Hydraulic fracture of a permeable thick-walled hollow sphere

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ECCM-ECFD 2018, Glasgow

Background

Hydraulic fracture processes in permeable geomaterials (concrete, rock, stiff soils) with stationary fluid flow.

Aim

Study the importance of Biot coefficient for hydraulic fracture processes.

Methodology

Investigate initial fracture (hardening) of permeable thick-walled hollow sphere subjected to inner fluid pressure.

Assumptions and notations

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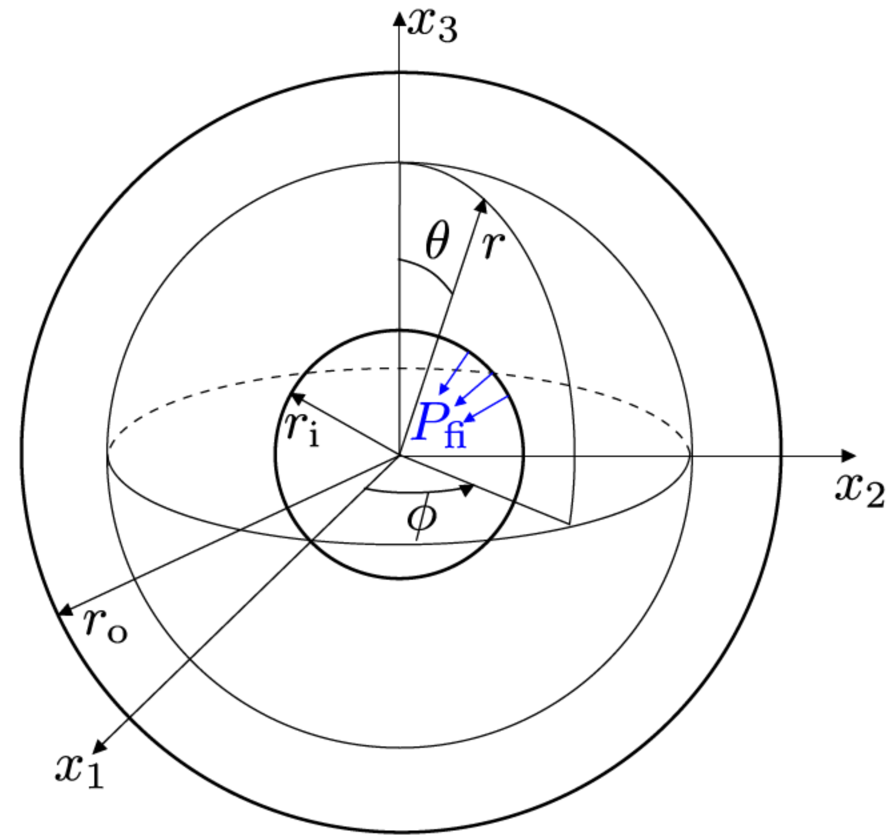
- Spherical symmetry
- Stationary flow
- Constant viscosity and permeability
- Fluid is incompressible
- Small displacements
- Influence of gravity neglected



P_{fi} tension positive

Fluid driven loading

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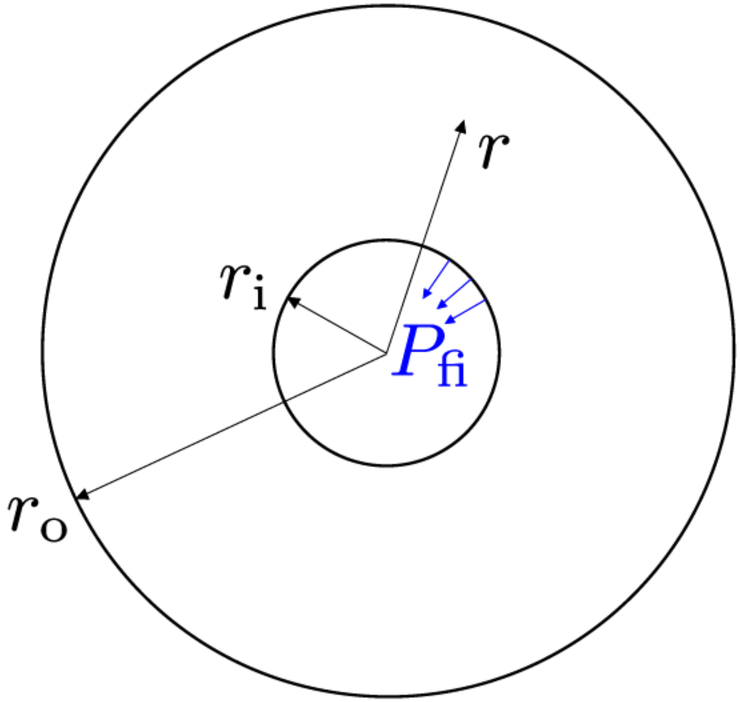


$$\dot{V} = \dot{V}_\mathrm{i} + Q \qquad \dot{V}_\mathrm{i} = \dot{u}_\mathrm{i} 4\pi r_\mathrm{i}^2$$

$$q = \frac{Q}{4\pi r^2} \qquad q = \frac{\kappa}{\mu}\frac{dP_\mathrm{f}}{dr}$$

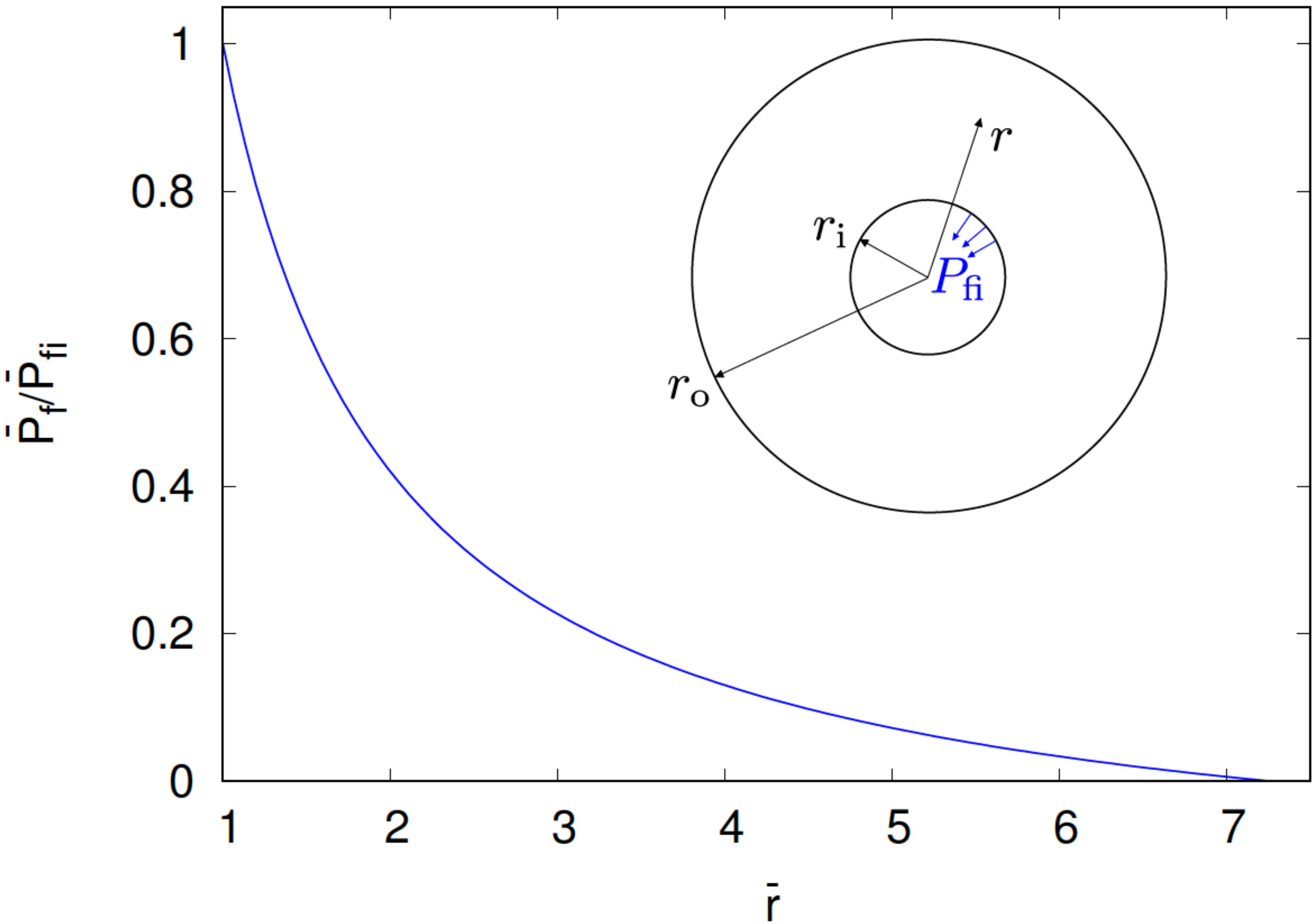
Pressure distribution

$$\bar{P}_\mathrm{f} = \bar{P}_\mathrm{fi} \frac{\bar{r} - \bar{r}_\mathrm{o}}{\bar{r}\left(1 - \bar{r}_\mathrm{o}\right)}$$

with dimensionless variables

$$\bar{P}_\mathrm{fi} = P_\mathrm{fi}/E \qquad \bar{P}_\mathrm{f} = P_\mathrm{f}/E \qquad \bar{r} = r/r_\mathrm{i} \qquad \bar{r}_\mathrm{o} = r_\mathrm{o}/r_\mathrm{i}$$

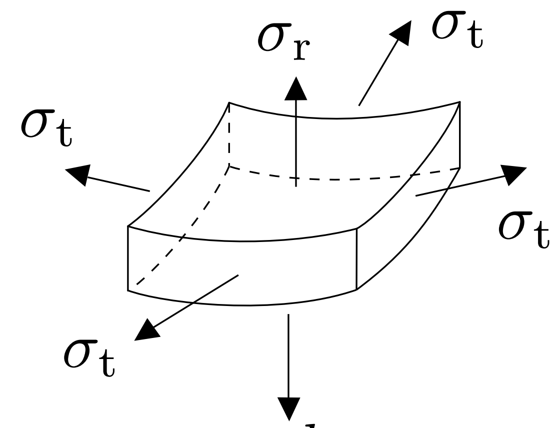
Pressure versus radius



Elastic response

Equilibrium condition

$$\frac{d\sigma_{\mathrm{r}}}{dr} + 2\frac{\sigma_{\mathrm{r}} - \sigma_{\mathrm{t}}}{r} = 0$$



Stress definition

$$\sigma_{\mathrm{r}} = \sigma_{\mathrm{r}}^{\mathrm{m}} + bP_{\mathrm{f}}$$

$$\sigma_{\mathrm{t}} = \sigma_{\mathrm{t}}^{\mathrm{m}} + bP_{\mathrm{f}}$$

$$\sigma_{\theta} = \sigma_{\phi} = \sigma_{\mathrm{t}}$$

Constitutive laws

$$\varepsilon_{\mathrm{r}} = \frac{1}{E}\left(\sigma_{\mathrm{r}}^{\mathrm{m}} - 2\nu\sigma_{\mathrm{t}}^{\mathrm{m}}\right)$$

$$\varepsilon_{\mathrm{t}} = \frac{1}{E}\left((1-\nu)\sigma_{\mathrm{t}}^{\mathrm{m}} - \nu\sigma_{\mathrm{r}}^{\mathrm{m}}\right)$$

Kinematics

$$\varepsilon_{\mathrm{r}} = \frac{du}{dr} \qquad \varepsilon_{\mathrm{t}} = \frac{u}{r}$$

Timoshenko and Goodier (1970), Coussy (2010)

Dimensionless ODE

$$\frac{d^2\bar{u}}{d\bar{r}^2} + 2\frac{d\bar{u}}{d\bar{r}}\frac{1}{\bar{r}} - 2\frac{\bar{u}}{\bar{r}^2} + b\bar{P}_{\mathrm{fi}}\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar{r}_{\mathrm{o}}}{1-\bar{r}_{\mathrm{o}}}\frac{1}{\bar{r}^2} = 0$$

with dimensionless radial displacement

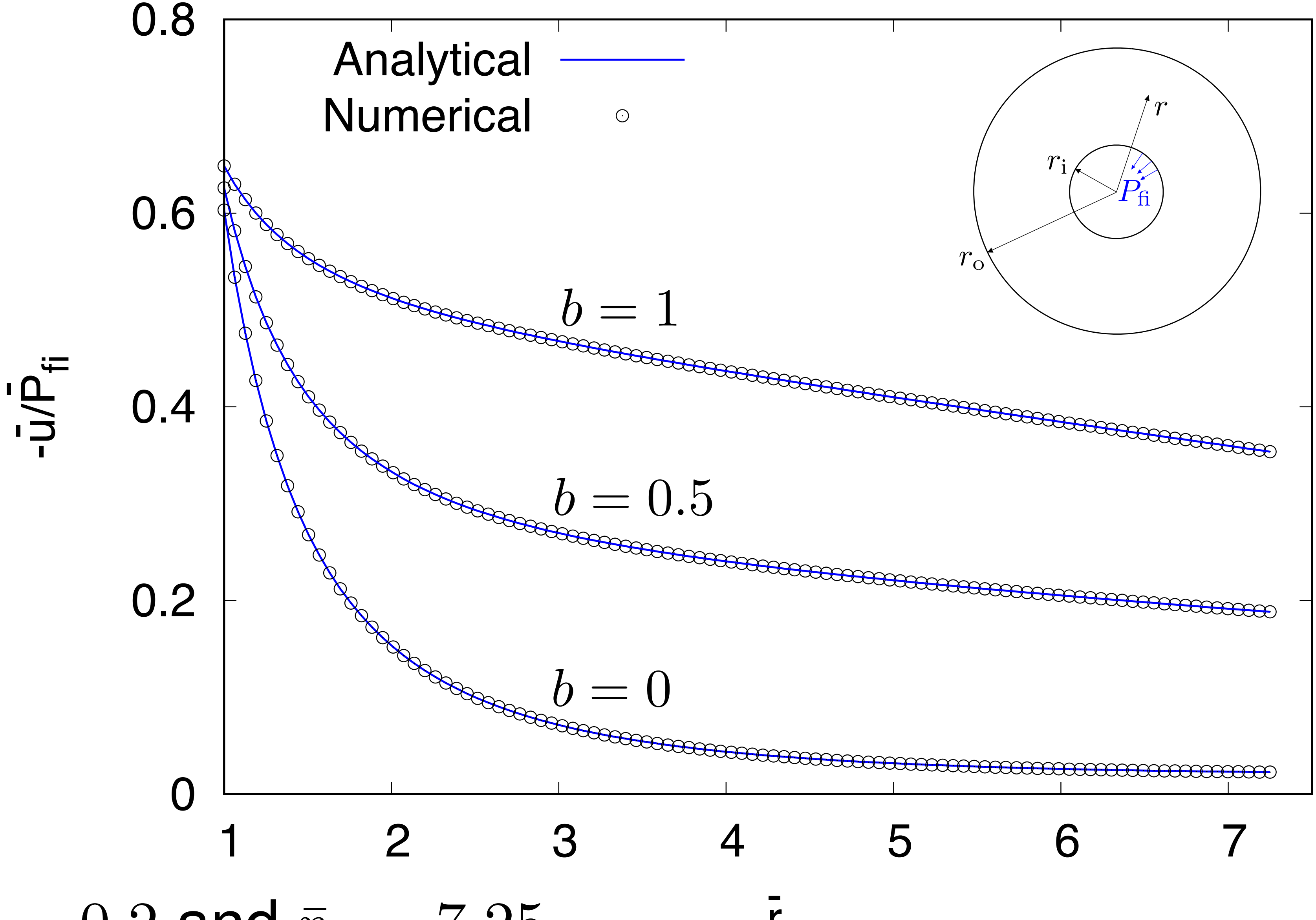
$$\bar{u} = \frac{u}{r_{\mathrm{i}}}$$

Solve analytically and numerically for boundary conditions:

$\bar{\sigma}_{\mathrm{r}}^{\mathrm{m}} = (1-b)\bar{P}_{\mathrm{fi}}$ at $\bar{r} = \bar{r}_{\mathrm{i}}$

$\bar{\sigma}_{\mathrm{r}}^{\mathrm{m}} = 0$ at $\bar{r} = \bar{r}_{\mathrm{o}}$

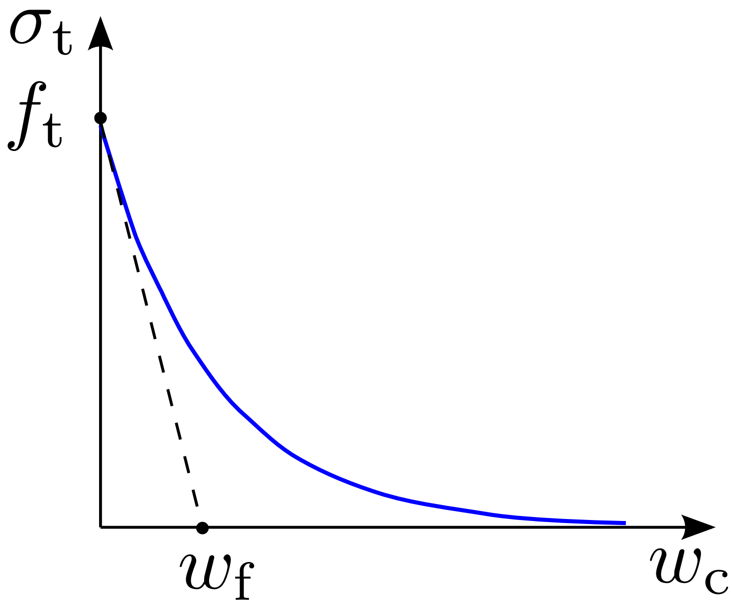
Radial displacement versus radius

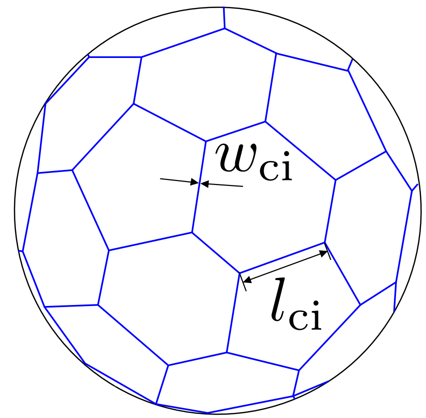


$\nu = 0.2$ and $\bar{r}_\mathrm{o} = 7.25$

Nonlinear fracture response

Nonlinear fracture mechanics



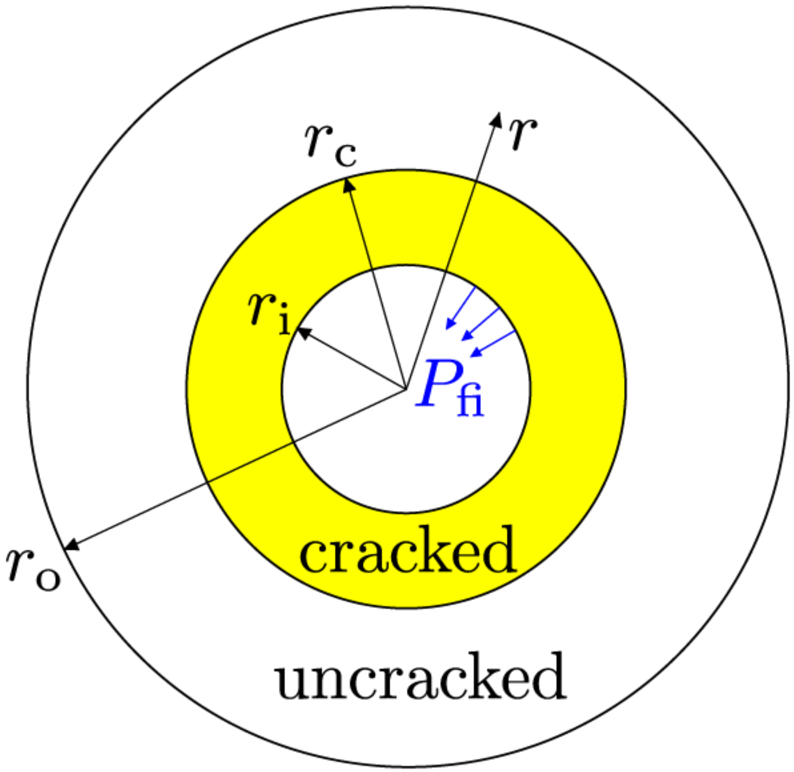


Possible fracture pattern

$$\bar{\sigma}_{\mathrm{t}}^{\mathrm{m}} = \varepsilon_0 \exp\left(-\frac{\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{\varepsilon_{\mathrm{f}}}\right) \quad \text{with} \quad \varepsilon_0 = f_{\mathrm{t}}/E$$

$$\varepsilon_{\mathrm{f}} = w_{\mathrm{f}}\frac{l_{\mathrm{c}}}{2S_{\mathrm{c}}} = w_{\mathrm{f}}\frac{\alpha}{r} = \bar{w}_{\mathrm{f}}\frac{\alpha}{\bar{r}} \quad \text{with} \quad \bar{w}_{\mathrm{f}} = \frac{w_{\mathrm{f}}}{r_{\mathrm{i}}}$$

Constitutive law for cracking



$$\varepsilon_r = \frac{1}{E}\left(\sigma_r^m - 2\nu\sigma_t^m\right)$$

$$\varepsilon_t = \varepsilon_t^e + \varepsilon_t^c$$

$$\varepsilon_t^e = \frac{1}{E}\left(\left(1-\nu\right)\sigma_t^m - \nu\sigma_r^m\right)$$

Nonlinear ODE

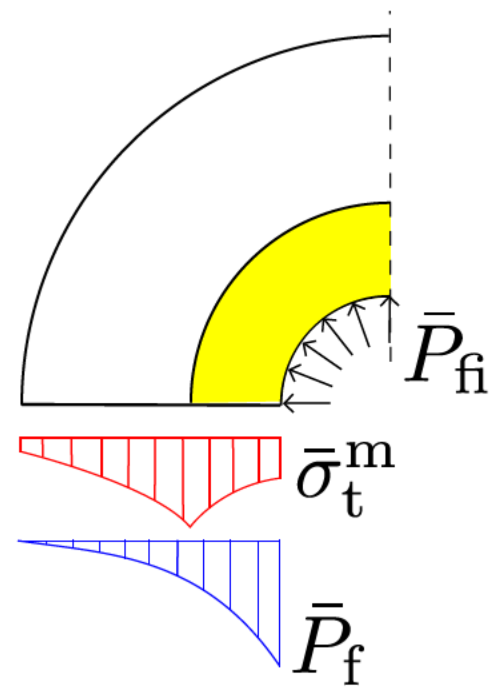
$$\frac{d^2\bar{u}}{d\bar{r}^2} + 2\frac{d\bar{u}}{d\bar{r}}\frac{1}{\bar{r}} - 2\frac{\bar{u}}{\bar{r}^2} - \frac{2\nu}{1-\nu}\frac{d\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{d\bar{r}} + \frac{2(1-2\nu)}{1-\nu}\frac{\varepsilon_{\mathrm{t}}^{\mathrm{c}}}{\bar{r}} +$$

$$+ b\frac{P_{\mathrm{fi}}}{E}\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar{r}_{\mathrm{o}}}{1-\bar{r}_{\mathrm{o}}}\frac{1}{\bar{r}^2} = 0$$

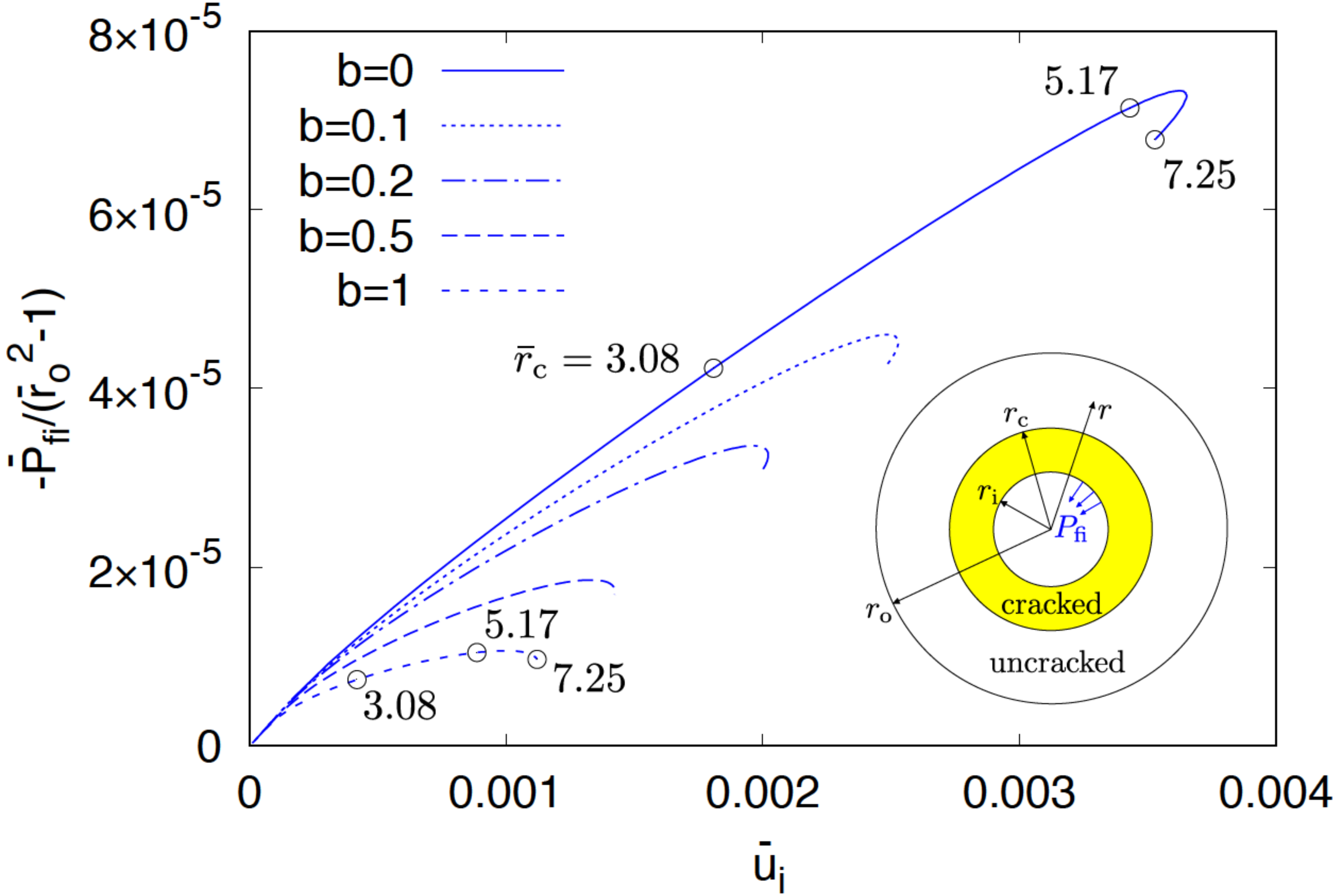
Equilibrium:

$$\bar{P}_{\mathrm{fi}} = -2\int_1^{\bar{r}_{\mathrm{o}}} (\bar{\sigma}_{\mathrm{t}}^{\mathrm{m}} + b\bar{P}_{\mathrm{f}})\,\bar{r}\,\mathrm{d}\bar{r}$$

Show $\dfrac{\bar{P}_{\mathrm{fi}}}{\bar{r}_{\mathrm{o}}^2 - 1}$ versus $\bar{u}_{\mathrm{i}}$

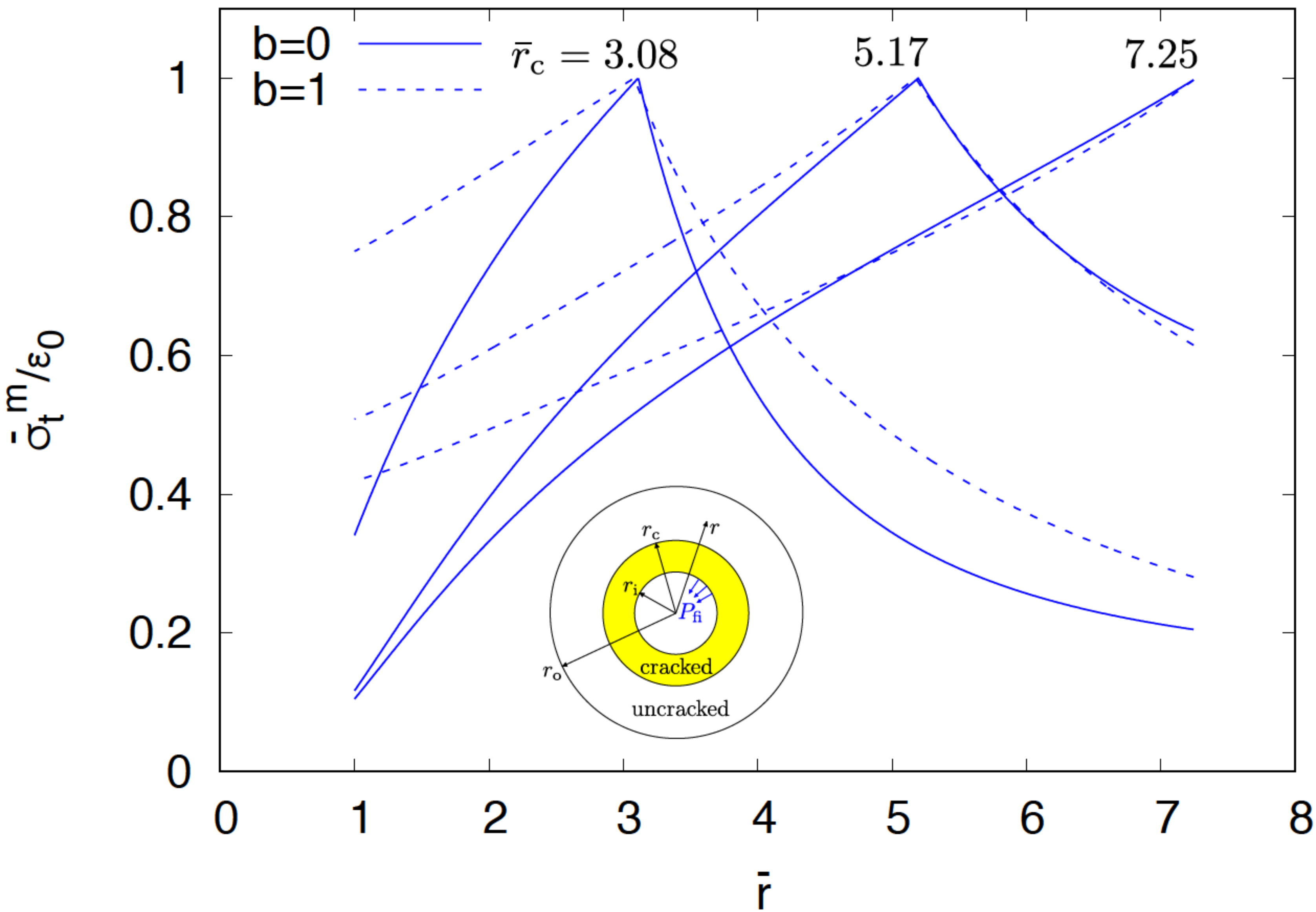


Pressure versus inner displacement



$\nu = 0.2$, $\bar{r}_\mathrm{o} = 7.25$ and $\alpha \bar{w}_\mathrm{f} = 0.01$

Tangential effective stress versus radius



$\nu = 0.2$, $\bar{r}_o = 7.25$ and $\alpha \bar{w}_f = 0.01$

Size effect

Size effect

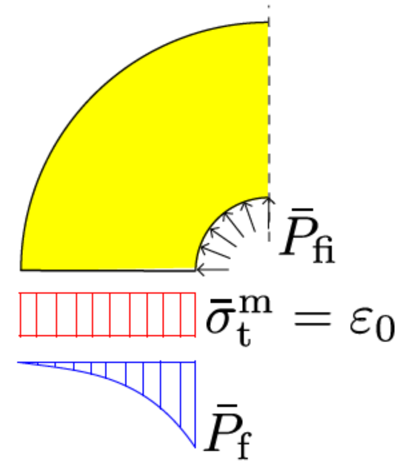
Investigate effect of inner radius r_{i} on strength for constant $\bar{r}_{\mathrm{o}} = r_{\mathrm{o}}/r_{\mathrm{i}}$

Dimensionless input affected by change of r_{i}:

$\bar{w}_{\mathrm{f}} = \dfrac{w_{\mathrm{f}}}{r_{\mathrm{i}}}$ with $w_{\mathrm{f}} = \mathrm{const}$

Limits

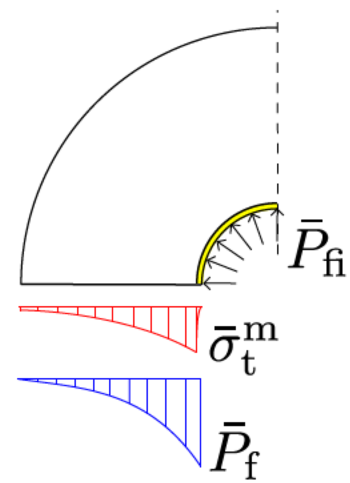
Plastic limit:



$$r_{\mathrm{i}} \to 0 \;\Rightarrow\; \bar{w}_{\mathrm{f}} = \frac{w_{\mathrm{f}}}{r_{\mathrm{i}}} \to \infty$$

$$\frac{\bar{P}_{\mathrm{fi,pl}}}{(\bar{r}_{\mathrm{o}}^2 - 1)} = -\frac{\varepsilon_0}{1 + b\,(\bar{r}_{\mathrm{o}} - 1)}$$

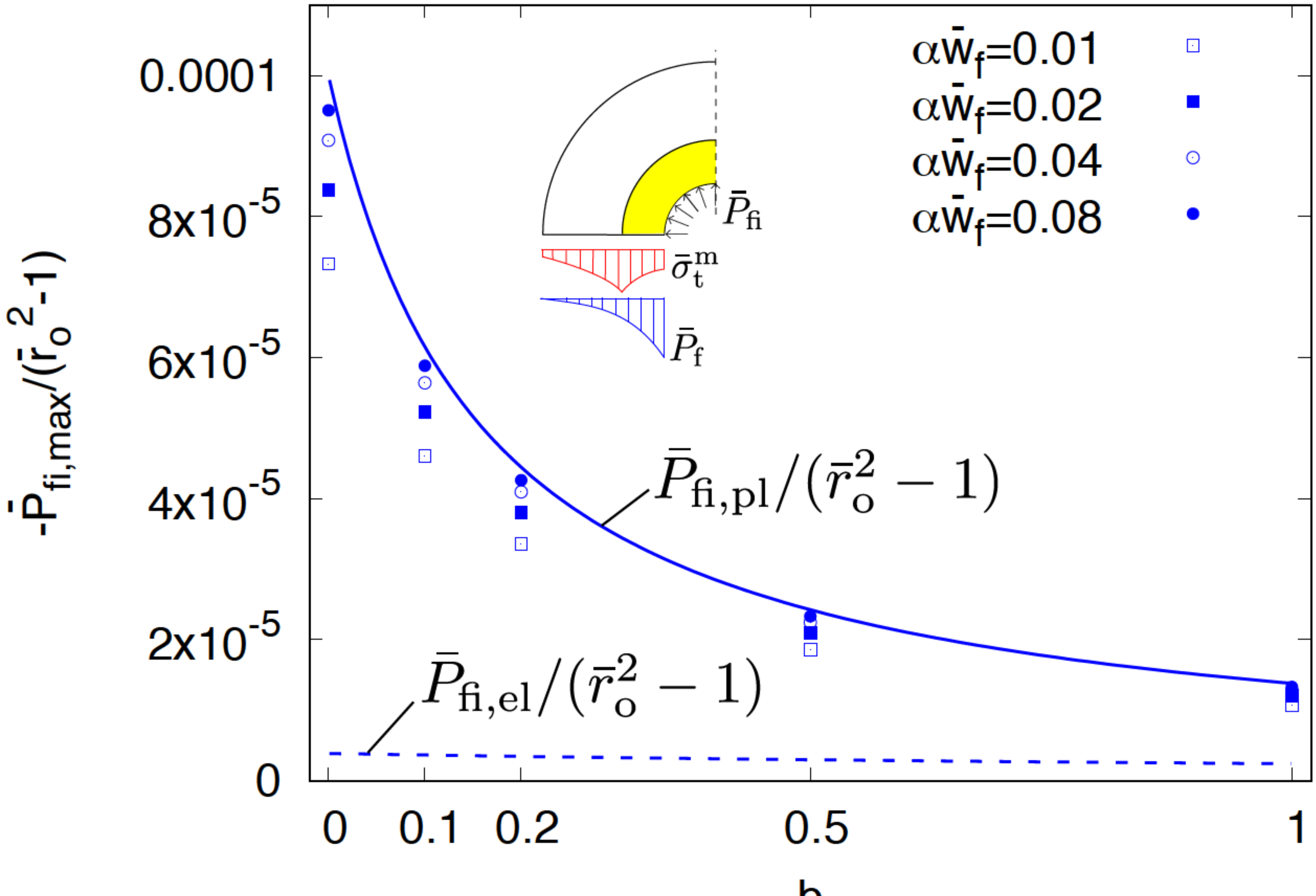
Onset of cracking:



$$r_{\mathrm{i}} \to \infty \;\Rightarrow\; \bar{w}_{\mathrm{f}} = \frac{w_{\mathrm{f}}}{r_{\mathrm{i}}} \to 0$$

$$\frac{\bar{P}_{\mathrm{fi,el}}}{\bar{r}_{\mathrm{o}}^2 - 1} = -\frac{1}{\bar{r}_{\mathrm{o}}^2 - 1} \frac{2\varepsilon_0}{\left(1 - b\frac{1-2\nu}{1-\nu}\right)\frac{\bar{r}_{\mathrm{o}}^3 + 2}{\bar{r}_{\mathrm{o}}^3 - 1} + b\frac{1}{1-\nu}\frac{\bar{r}_{\mathrm{o}} - 2\nu}{\bar{r}_{\mathrm{o}} - 1}}$$

Size effect for $\bar{r}_{\mathrm{o}} = 7.25$



$\nu = 0.2$

Conclusions

- Model for fluid driven fracture in a thick-walled sphere based on nonlinear fracture mechanics

- Strong effect of Biot-coefficient on strength

- Strong effect of size on strength, which decreases with increasing Biot-coefficient and decreasing thickness of the sphere.