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Hydro-mechanical lattice approach for hydraulic fracture

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Reference: Grassl, Fahy, Gallipoli and Wheeler (2015), Journal of the Mechanics and Physics of Solids, Vol. 75, Feb. 2015, Pages 104–118

Background

Hydraulic fracture: oil and gas extraction, failure of flood defense embankments, and earth and concrete dams, injection of sills and clastic dykes.

Aim

Propose a coupled hydro-mechanical lattice approach for modelling fracture in saturated porous materials.

Outline

2D lattice model

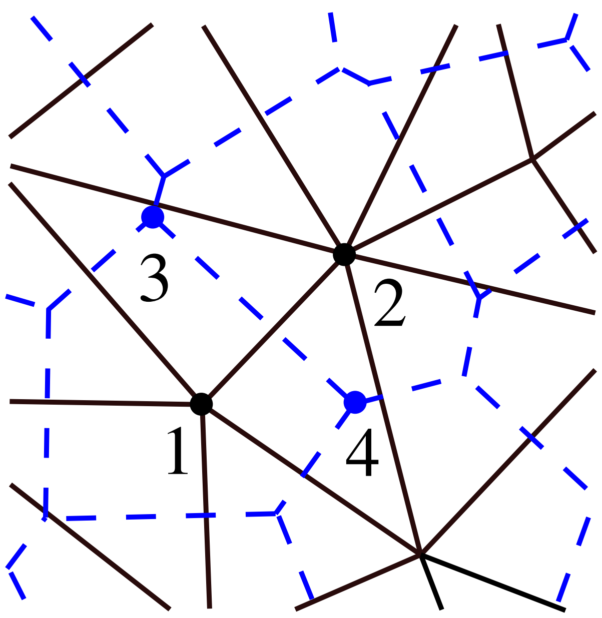
Elastic benchmark

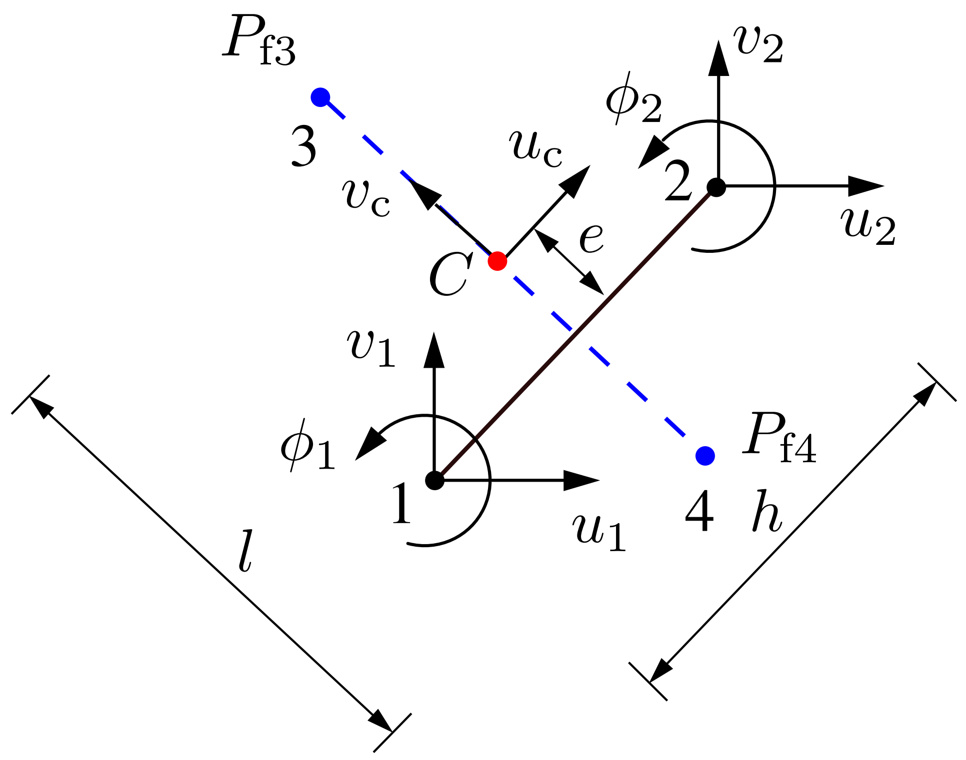
Fracture analysis

3D Extension

2D lattice model

Discretisation





References: Bolander and Saito (1998), Bolander and Berton (2004), Grassl (2009)

Coupled constitutive model

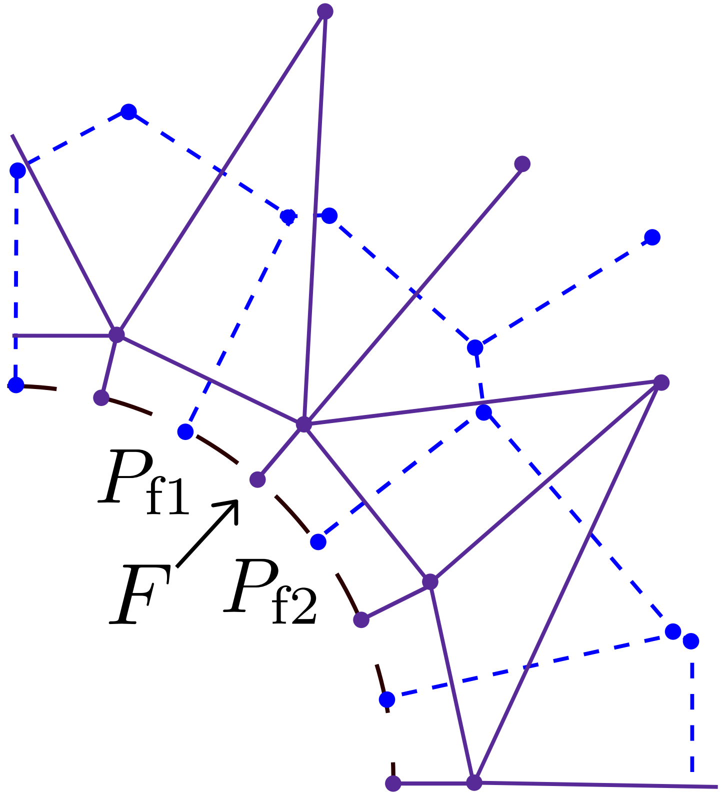
$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathrm{m}} + b\boldsymbol{\sigma}^{\mathrm{f}}$$

$$\boldsymbol{\sigma}^{\mathrm{m}} = (1-\omega)\mathbf{D}\boldsymbol{\varepsilon} \qquad \mathbf{q} = \frac{\kappa}{\mu}\mathrm{grad}P_{\mathrm{f}}$$

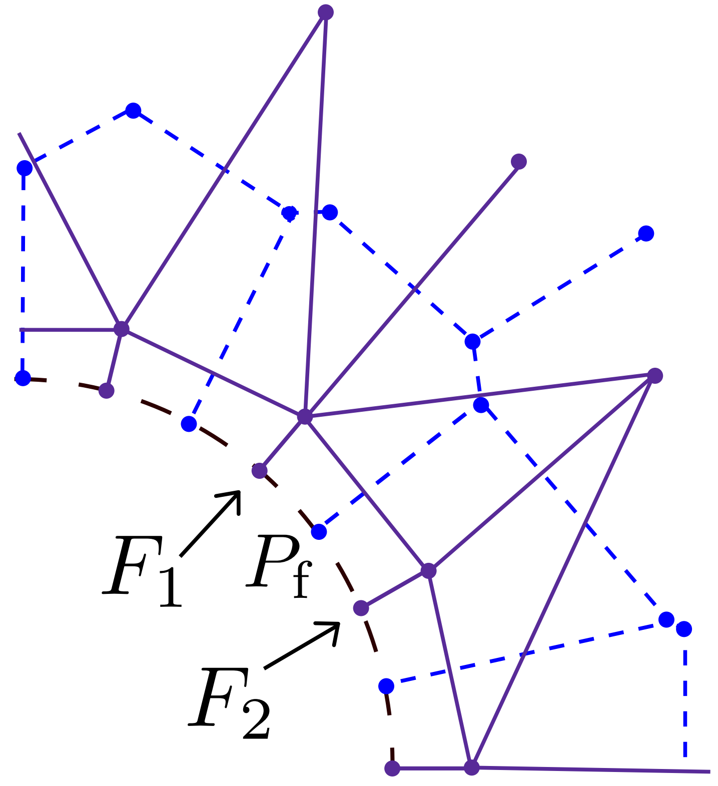
Assumptions for transport:

- Steady state conditions
- Independent of mechanical response
- Fluid is incompressible

Boundaries



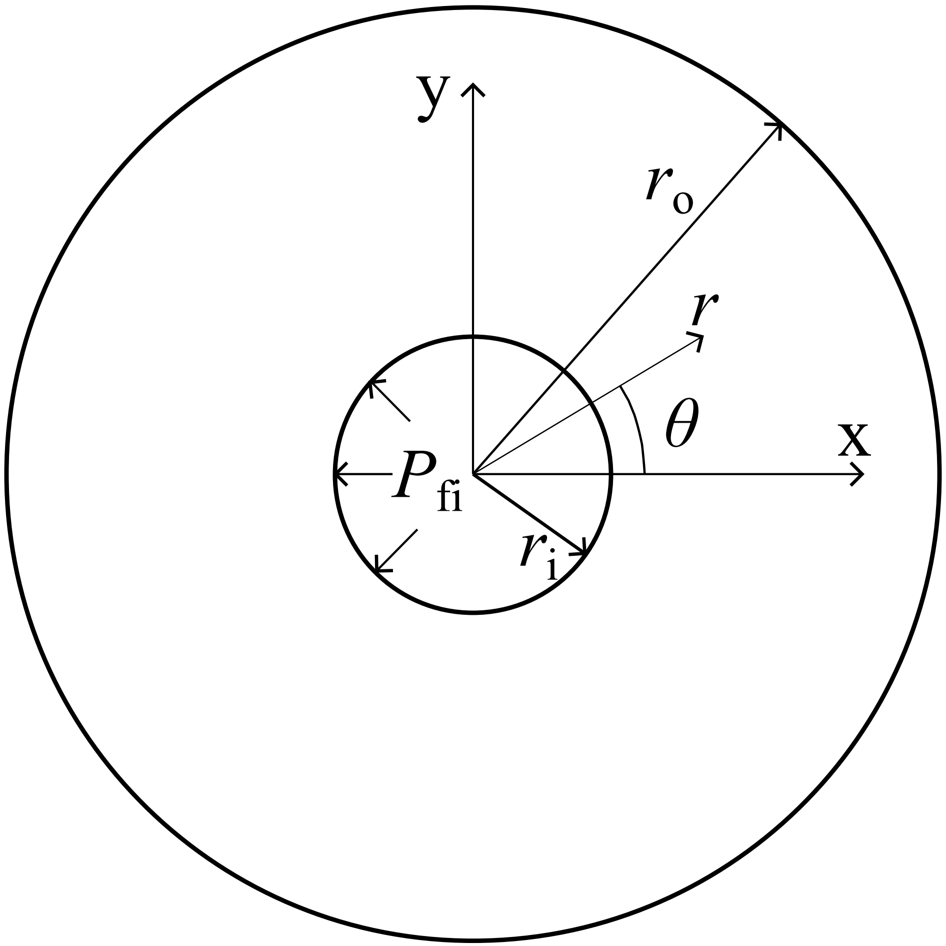
Approach 1



Approach 2

Elastic Benchmark

Thick Walled Cylinder



Dimensionless variables:

$$\bar{r} = r/r_i$$

$$\bar{r}_o = r_o/r_i$$

$$\bar{P}_f = P_f/E_c$$

$$\bar{P}_{fi} = P_{fi}/E_c$$

Analytical solution

Transport

$$\bar{P}_{\mathrm{f}} = -\bar{P}_{\mathrm{fi}} \frac{\ln \dfrac{\bar{r}}{\bar{r}_{\mathrm{o}}}}{\ln \bar{r}_{\mathrm{o}}}$$

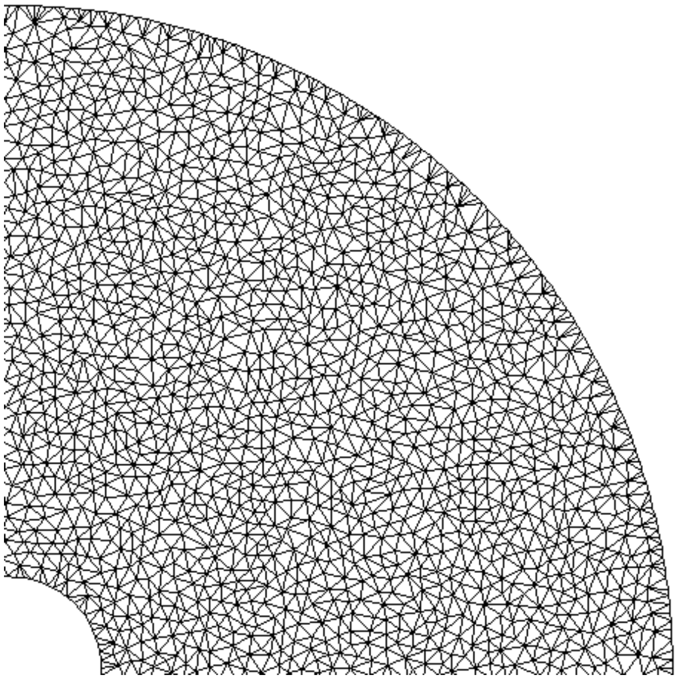
Mechanical

$$\bar{u} = -\, b\bar{P}_{\mathrm{fi}} \frac{1-\nu^2}{2} \left[\frac{\bar{r}_{\mathrm{o}}^2}{\bar{r}_{\mathrm{o}}^2 - 1} \left(\frac{1+\nu}{1-\nu} \frac{1}{\bar{r}} + \bar{r} \right) + \bar{r} \frac{\dfrac{1}{1+\nu} - \ln \bar{r}}{\ln \bar{r}_{\mathrm{o}}} \right] -$$

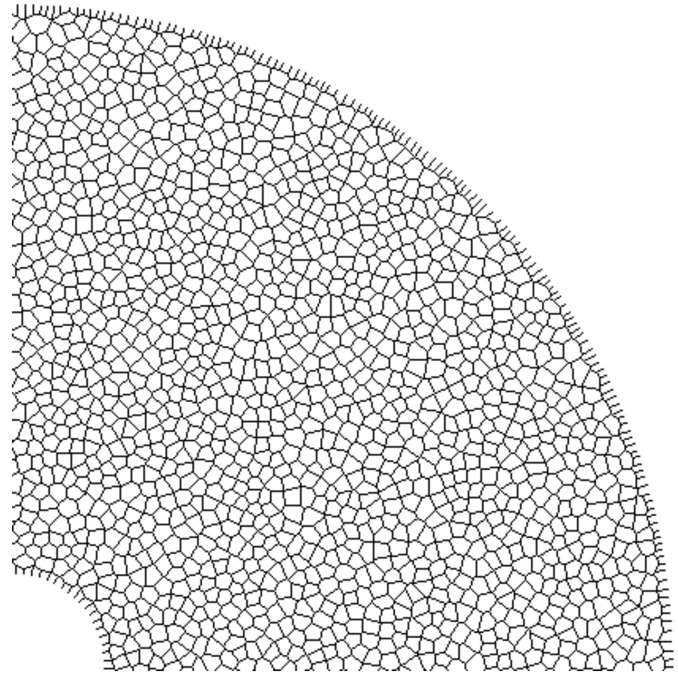
$$(1-b)\bar{P}_{\mathrm{fi}} \frac{\bar{r}_{\mathrm{o}}^2}{\bar{r}_{\mathrm{o}}^2 - 1} \left(\frac{1+\nu}{\bar{r}} + \frac{\bar{r}(1-\nu)}{\bar{r}_{\mathrm{o}}^2} \right)$$

References: Grassl et al. (2015), Hill (1950), Shawki and Elwahi (1970)

Lattice analysis



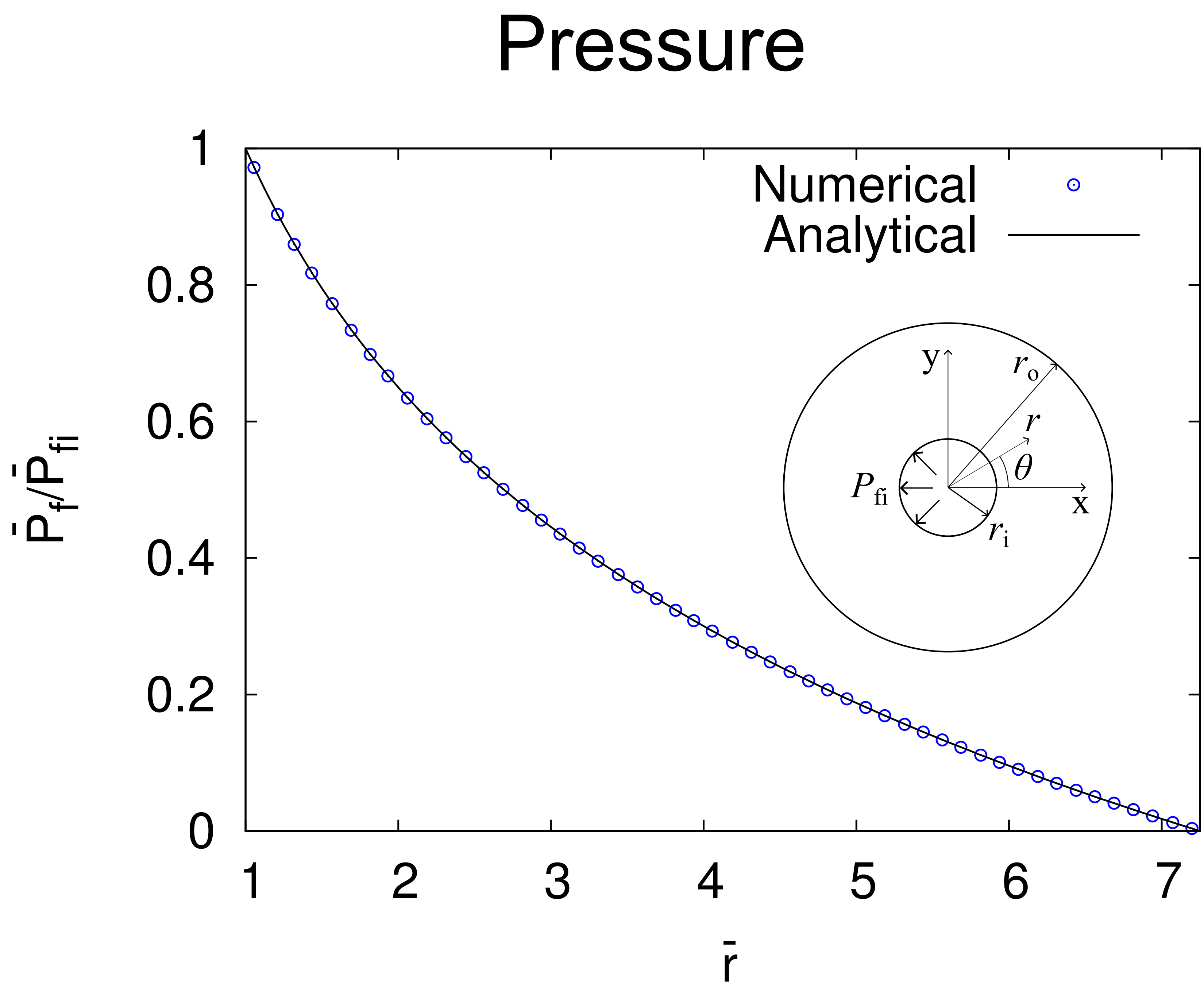
Mechanical



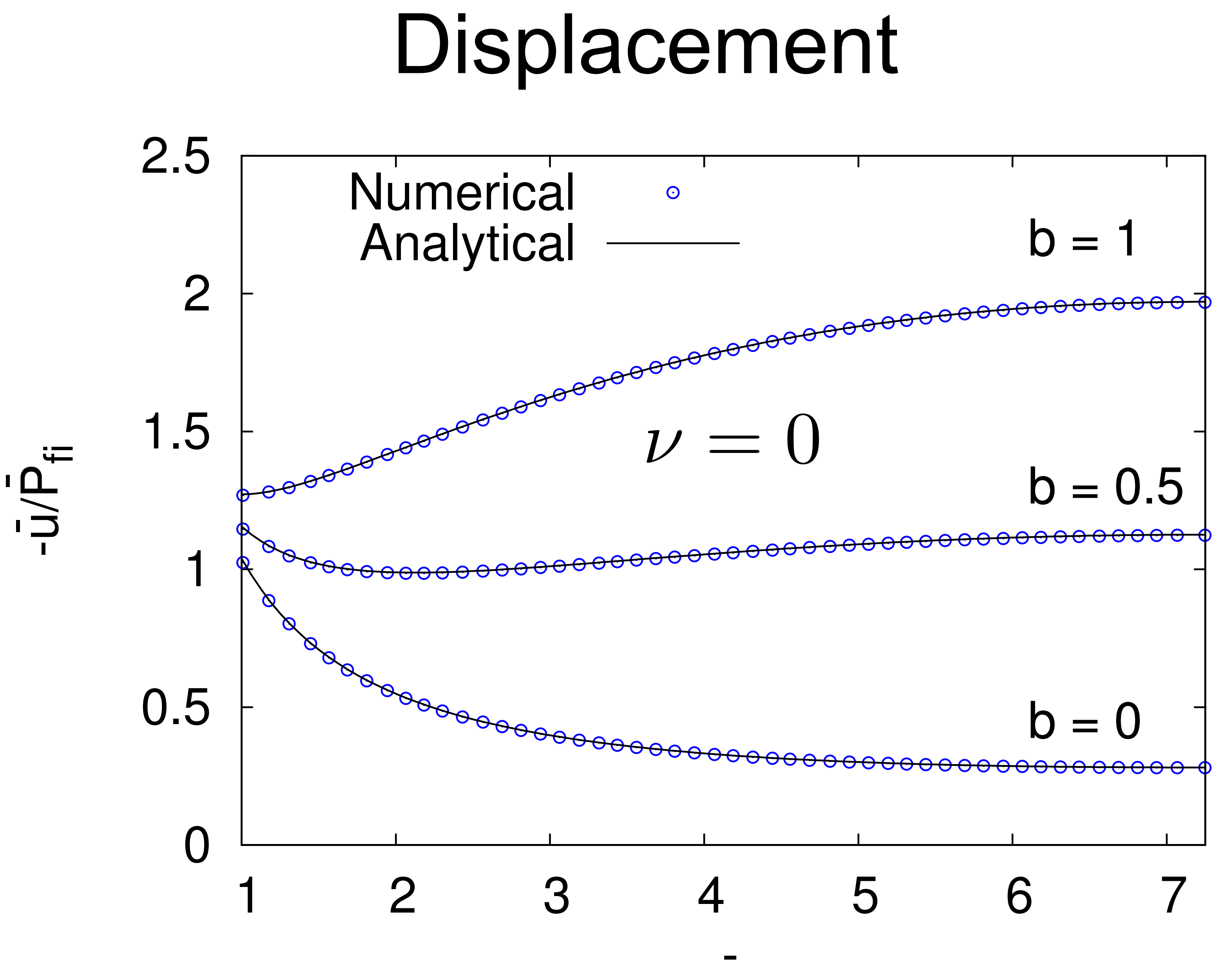
Transport

Boundary Approach 1

Input: $\bar{r}_{\mathrm{o}} = \dfrac{r_{\mathrm{o}}}{r_{\mathrm{i}}} = 7.25 \quad \bar{d}_{\mathrm{min}} = \dfrac{d_{\mathrm{min}}}{r_{\mathrm{i}}} = 0.123$



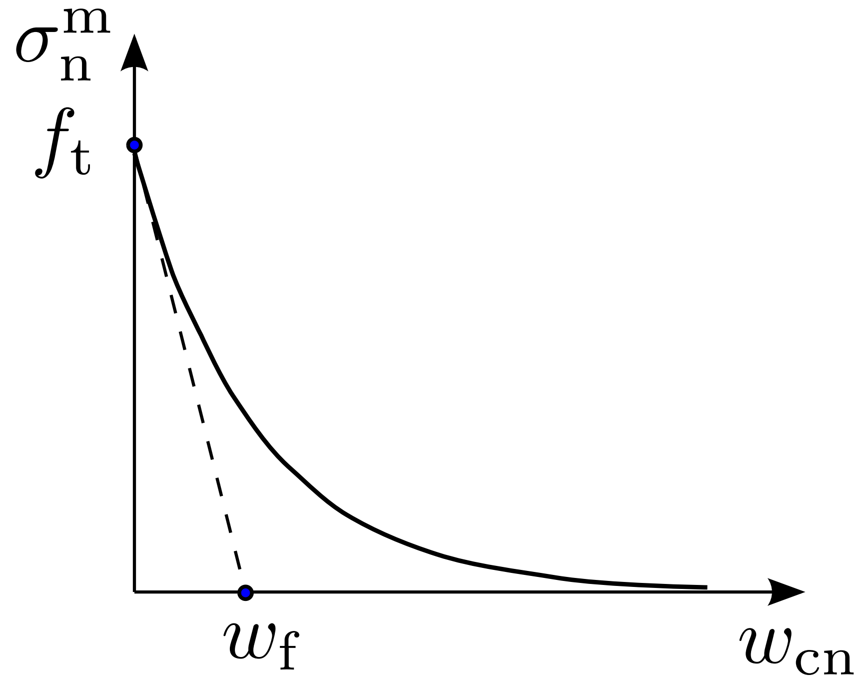
Pressure
Numerical
Analytical
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
$\bar{P}_f/\bar{P}_{fi}$
$\bar{r}$
y
x
r_o
r
θ
P_{fi}
r_i


Displacement
Numerical
Analytical
b = 1
b = 0.5
b = 0
$\nu = 0$
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
$-\bar{u}/\bar{P}_{fi}$
$\bar{r}$

Fracture analysis

Fracture analysis

- Fracture process
- Size effect

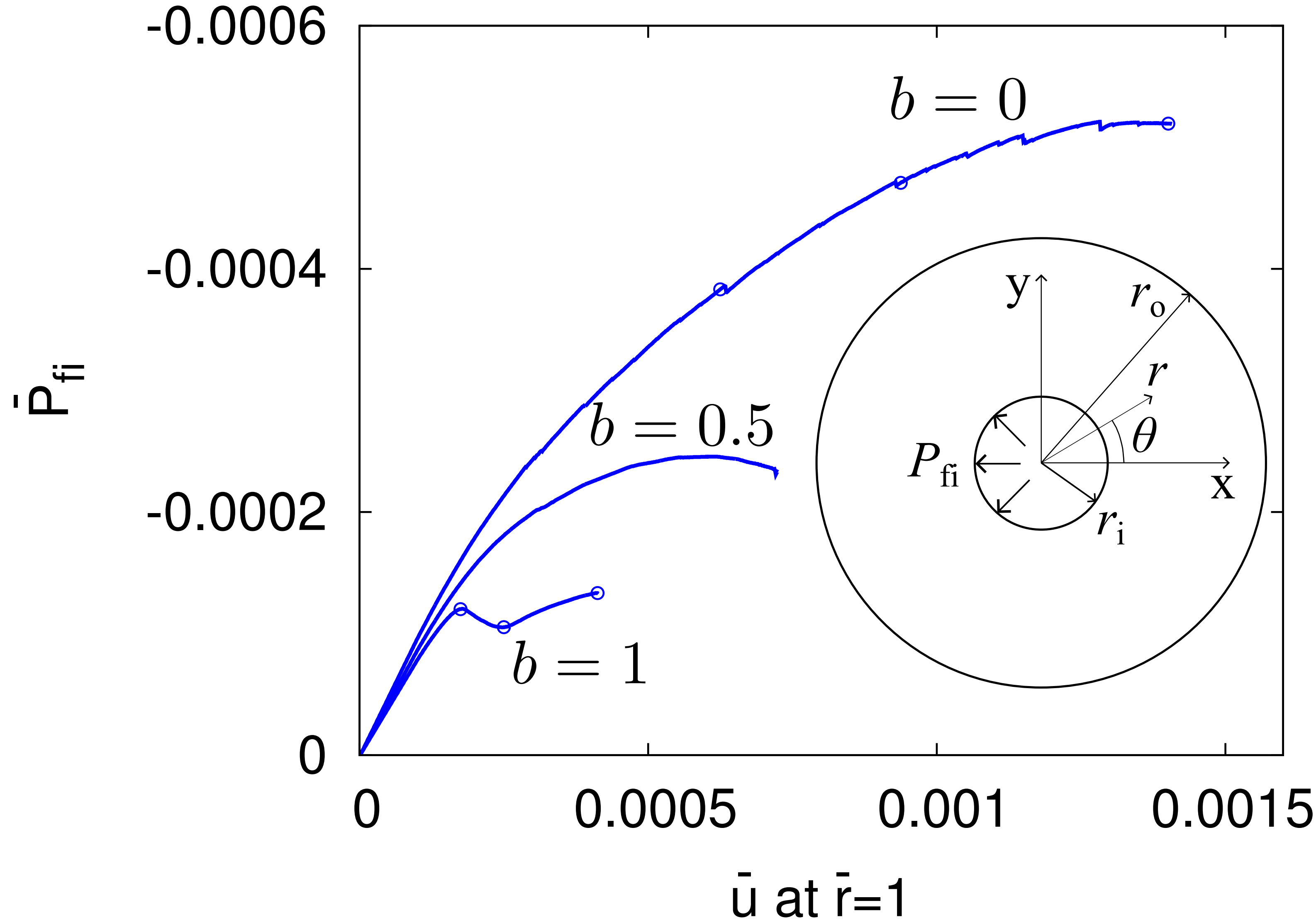


Boundary approach 2

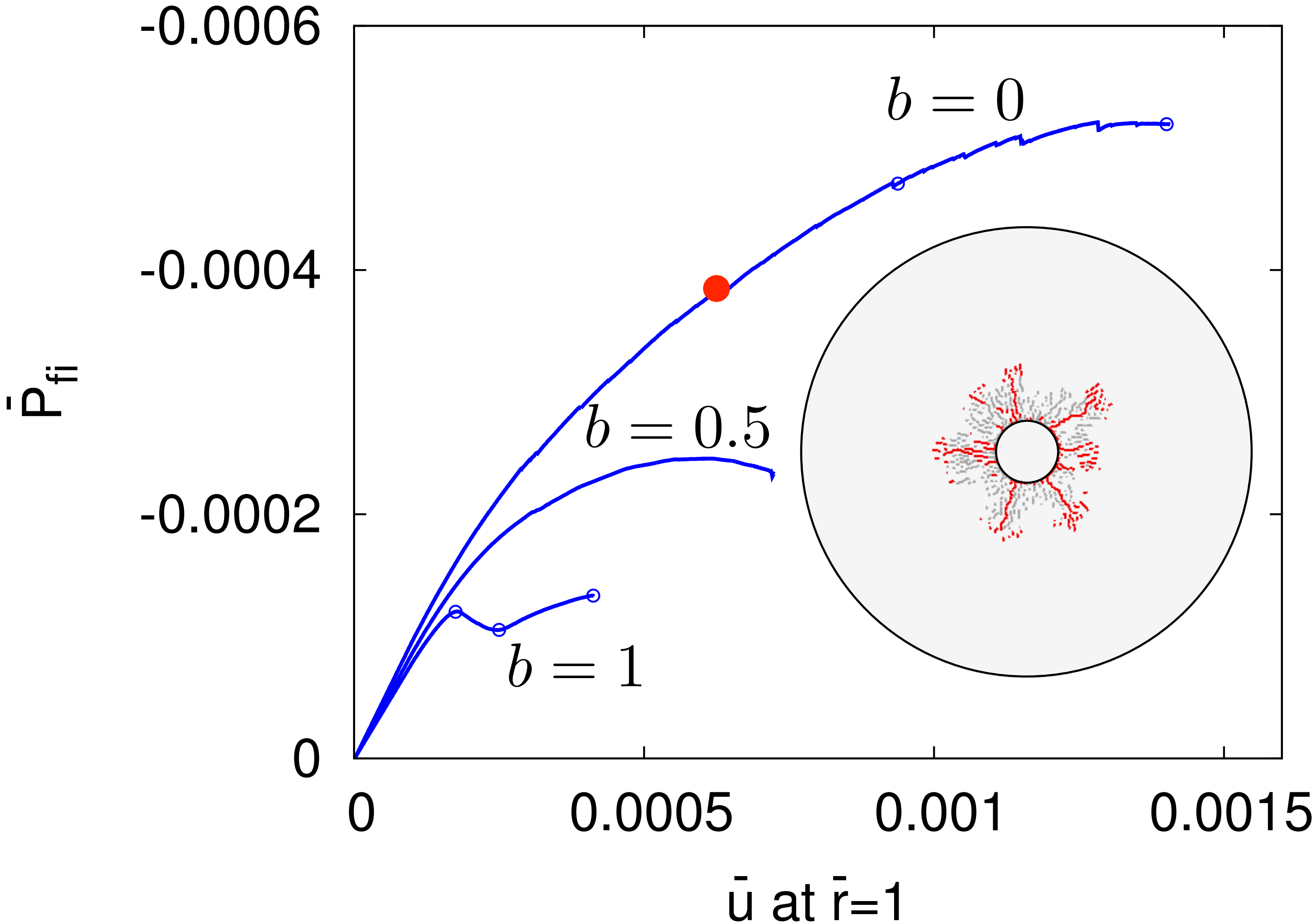
Input: $\varepsilon_0 = 0.0001$ $q = 2$

$c = 20$ $\bar{w}_{\mathrm{f}} = w_{\mathrm{f}}/r_{\mathrm{i}} = 0.00625$

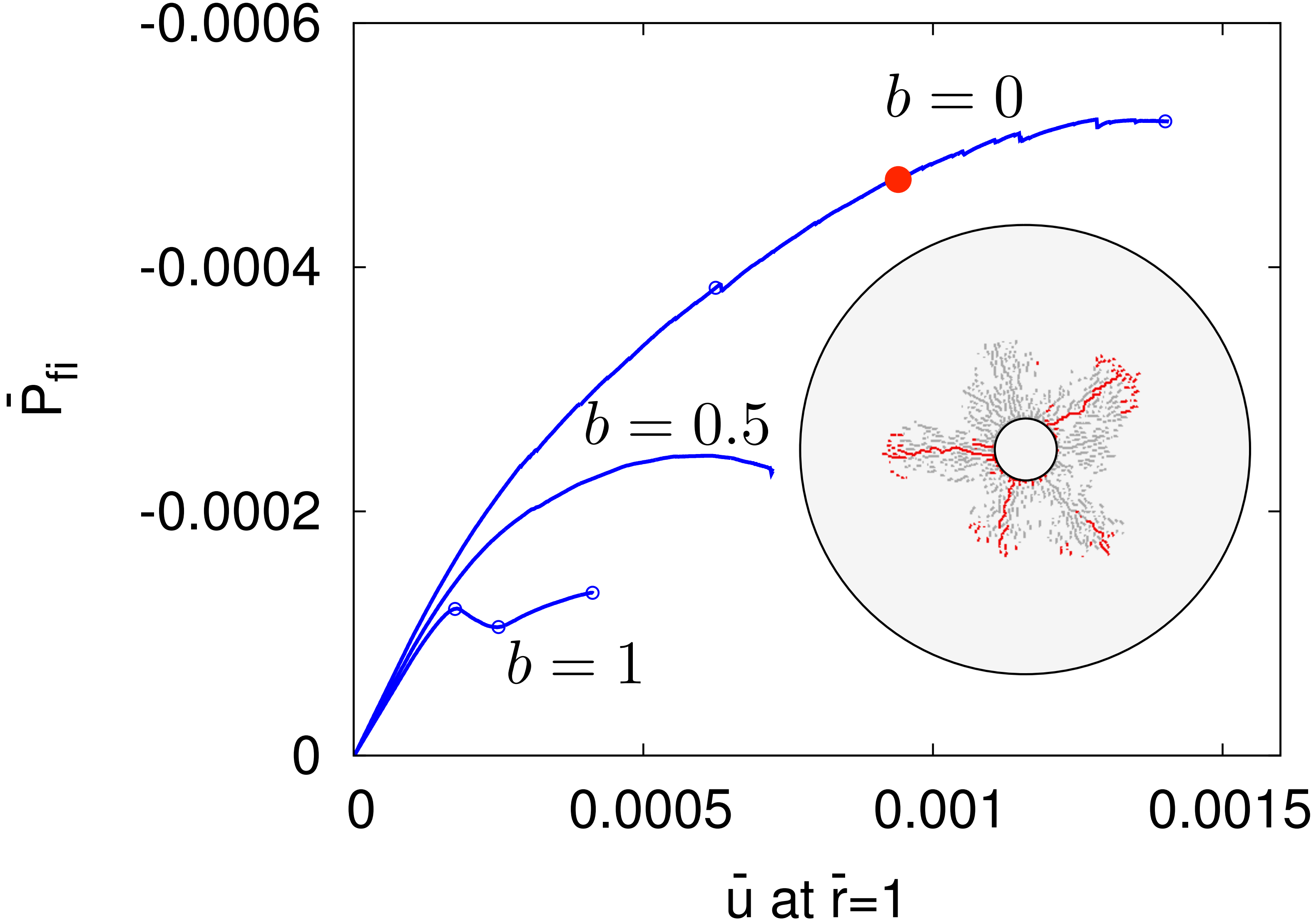
Fracture process



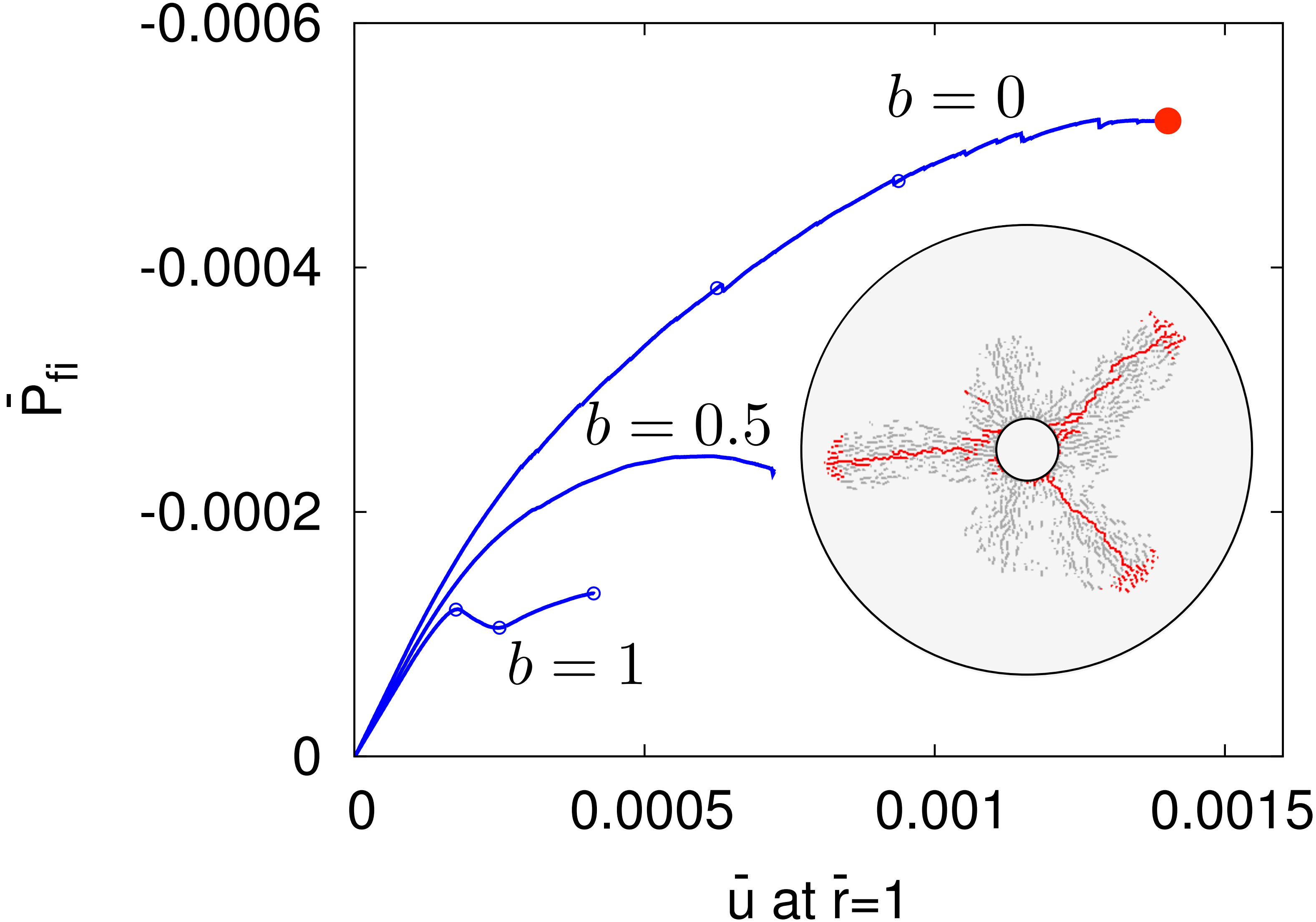
Fracture process



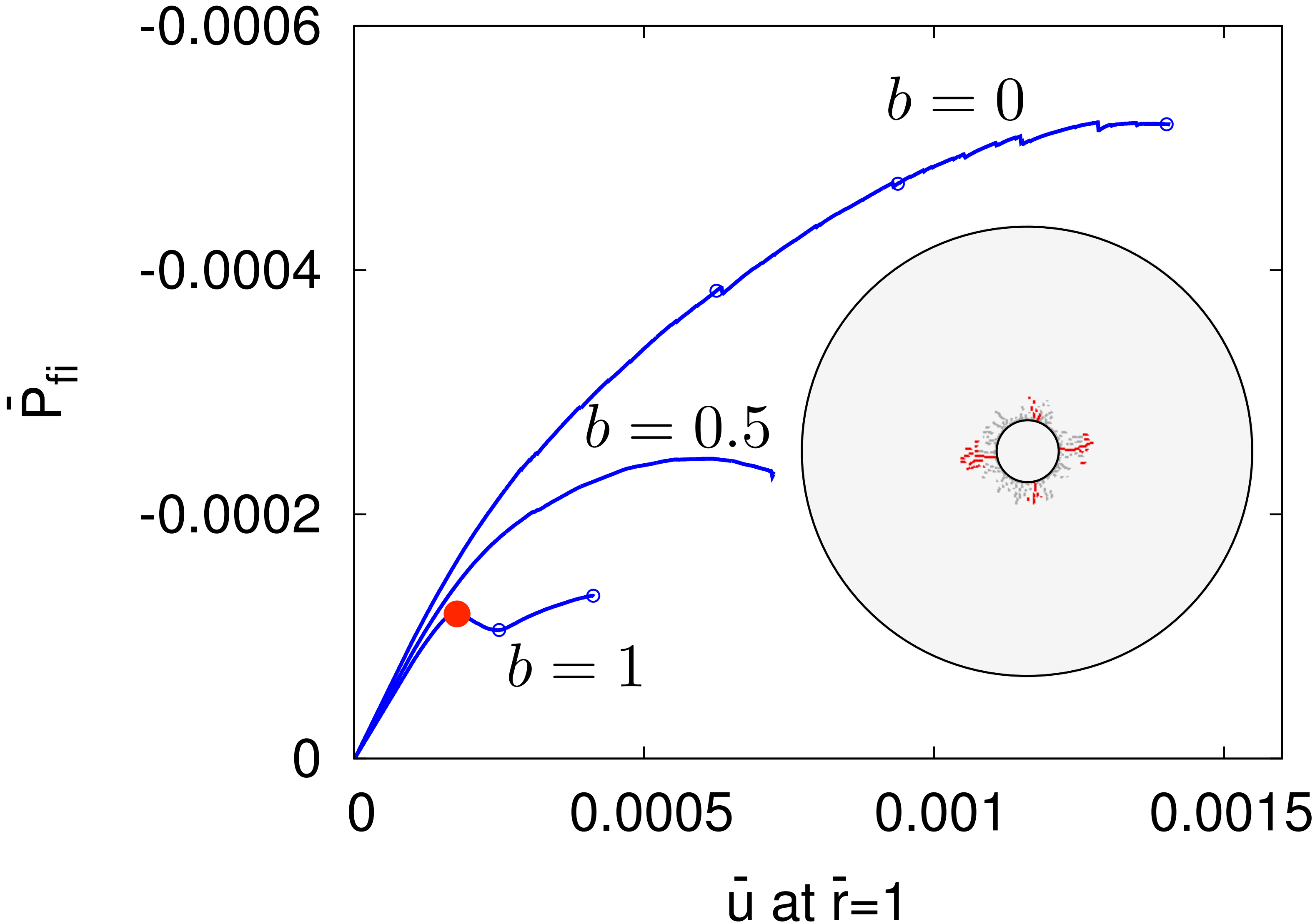
Fracture process



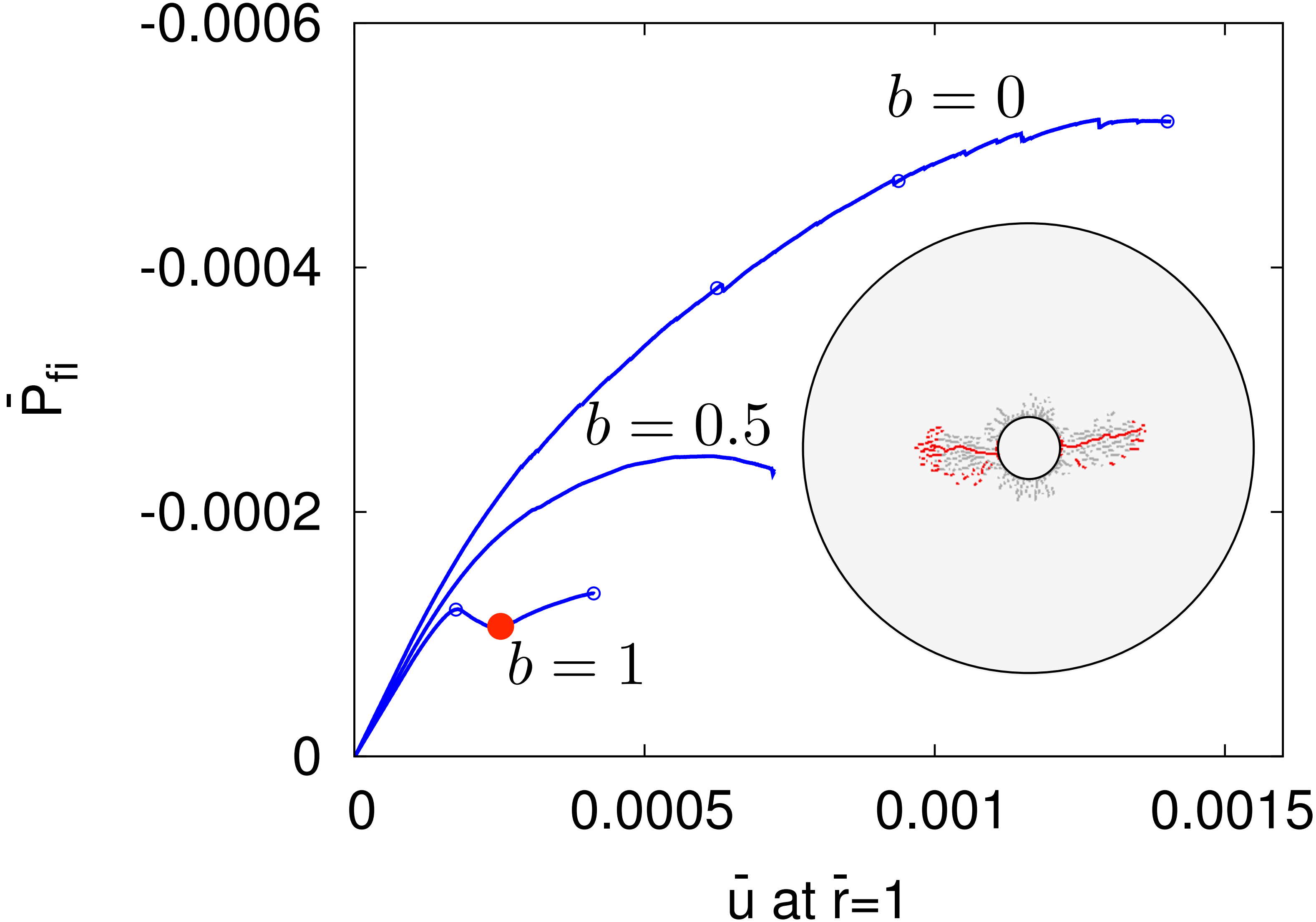
Fracture process



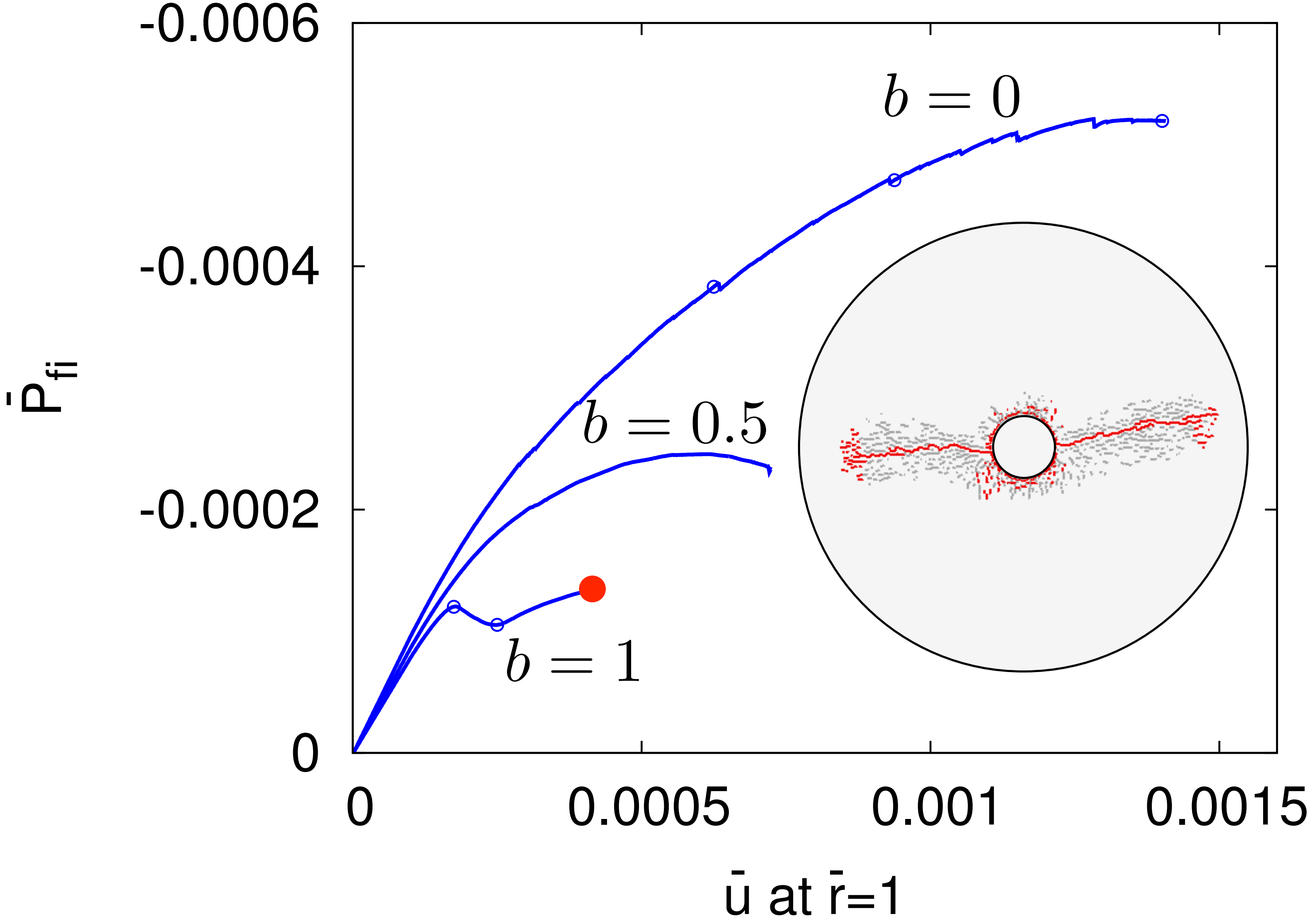
Fracture process



Fracture process



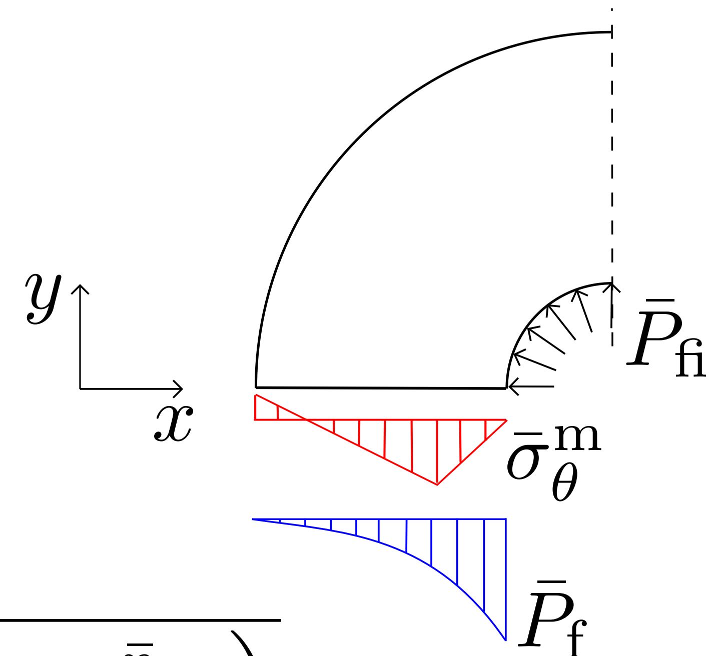
Fracture process



Size effect

Force equilibrium

$$\bar{P}_{\mathrm{fi}} = -\int_1^{\bar{r}_{\mathrm{o}}} \left(\bar{\sigma}_\theta^{\mathrm{m}} + \bar{P}_{\mathrm{f}}\right) d\bar{r}$$



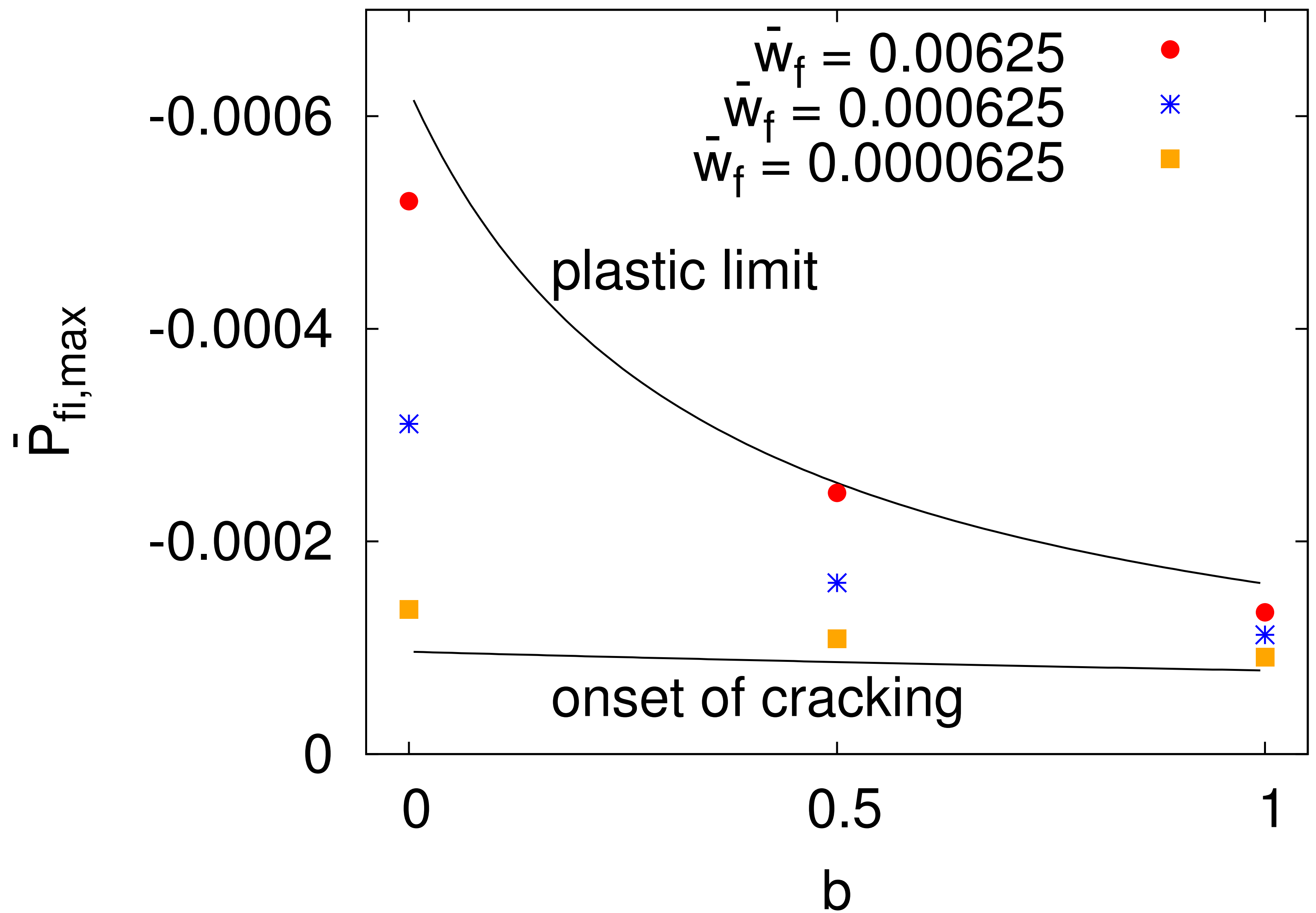
Plastic limit $\bar{w}_{\mathrm{f}} \to \infty$

$$\bar{P}_{\mathrm{fi,max}} = \frac{(\bar{r}_{\mathrm{o}} - 1)\varepsilon_0}{1 - b\left(\dfrac{1}{\ln \bar{r}_{\mathrm{o}}} + \dfrac{1}{(\ln \bar{r}_{\mathrm{o}})^2} - \dfrac{\bar{r}_{\mathrm{o}}}{\ln \bar{r}_{\mathrm{o}}}\right)}$$

Onset of cracking $\bar{w}_{\mathrm{f}} \to 0$

$$\bar{P}_{\mathrm{fi}} = -\frac{\varepsilon_0}{\left[b\left(\dfrac{\bar{r}_{\mathrm{o}}^2}{\bar{r}_{\mathrm{o}}^2 - 1} + \dfrac{1}{2\ln \bar{r}_{\mathrm{o}}}\right) + (1-b)\dfrac{1+\bar{r}_{\mathrm{o}}^2}{\bar{r}_{\mathrm{o}}^2 - 1}\right]}$$

Size effect



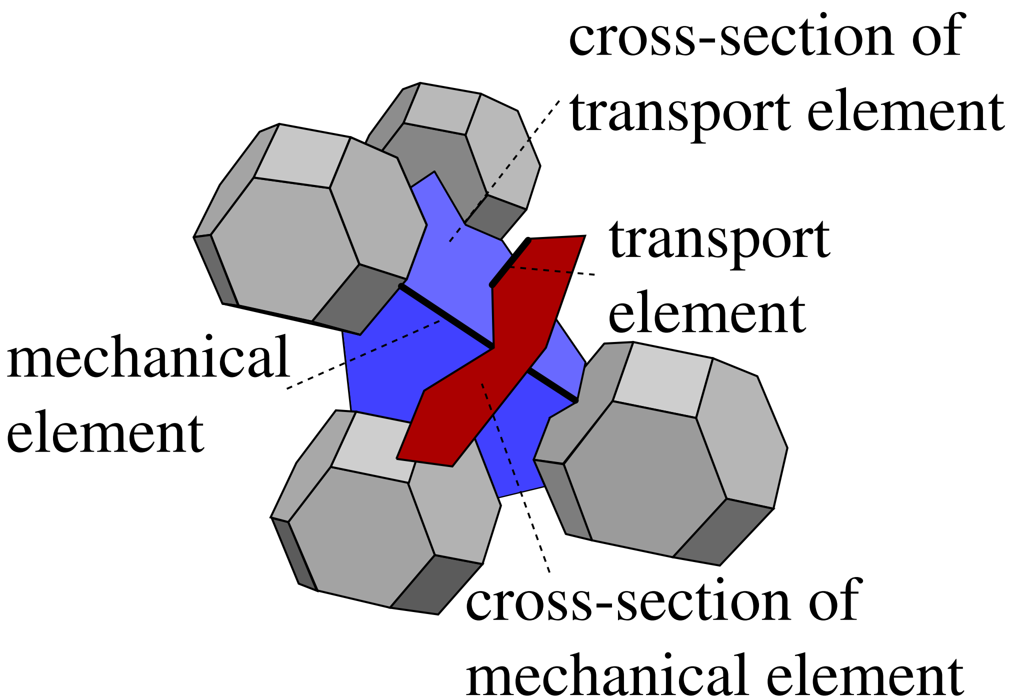
$\bar{w}_f$ = 0.00625
$\bar{w}_f$ = 0.000625
$\bar{w}_f$ = 0.0000625
plastic limit
onset of cracking
-0.0006
-0.0004
-0.0002
0
0
0.5
1
$\bar{P}_{fi,max}$
b

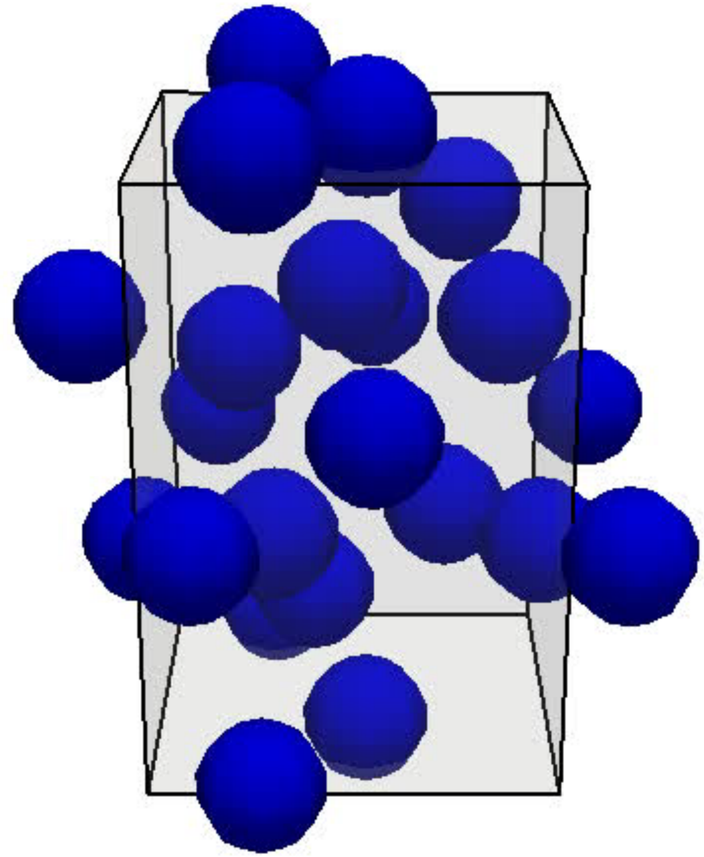
Conclusions

Lattice analysis of the hydro-mechanical response of a thick-walled cylinder:

- Good agreement with the analytical solution for varying Biot's coefficient.
- Hydraulic fracturing is described mesh-size independently.
- Thick walled cylinder exhibits strong size effect which decreases with an increase in Biot's coefficient.

3D extension





Shrinkage
induced cracking

Acknowledgements

The numerical analyses are performed with OOFEM (www.oofem.org) extended by the authors.

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Scalar damage model

Stress-strain law

$$\boldsymbol{\sigma}^{\mathrm{m}} = (1-\omega)\mathbf{D}_{\mathrm{e}}\boldsymbol{\varepsilon}$$

Loading function

$$f\left(\boldsymbol{\varepsilon},\kappa\right) = \varepsilon_{\mathrm{eq}}\left(\boldsymbol{\varepsilon}\right) - \kappa$$

Loading-unloading conditions

$$f \leq 0, \qquad \dot{\kappa} \geq 0, \qquad \dot{\kappa} f = 0$$

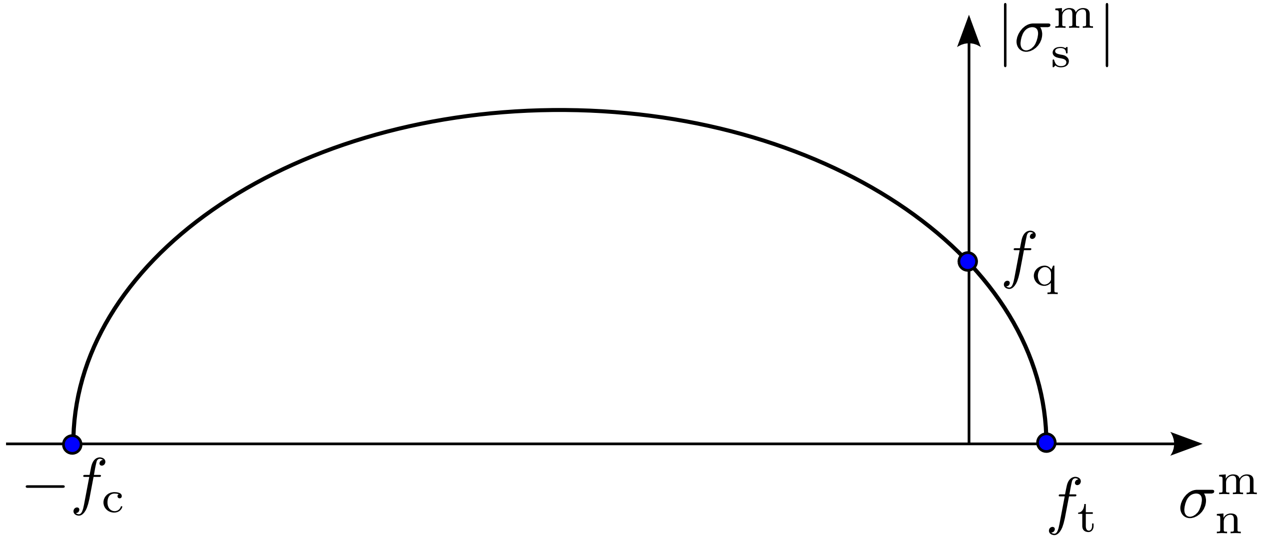
Damage function

$$\omega = g_{\mathrm{d}}\left(\kappa\right)$$

Equivalent strain definition

$$\varepsilon_{\mathrm{eq}}(\varepsilon_{\mathrm{n}}, \varepsilon_{\mathrm{s}}) = \frac{1}{2}\varepsilon_0\,(1-c) + \sqrt{\left(\frac{1}{2}\varepsilon_0(c-1) + \varepsilon_{\mathrm{n}}\right)^2 + \frac{c\gamma^2\varepsilon_{\mathrm{s}}^2}{q^2}}$$

Strength envelope

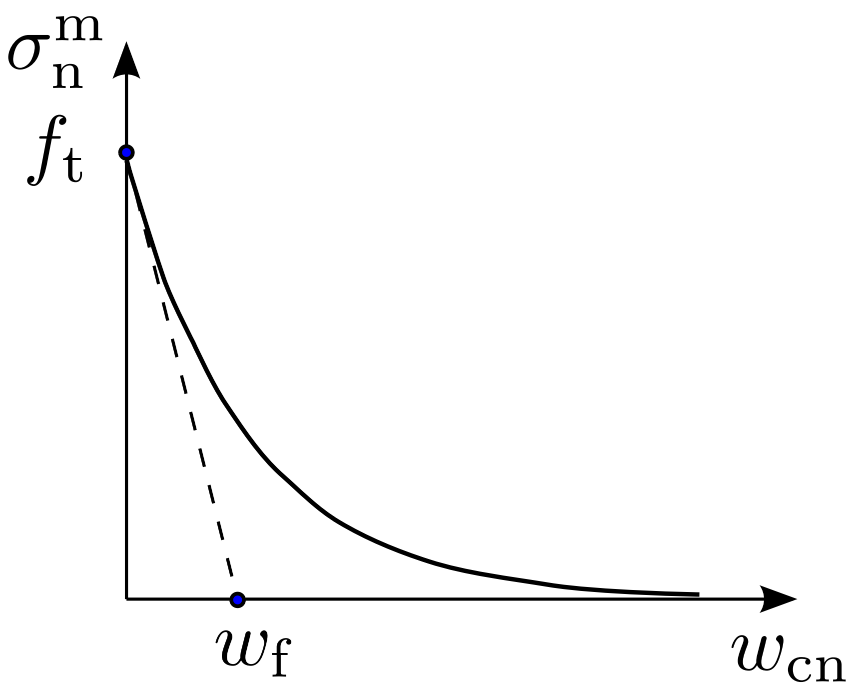


$$f_{\mathrm{t}} = E\varepsilon_0$$

$$f_{\mathrm{q}} = qf_{\mathrm{t}}$$

$$f_{\mathrm{c}} = cf_{\mathrm{t}}$$

Damage function



$$\sigma_{\mathrm{n}}^{\mathrm{m}} = f_{\mathrm{t}} \exp\left(-\frac{\omega\kappa h}{w_{\mathrm{f}}}\right)$$

$$\sigma_{\mathrm{n}}^{\mathrm{m}} = (1-\omega)E\kappa$$

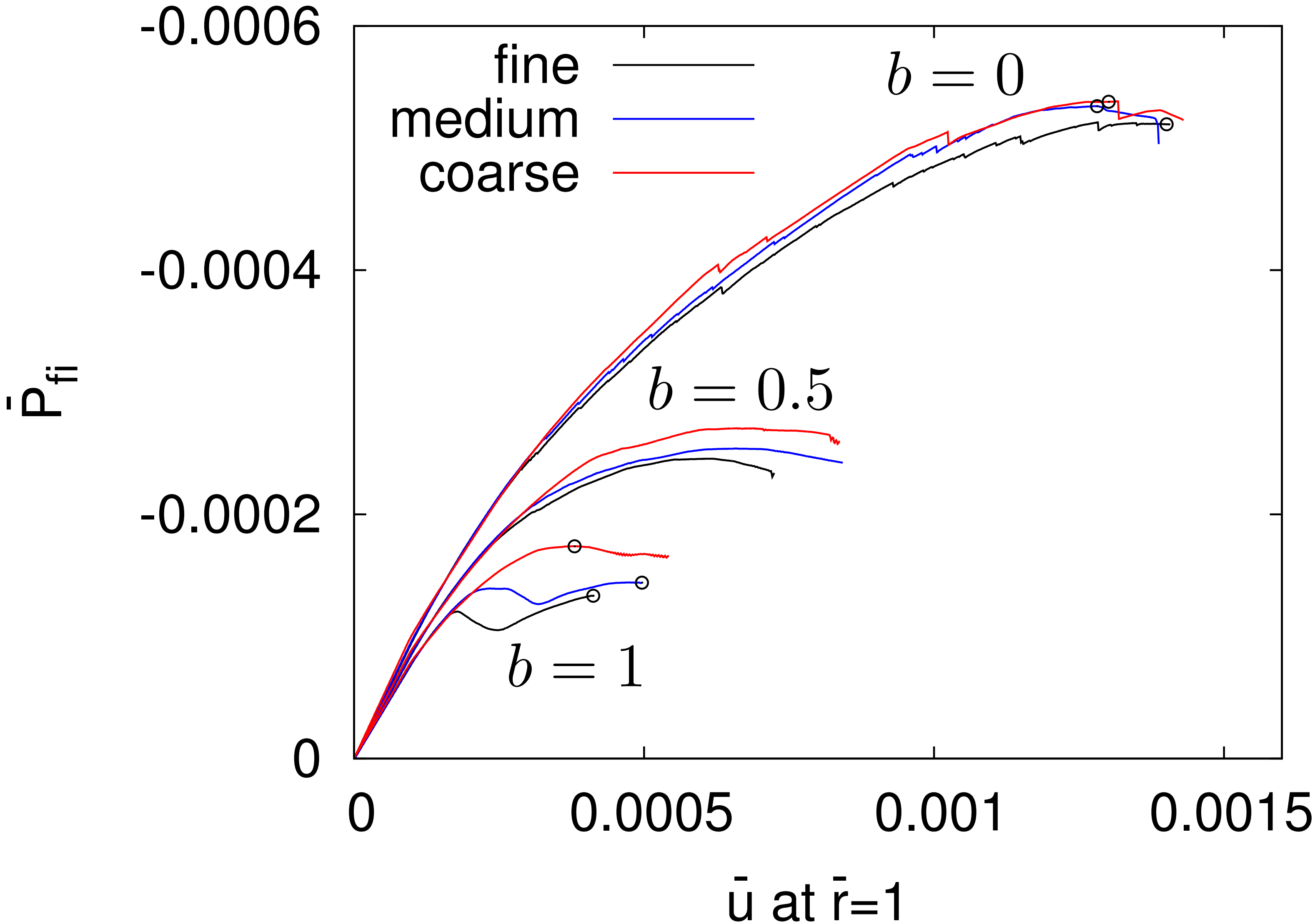
Increment of dissipated energy:

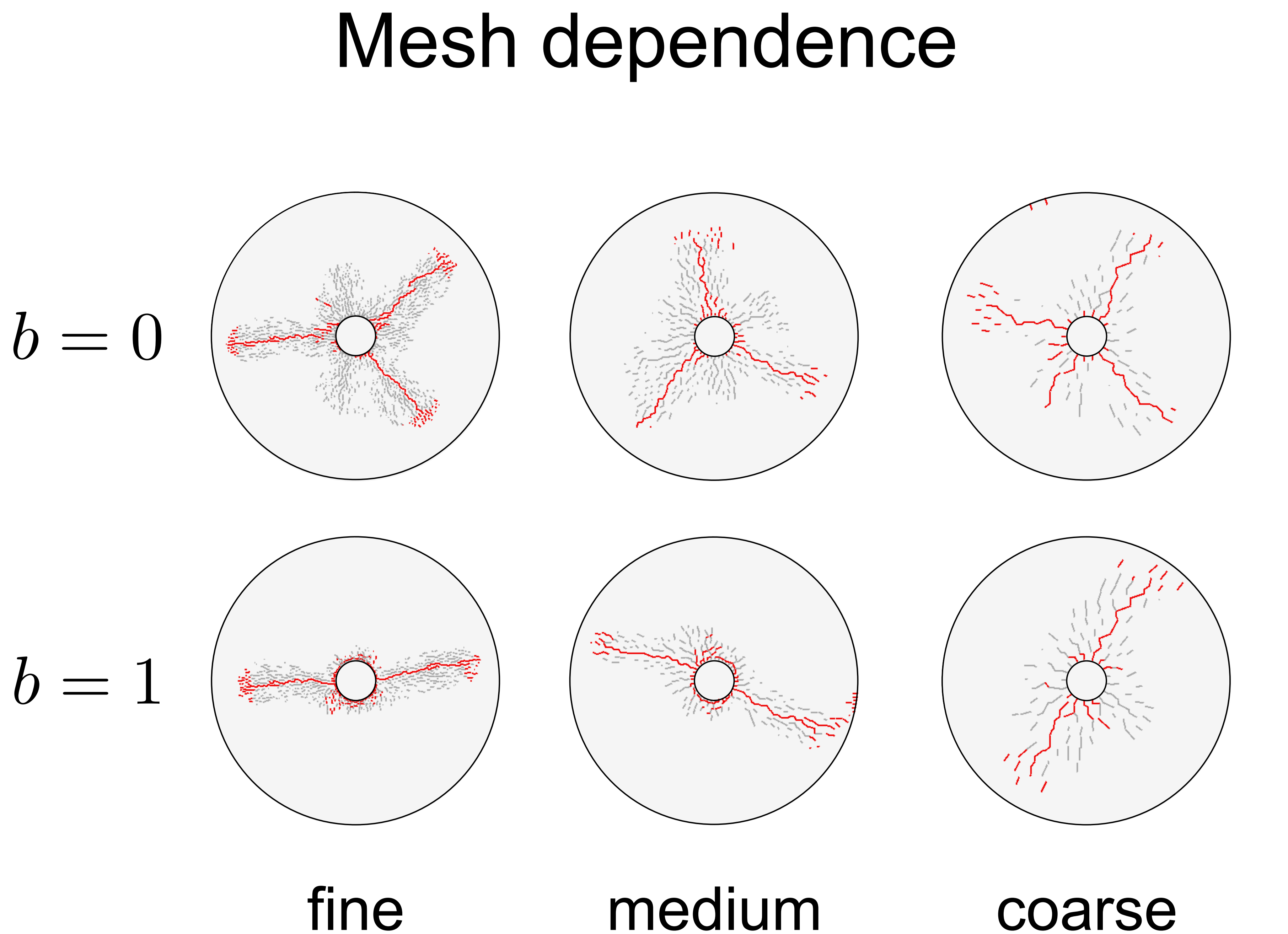
$$\Delta D_{\mathrm{d}} = \frac{1}{2}\boldsymbol{\varepsilon}\mathbf{D}\boldsymbol{\varepsilon}\Delta\omega h$$

Elements with $\Delta D_{\mathrm{d}} > 0$

Elements with $\Delta D_{\mathrm{d}} = 0$ and $D_{\mathrm{d}} > 0$

Mesh dependence




Mesh dependence
$b = 0$
$b = 1$
fine
medium
coarse