Hydraulic fracture of a porous thick-walled hollow sphere

Peter Grassl

University of Glasgow, UK

Domenico Gallipoli

University of Pau, France

Milan Jirásek

Czech Technical University, Czech Republic

Background

Hydraulic fracture processes in permeable geomaterials with stationary fluid flow.

Aim

Study the importance of Biot coefficient for hydraulic fracture processes.

Methodology

Investigate fracture of permeable thick-walled hollow sphere subjected to inner fluid pressure.

Outline

Assumptions and notations

Fluid driven loading

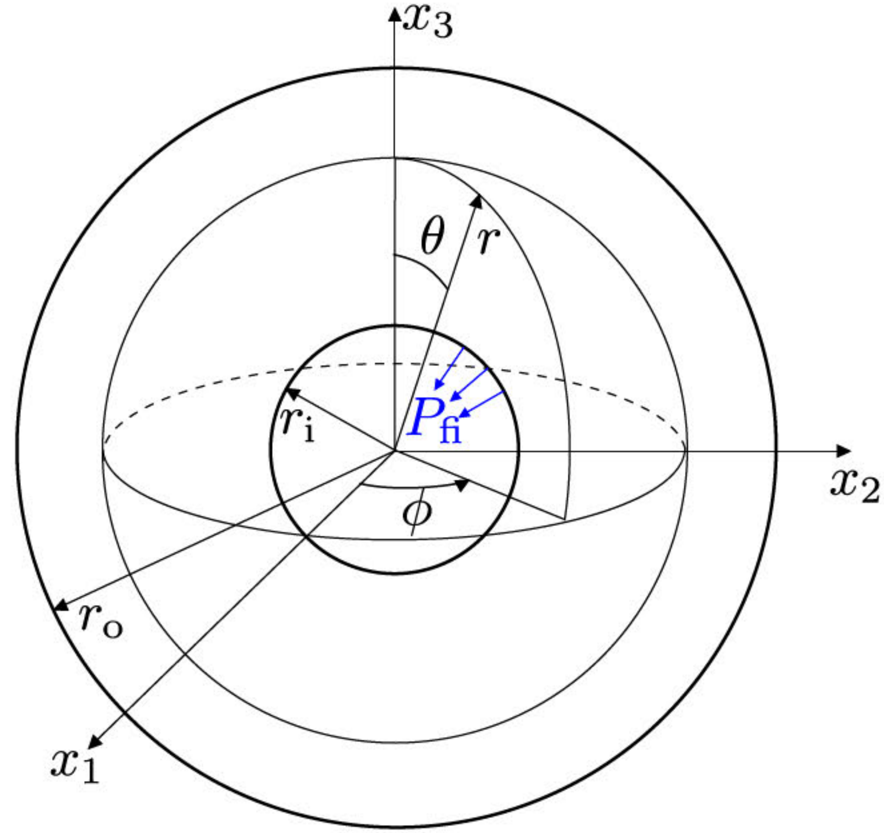
Elastic response

Nonlinear fracture response

Size effect

Assumptions and notations

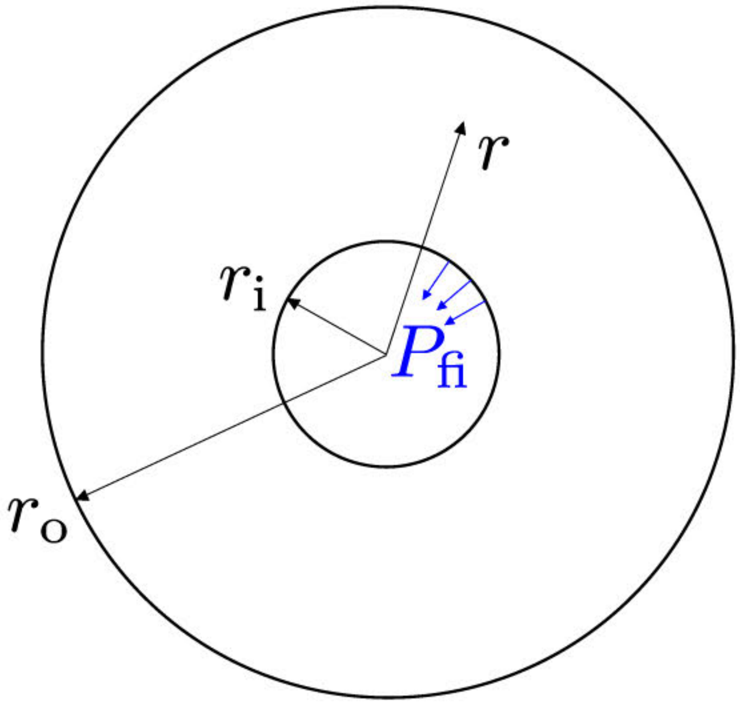
- Spherical symmetry
- Stationary flow
- Constant viscosity and permeability
- Small displacements
- Influence of gravity neglected



P_fi tension positive

Fluid driven loading

Fluid driven loading



$$\dot{V} = \dot{V}_{\mathrm{i}} + Q \qquad \dot{V}_{\mathrm{i}} = \dot{u}_{\mathrm{i}} 4\pi r_{\mathrm{i}}^2$$

$$q = \frac{Q}{4\pi r^2} \qquad q = \frac{\kappa}{\mu} \frac{dP_{\mathrm{f}}}{dr}$$

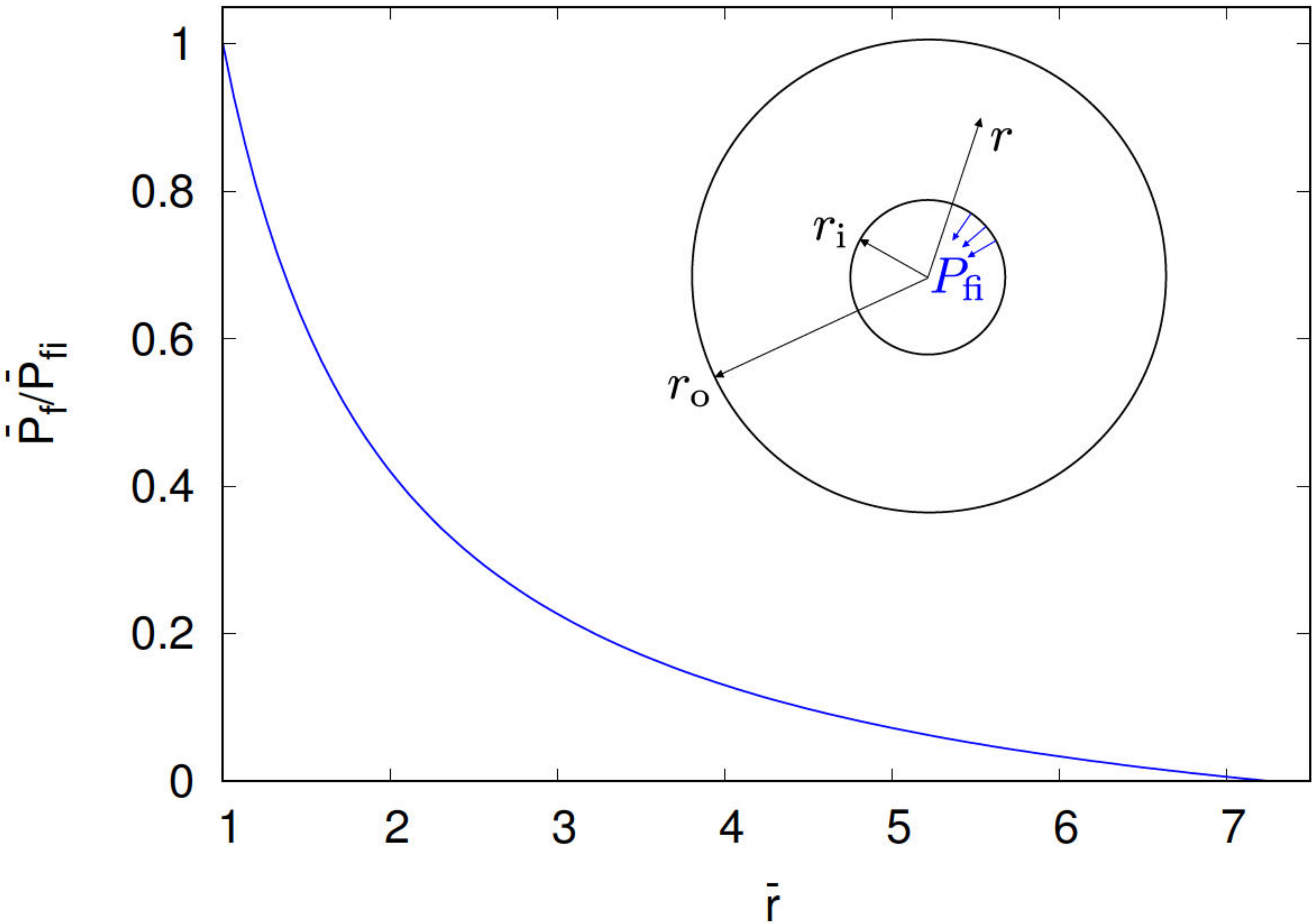
Pressure distribution

$$\bar{P}_{\mathrm{f}} = \bar{P}_{\mathrm{fi}} \frac{\bar{r} - \bar{r}_{\mathrm{o}}}{\bar{r}\,(1 - \bar{r}_{\mathrm{o}})}$$

with dimensionless variables

$$\bar{P}_{\mathrm{fi}} = P_{\mathrm{fi}}/E \qquad \bar{P}_{\mathrm{f}} = P_{\mathrm{f}}/E \qquad \bar{r} = r/r_{\mathrm{i}} \qquad \bar{r}_{\mathrm{o}} = r_{\mathrm{o}}/r_{\mathrm{i}}$$

Pressure versus radius



Elastic response

Equilibrium condition

$$\frac{d\sigma_{\mathrm{r}}}{dr} + 2\frac{\sigma_{\mathrm{r}} - \sigma_{\mathrm{t}}}{r} = 0$$

Stress definition

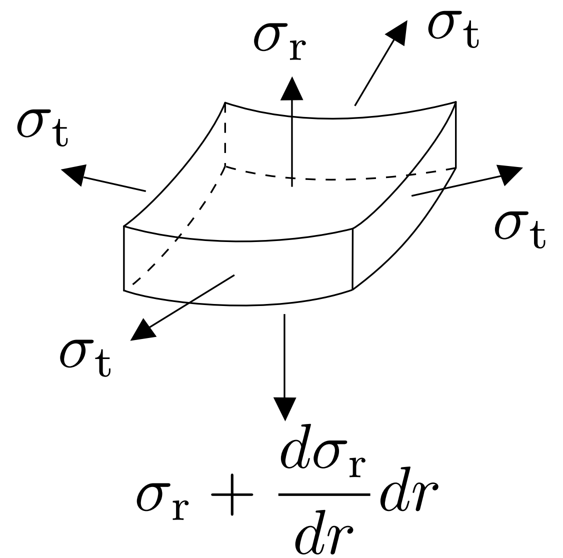
$$\sigma_{\mathrm{r}} = \sigma_{\mathrm{r}}^{\mathrm{m}} + bP_{\mathrm{f}}$$

$$\sigma_{\mathrm{t}} = \sigma_{\mathrm{t}}^{\mathrm{m}} + bP_{\mathrm{f}}$$

Constitutive laws

$$\varepsilon_{\mathrm{r}} = \frac{1}{E}\left(\sigma_{\mathrm{r}}^{\mathrm{m}} - 2\nu\sigma_{\mathrm{t}}^{\mathrm{m}}\right)$$

$$\varepsilon_{\mathrm{t}} = \frac{1}{E}\left((1-\nu)\sigma_{\mathrm{t}}^{\mathrm{m}} - \nu\sigma_{\mathrm{r}}^{\mathrm{m}}\right)$$



$$\sigma_{\theta} = \sigma_{\phi} = \sigma_{\mathrm{t}}$$

Kinematics

$$\varepsilon_{\mathrm{r}} = \frac{du}{dr} \qquad \varepsilon_{\mathrm{t}} = \frac{u}{r}$$

Timoshenko and Goodier (1970), Coussy (2010)

Dimensionless ODE

$$\frac{d^2\bar{u}}{d\bar{r}^2} + 2\frac{d\bar{u}}{d\bar{r}}\frac{1}{\bar{r}} - 2\frac{\bar{u}}{\bar{r}^2} + b\bar{P}_{\mathrm{fi}}\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar{r}_{\mathrm{o}}}{1-\bar{r}_{\mathrm{o}}}\frac{1}{\bar{r}^2} = 0$$

with dimensionless radial displacement

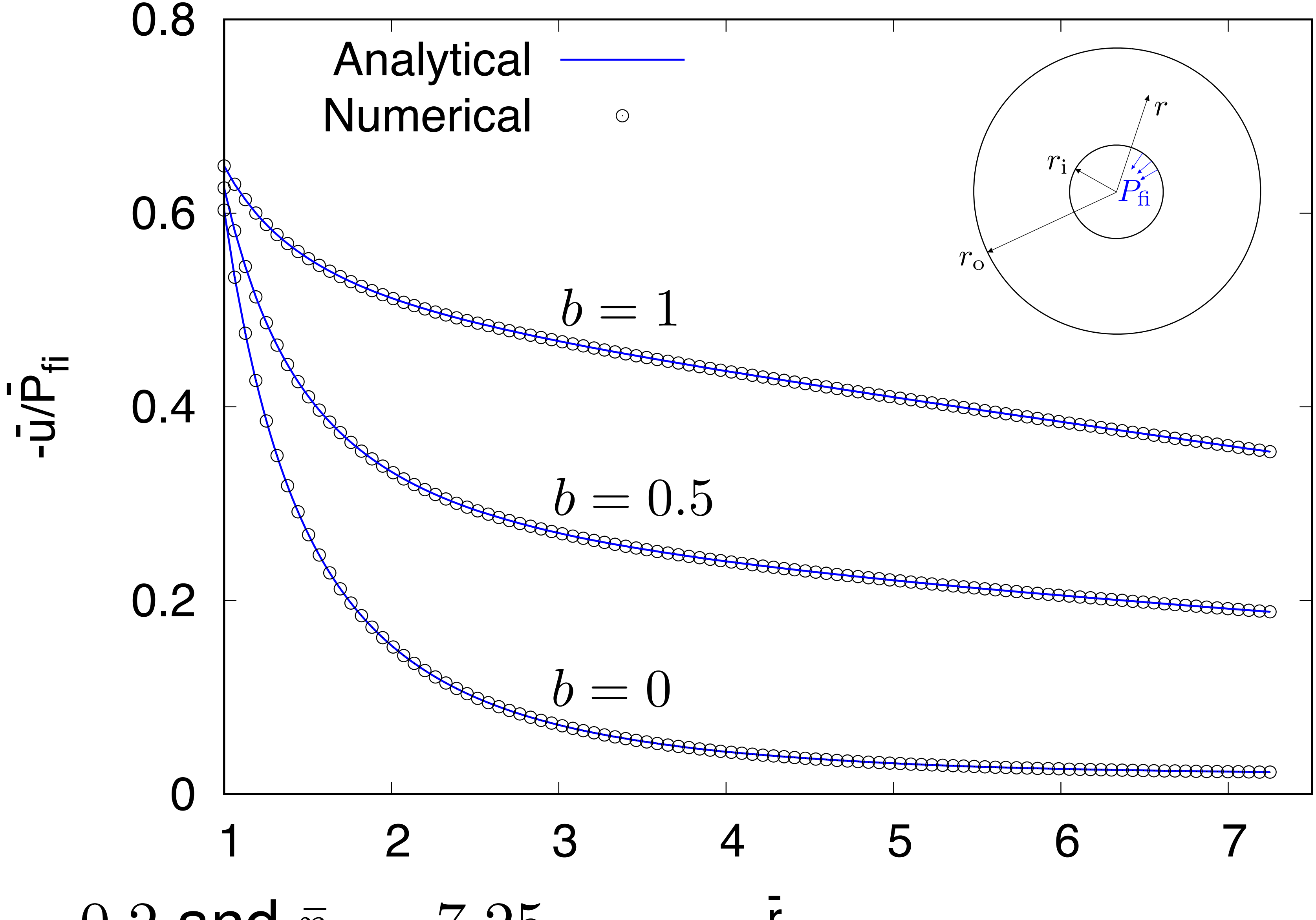
$$\bar{u} = \frac{u}{r_{\mathrm{i}}}$$

Solve analytically and numerically for boundary conditions:

$\bar{\sigma}_{\mathrm{r}}^{\mathrm{m}} = (1-b)\bar{P}_{\mathrm{fi}}$ at $\bar{r} = \bar{r}_{\mathrm{i}}$

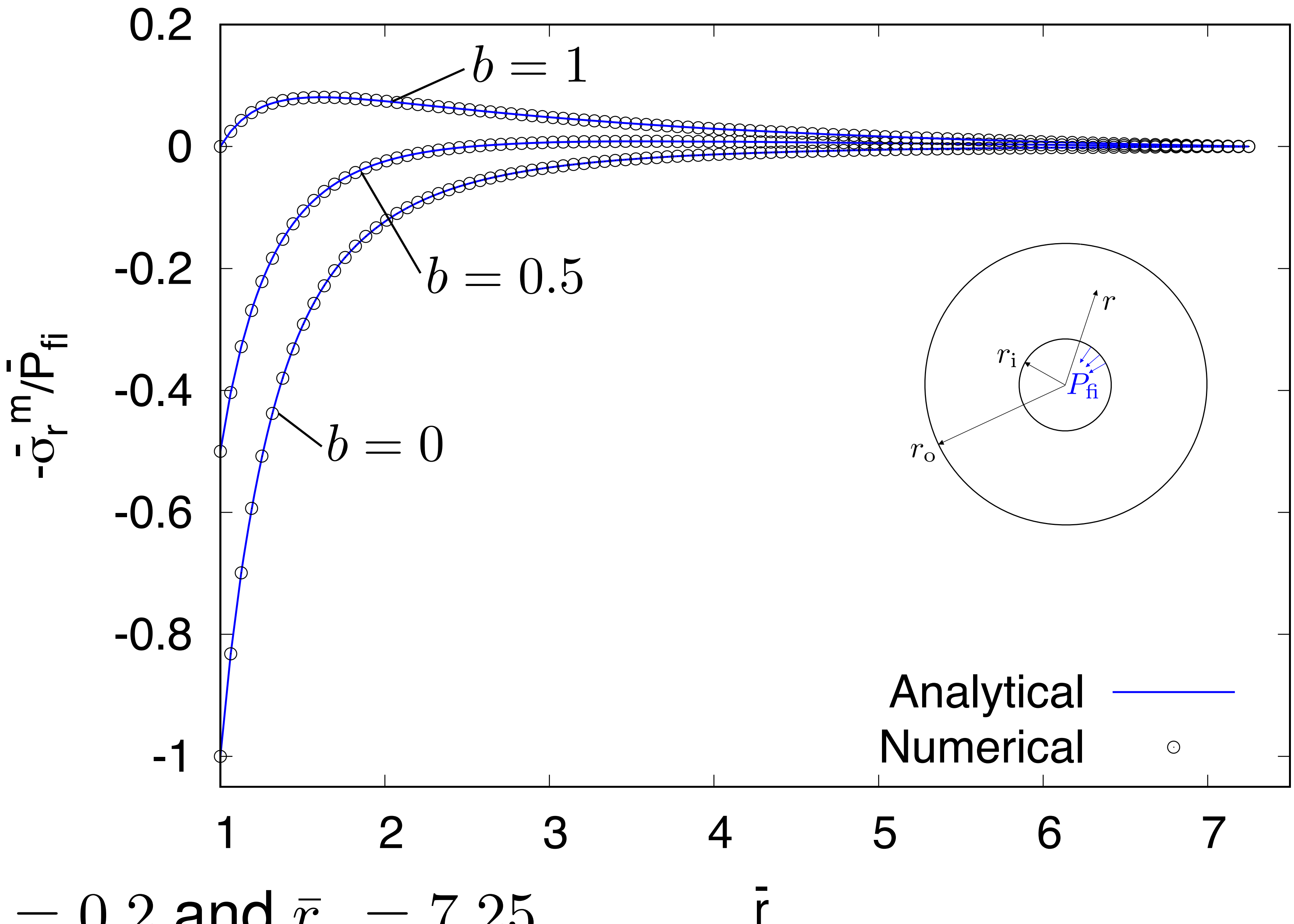
$\bar{\sigma}_{\mathrm{r}}^{\mathrm{m}} = 0$ at $\bar{r} = \bar{r}_{\mathrm{o}}$

Radial displacement versus radius



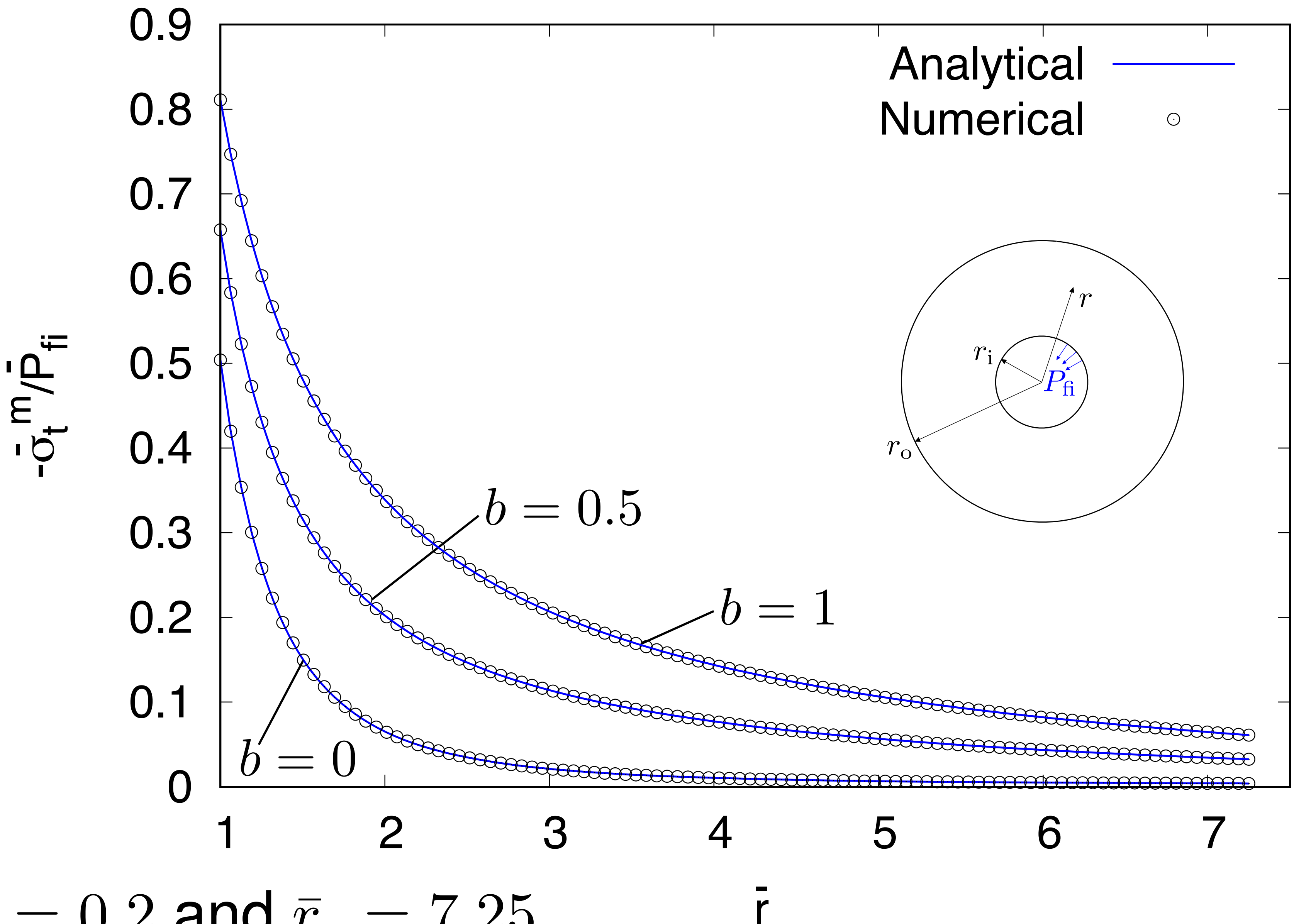
$\nu = 0.2$ and $\bar{r}_\mathrm{o} = 7.25$

Radial stress versus radius



$\nu = 0.2$ and $\bar{r}_o = 7.25$

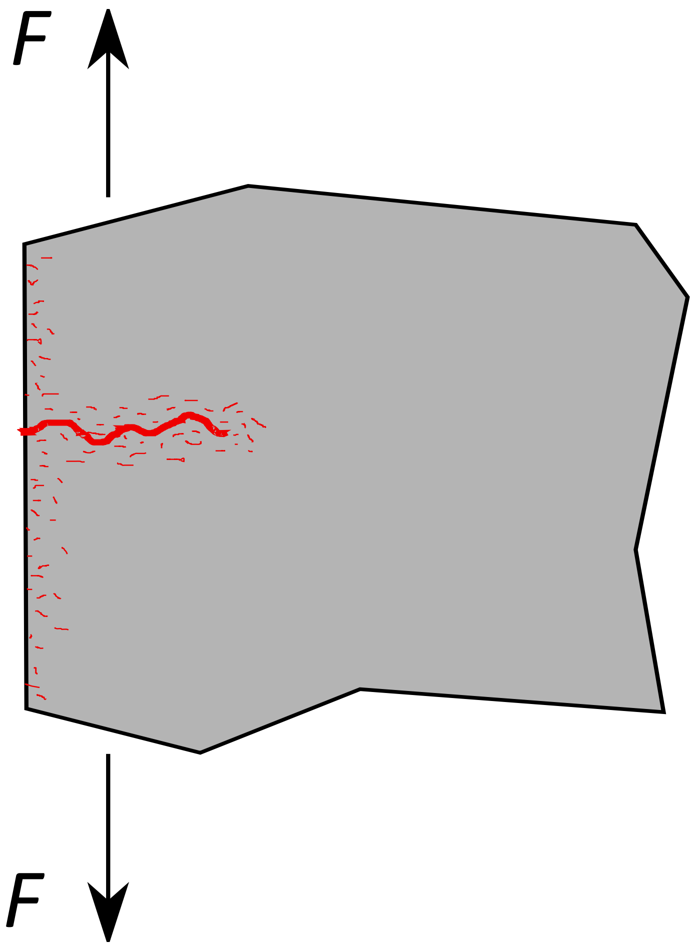
Tangential stress versus radius



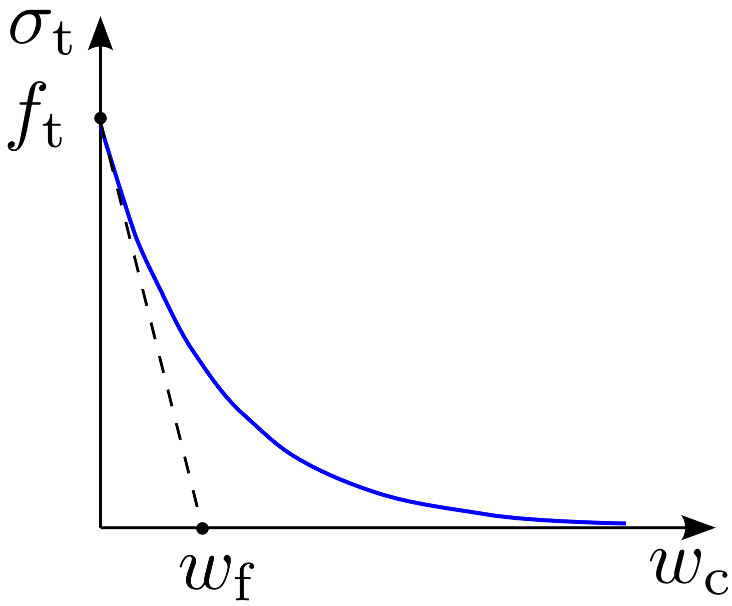
$\nu = 0.2$ and $\bar{r}_o = 7.25$

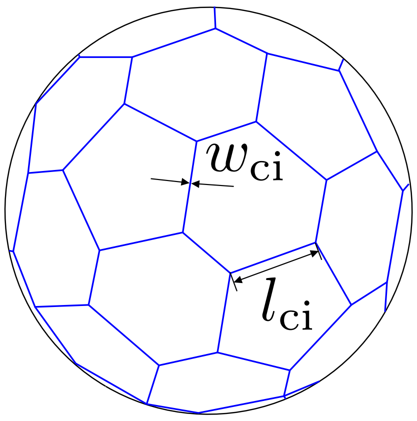
Nonlinear fracture response

Nonlinear fracture mechanics



Nonlinear fracture mechanics



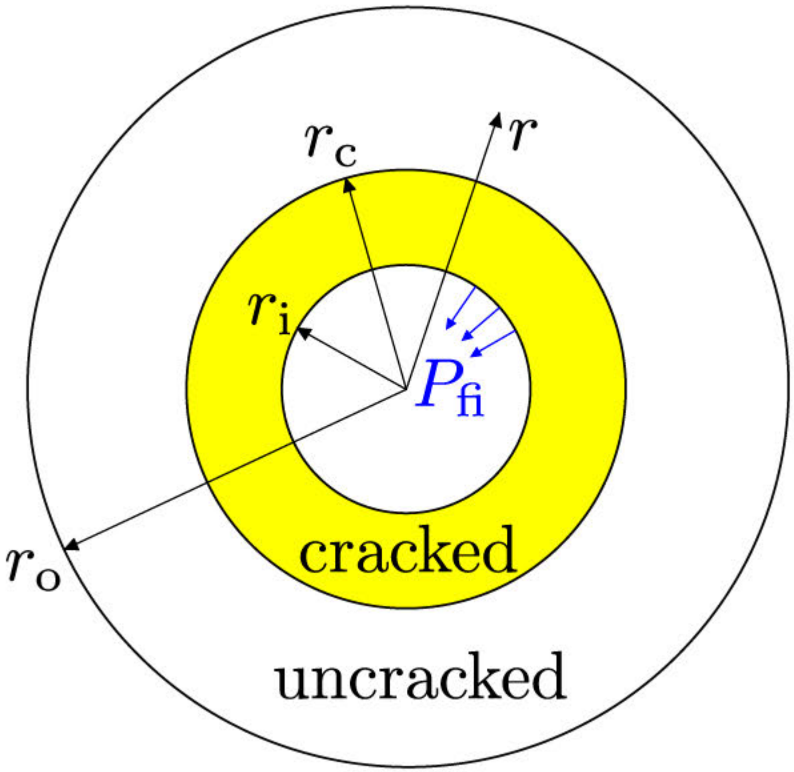


Possible fracture pattern

$$\bar{\sigma}_{\mathrm{t}}^{\mathrm{m}} = \varepsilon_0 \exp\left(-\frac{\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{\varepsilon_{\mathrm{f}}}\right) \quad \text{with} \quad \varepsilon_0 = f_{\mathrm{t}}/E$$

$$\varepsilon_{\mathrm{f}} = w_{\mathrm{f}}\frac{l_{\mathrm{c}}}{2S_{\mathrm{c}}} = w_{\mathrm{f}}\frac{\alpha}{r} = \bar{w}_{\mathrm{f}}\frac{\alpha}{\bar{r}} \quad \text{with} \quad \bar{w}_{\mathrm{f}} = \frac{w_{\mathrm{f}}}{r_{\mathrm{i}}}$$

Constitutive law for cracking



$$\varepsilon_r = \frac{1}{E}\left(\sigma_r^m - 2\nu\sigma_t^m\right)$$

$$\varepsilon_t = \varepsilon_t^e + \varepsilon_t^c$$

$$\varepsilon_t^e = \frac{1}{E}\left(\left(1-\nu\right)\sigma_t^m - \nu\sigma_r^m\right)$$

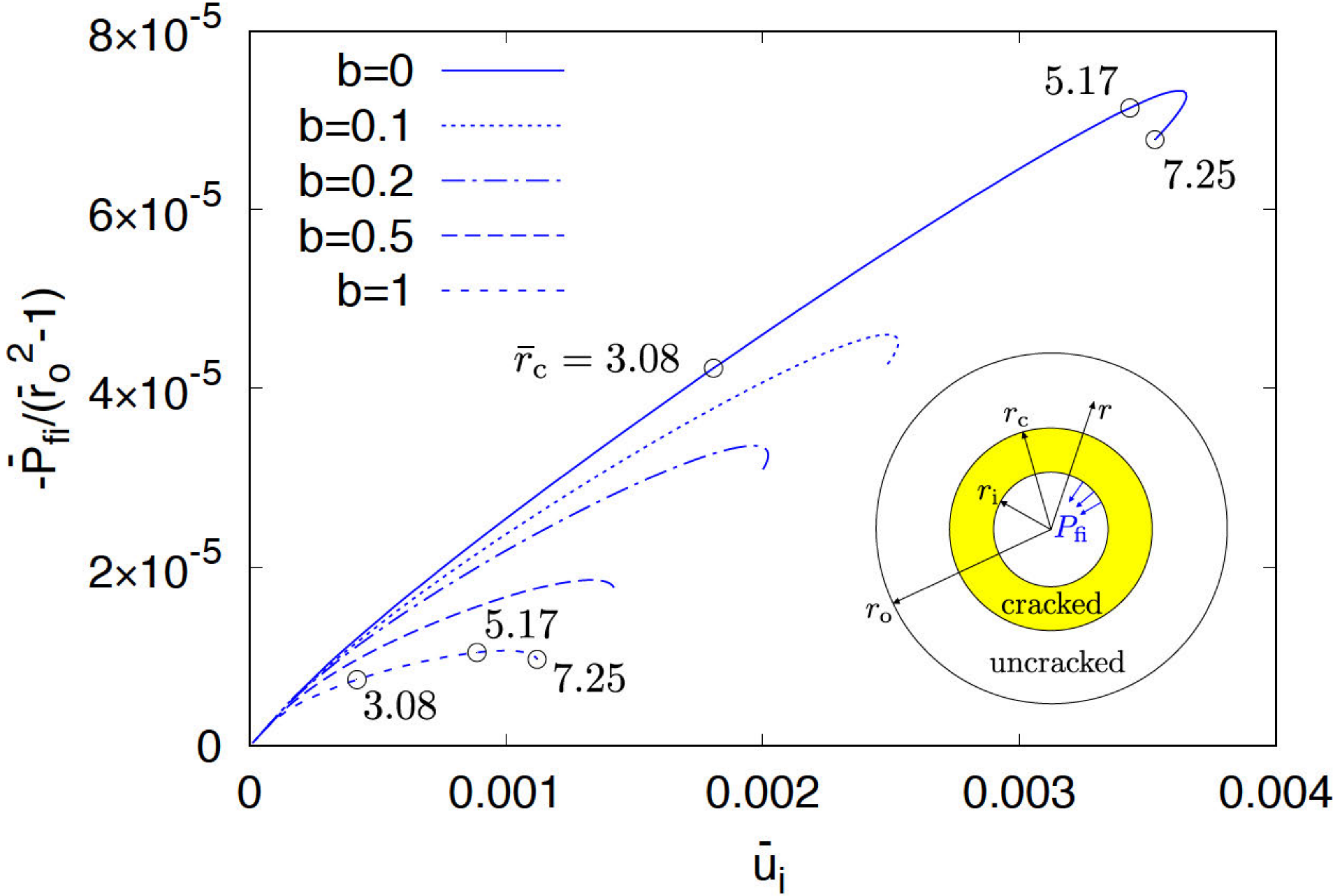
Nonlinear ODE

$$\frac{d^2\bar{u}}{d\bar{r}^2} + 2\frac{d\bar{u}}{d\bar{r}}\frac{1}{\bar{r}} - 2\frac{\bar{u}}{\bar{r}^2} - \frac{2\nu}{1-\nu}\frac{d\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{d\bar{r}} + \frac{2(1-2\nu)}{1-\nu}\frac{\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{\bar{r}} + $$
$$+ b\frac{P_{\mathrm{fi}}}{E}\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar{r}_{\mathrm{o}}}{1-\bar{r}_{\mathrm{o}}}\frac{1}{\bar{r}^2} = 0$$

Numerical solution:

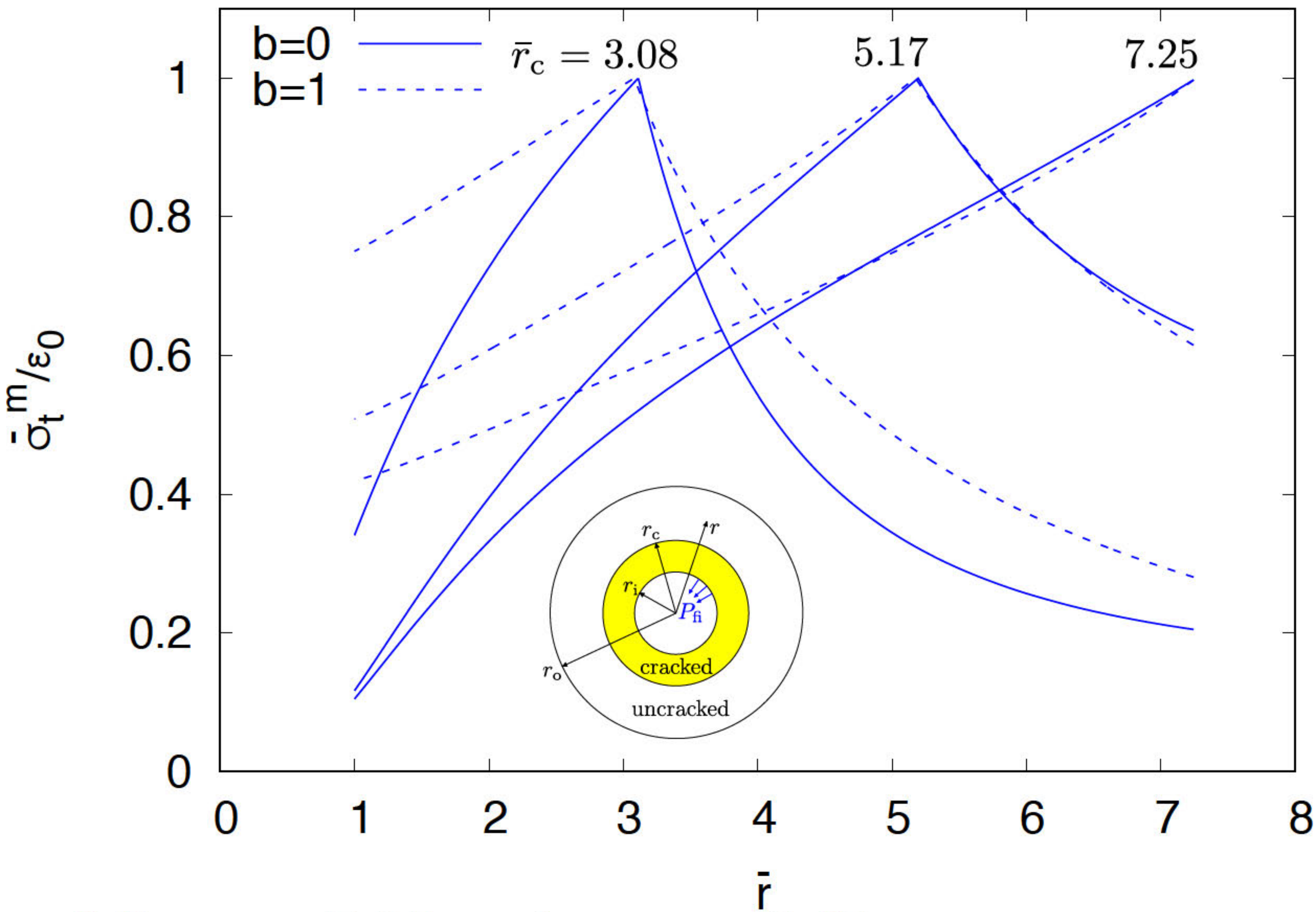
- Finite difference scheme
- Shooting method
- Newton method
- Outer displacement control

Pressure versus inner displacement



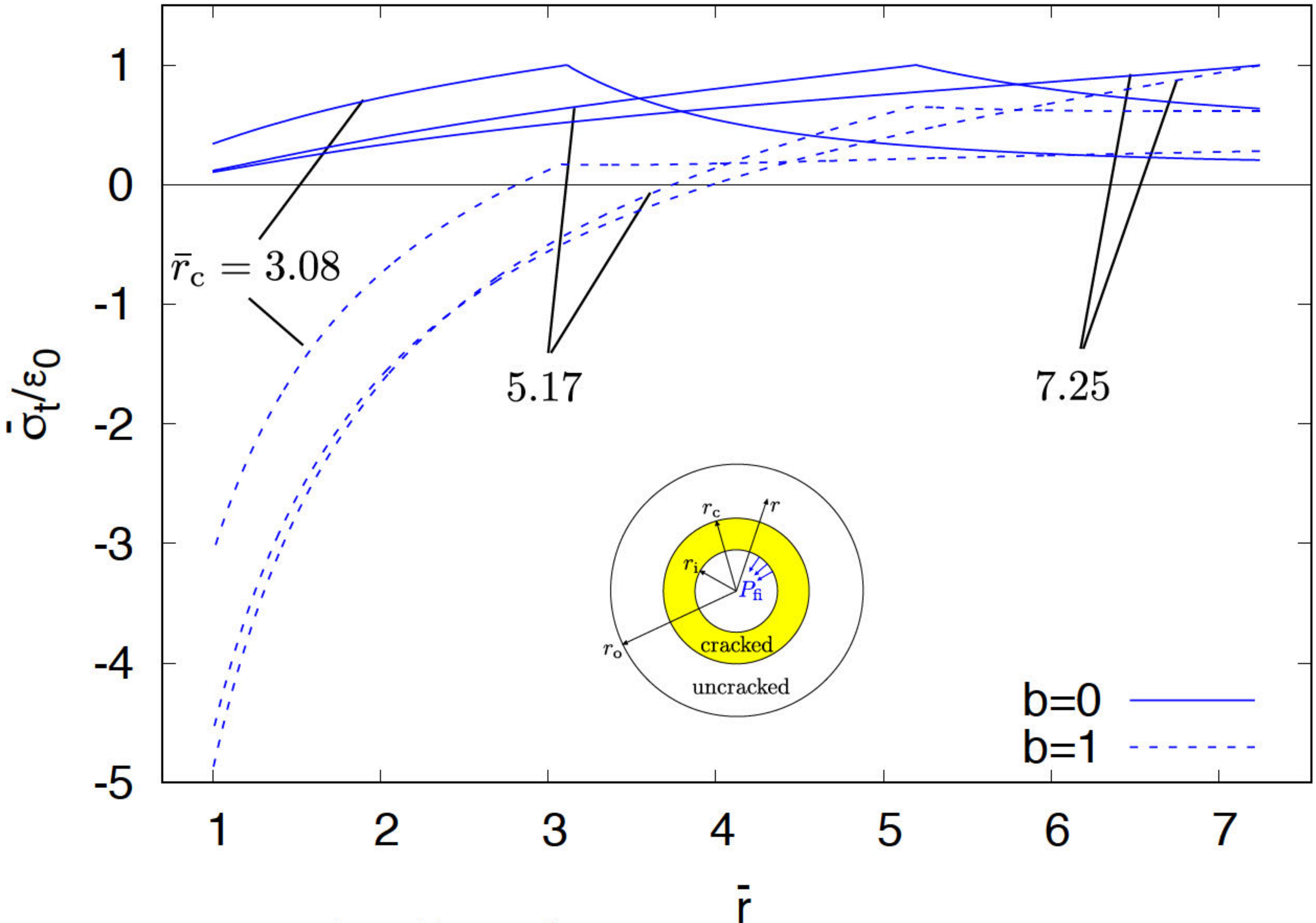
$\nu = 0.2$, $\bar{r}_\mathrm{o} = 7.25$ and $\alpha \bar{w}_\mathrm{f} = 0.01$

Tangential effective stress versus radius



$\nu = 0.2$, $\bar{r}_o = 7.25$ and $\alpha \bar{w}_f = 0.01$

Tangential stress versus radius



$\nu = 0.2$, $\bar{r}_o = 7.25$ and $\alpha \bar{w}_f = 0.01$

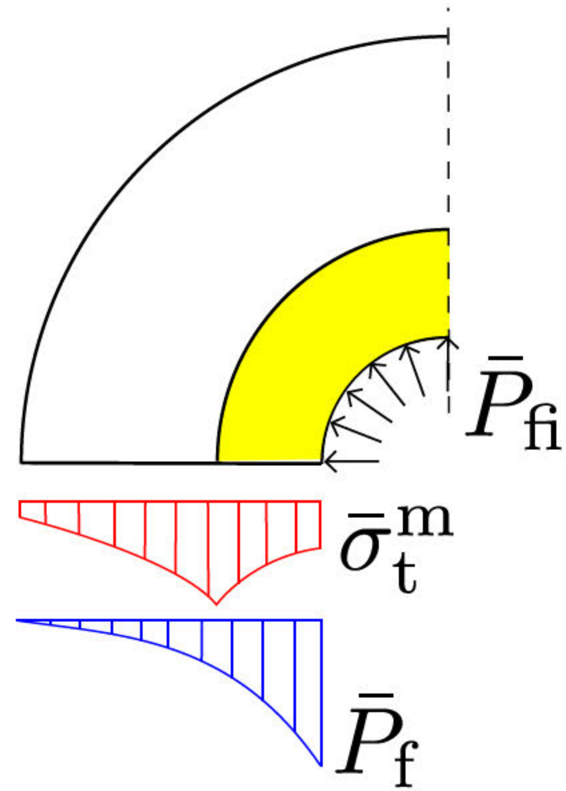
Size effect

Size effect

Investigate effect of inner radius r_i on strength for constant $\bar{r}_\mathrm{o} = r_\mathrm{o}/r_\mathrm{i}$

Dimensionless input affected by change of r_i:

$$\bar{w}_\mathrm{f} = \frac{w_\mathrm{f}}{r_\mathrm{i}} \quad \text{with} \quad w_\mathrm{f} = \mathrm{const}$$



Equilibrium

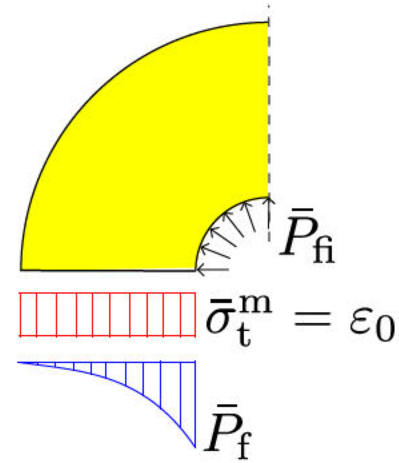
$$\bar{P}_\mathrm{fi} = -2\int_1^{\bar{r}_\mathrm{o}} (\bar{\sigma}_\mathrm{t}^\mathrm{m} + b\bar{P}_\mathrm{f})\,\bar{r}\,\mathrm{d}\bar{r}$$

Limits

Plastic limit:

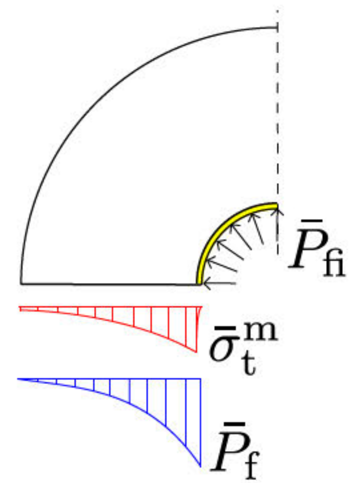
$$r_{\rm i} \to 0 \;\Rightarrow\; \bar{w}_{\rm f} = \frac{w_{\rm f}}{r_{\rm i}} \to \infty$$

$$\frac{\bar{P}_{\rm fi,pl}}{(\bar{r}_{\rm o}^2 - 1)} = -\frac{\varepsilon_0}{1 + b\,(\bar{r}_{\rm o} - 1)}$$



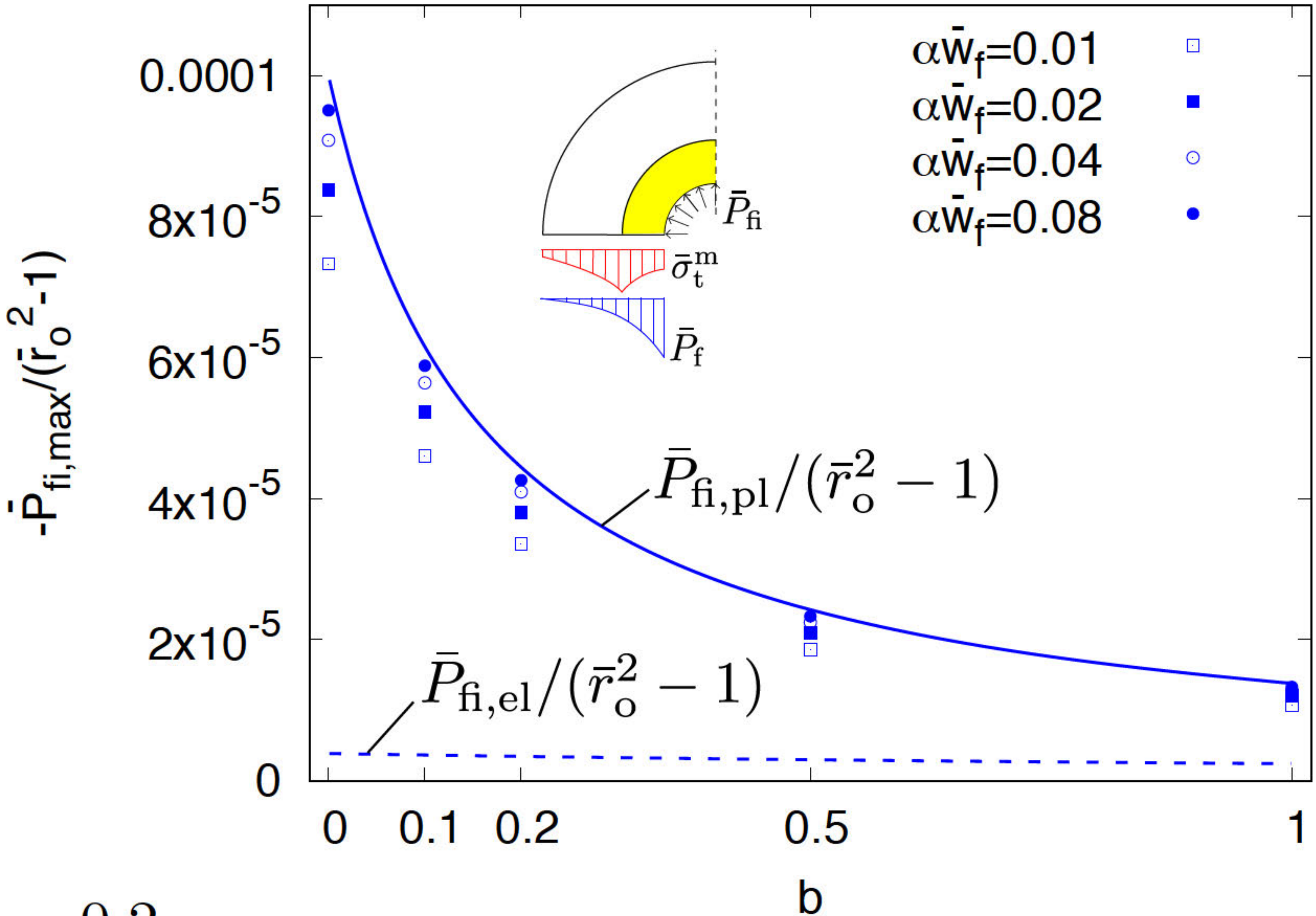
Onset of cracking:

$$r_{\rm i} \to \infty \;\Rightarrow\; \bar{w}_{\rm f} = \frac{w_{\rm f}}{r_{\rm i}} \to 0$$



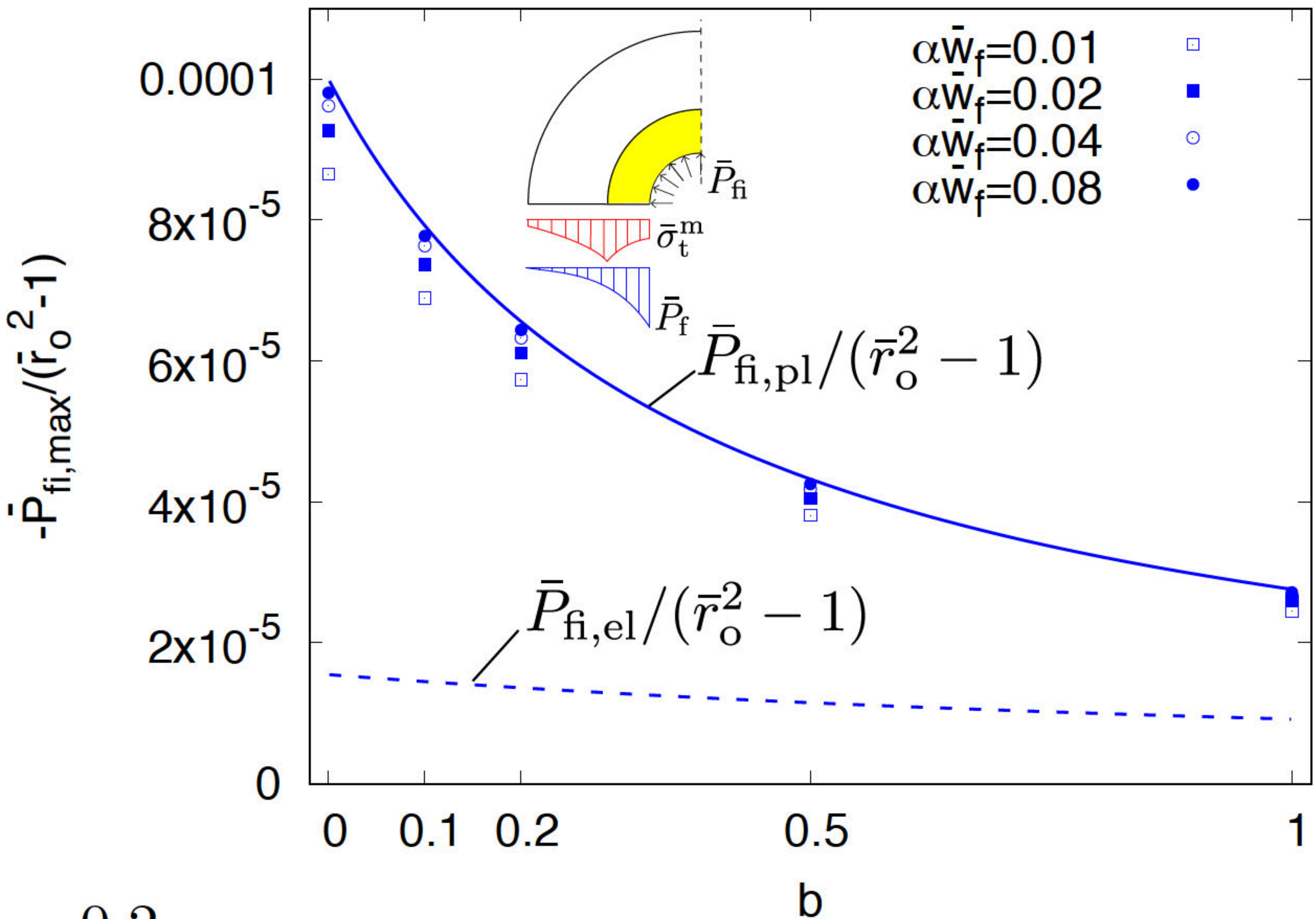
$$\frac{\bar{P}_{\rm fi,el}}{\bar{r}_{\rm o}^2 - 1} = -\frac{1}{\bar{r}_{\rm o}^2 - 1}\,\frac{2\varepsilon_0}{\left(1 - b\dfrac{1-2\nu}{1-\nu}\right)\dfrac{\bar{r}_{\rm o}^3 + 2}{\bar{r}_{\rm o}^3 - 1} + b\dfrac{1}{1-\nu}\,\dfrac{\bar{r}_{\rm o} - 2\nu}{\bar{r}_{\rm o} - 1}}$$

Size effect for $\bar{r}_o = 7.25$



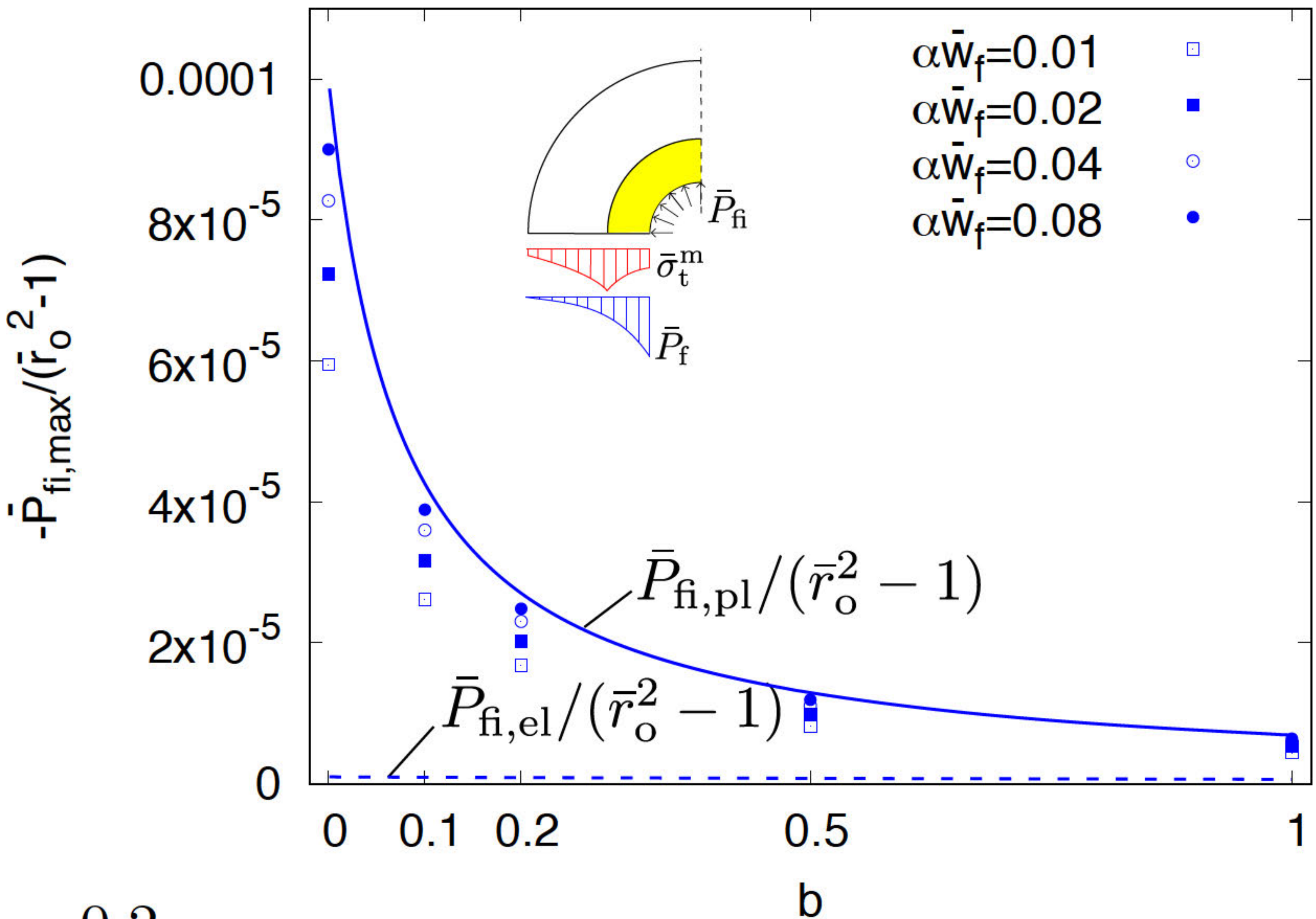
$\nu = 0.2$

Size effect for $\bar{r}_o = 3.125$



$\nu = 0.2$

Size effect for $\bar{r}_\mathrm{o} = 14.5$



$\nu = 0.2$

Conclusions

- Model for fluid driven fracture in a thick-walled sphere based on nonlinear fracture mechanics
- Strong effect of Biot-coefficient on strength
- Strong effect of size on strength, which decreases with increasing Biot-coefficient and decreasing thickness of the sphere.