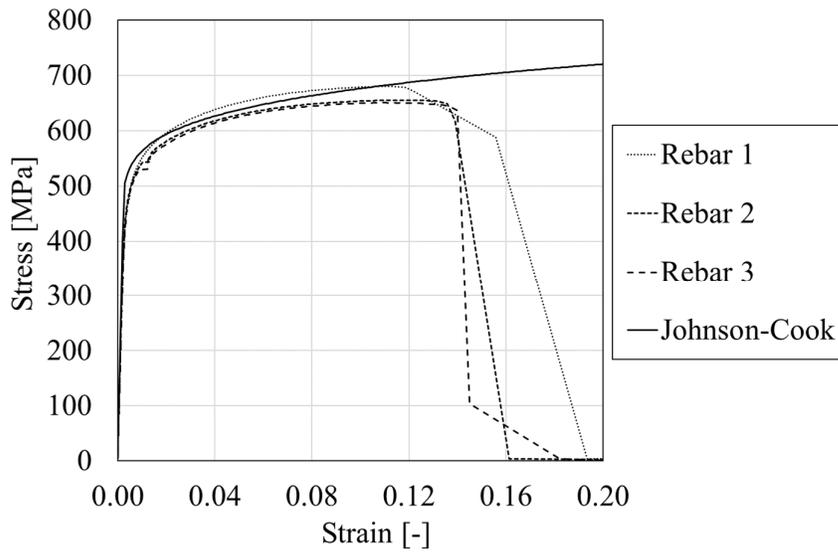
824 Geometry for the drop weight.

Length	260 [mm]
Diameter	80 [mm]
Weight	10.1 [kg]
Radius of the head	400 [mm]



825

826 **Fig. A.1.** Force – CMOD curve from RILEM beams, and results from adaptive inverse  
 827 analysis.



828

829 **Fig. A.2.** Stress-strain curve for the reinforcement: measured values (3 bars) vs. input data  
 830 to FE analyses for the material model Johnson-Cook.