# 3D network modelling of the fracture process zone in fibre-reinforced geomaterials

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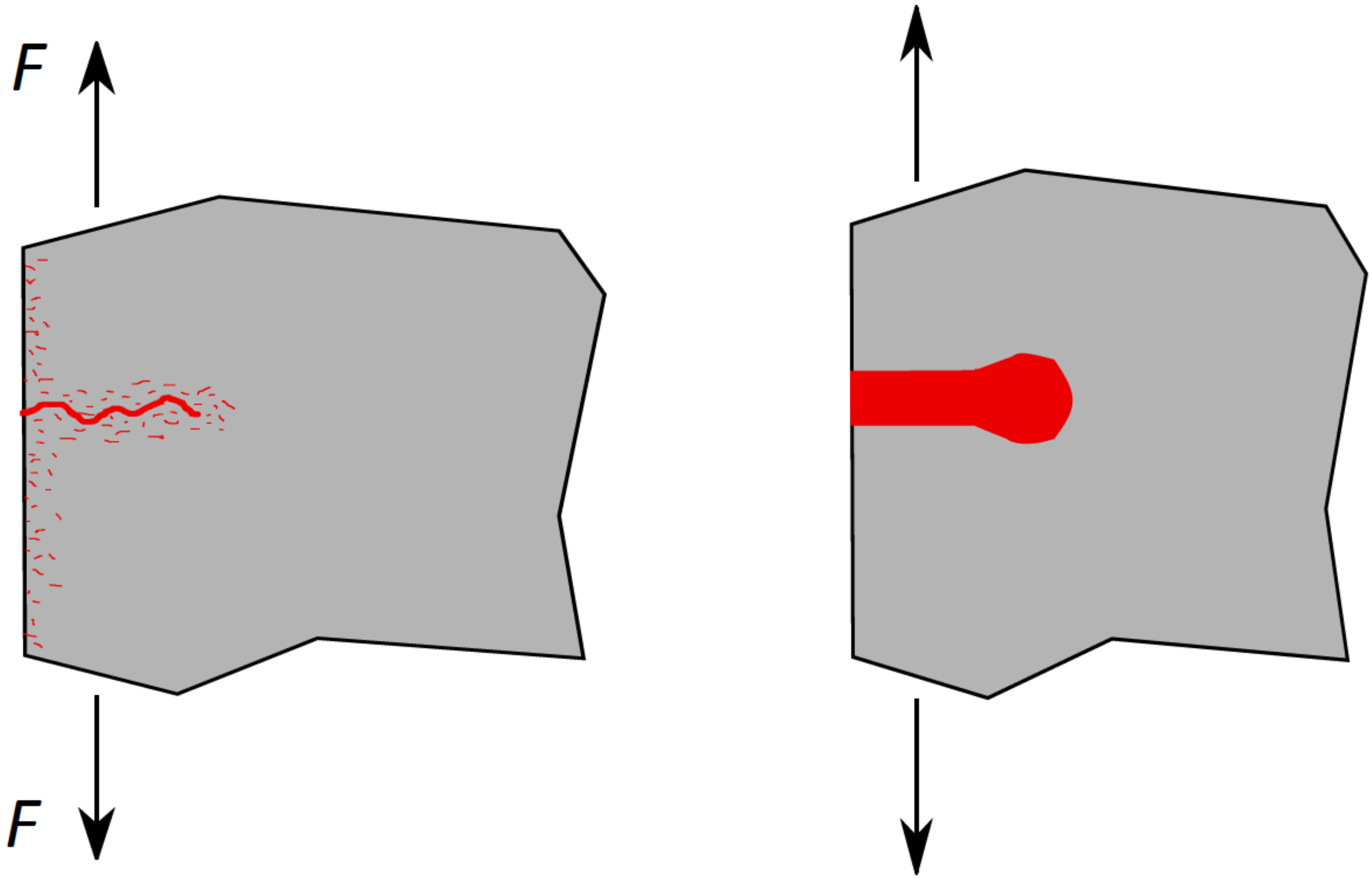
# Background




Ref: Grassl and Jirásek (2010), Grassl et al. (2012), Grégoire et al. (2015), Xenos et al. (2015)

# Background

## Aim

Improve the understanding of fracture processes at the meso-scale of fibre reinforced materials

## Approach

Use network model for periodic cells subjected to direct tension

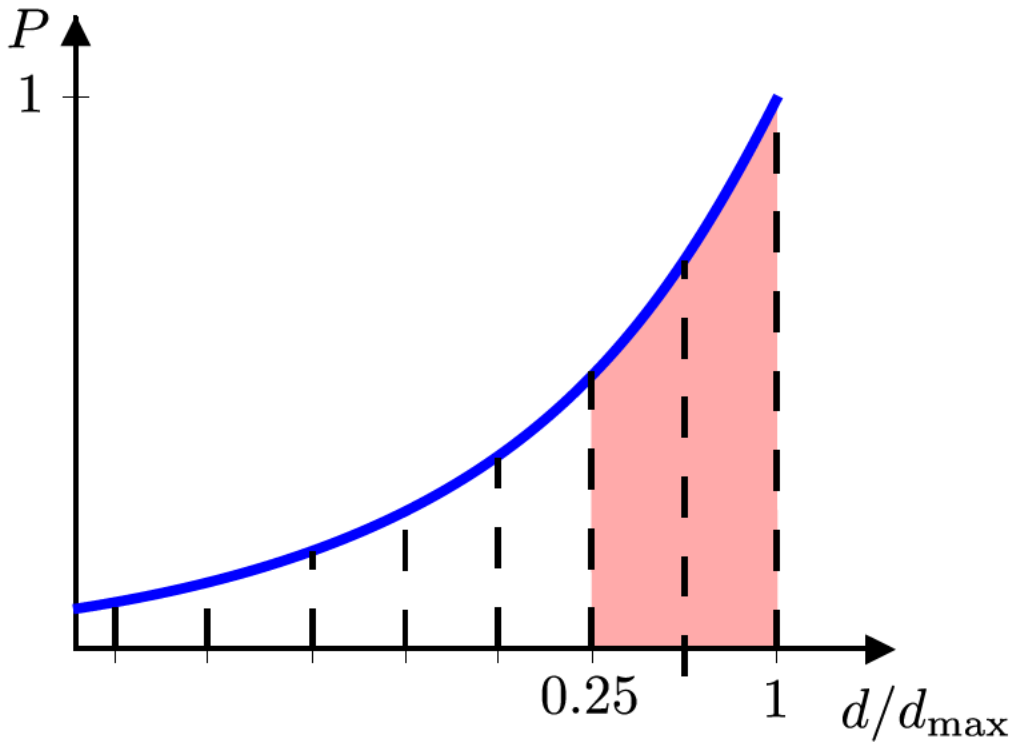
# Outline

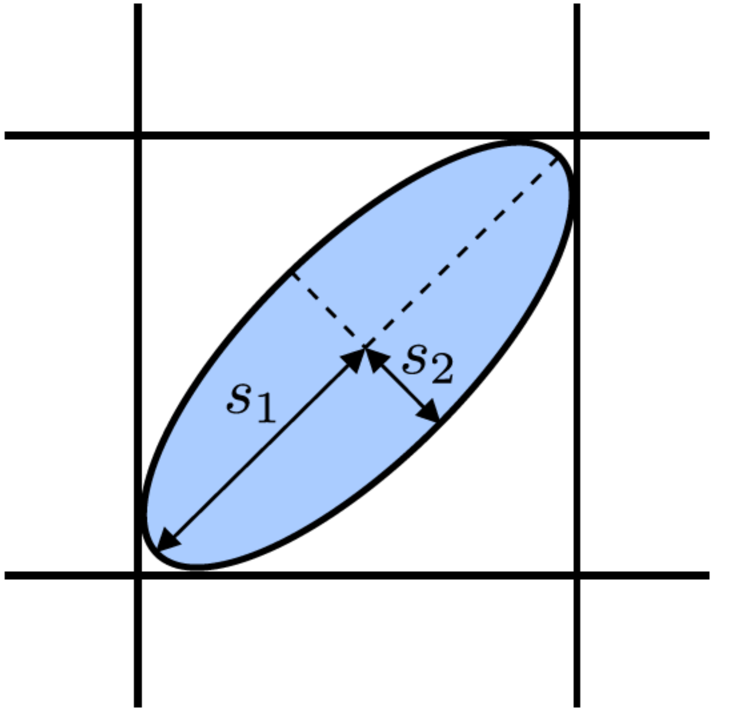
- Method
  - Meso-scale generation
  - Network model
  - Constitutive model
- Analyses
- Results
- Conclusions

# Meso-scale generation

# Meso-scale generation

Aggregates: polydispersed ellipsoids




Fibres: monodispersed line segments

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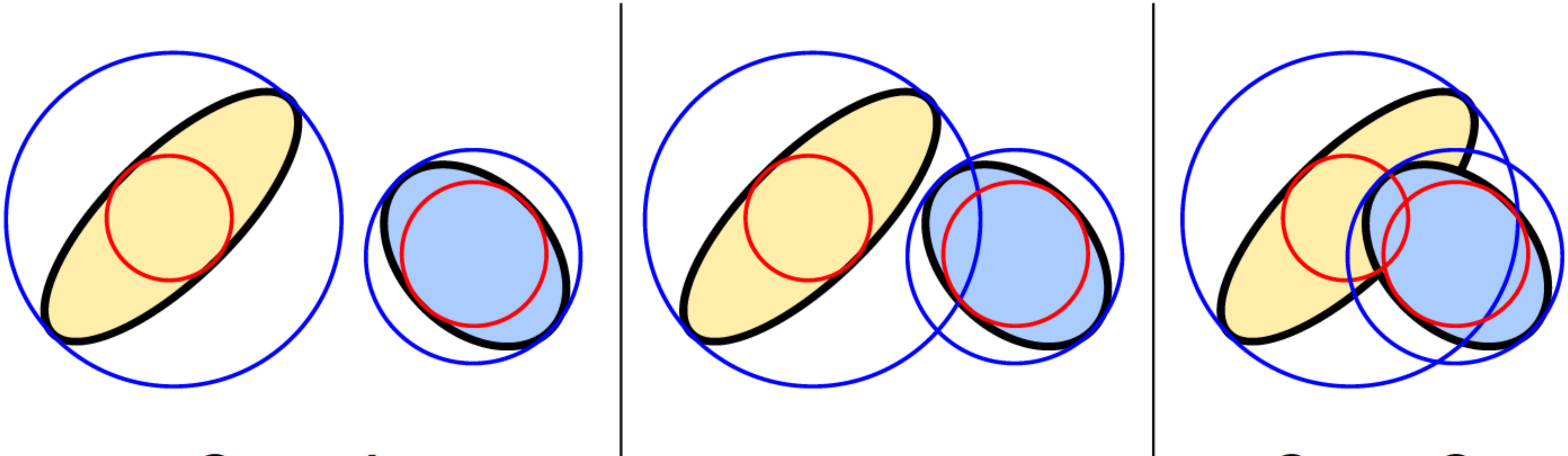
Wriggers and Moftah (2006), Leite et al. (2007)

# Meso-scale generation

Periodic random sequential addition approach

Algebraic overlap check:



Case A

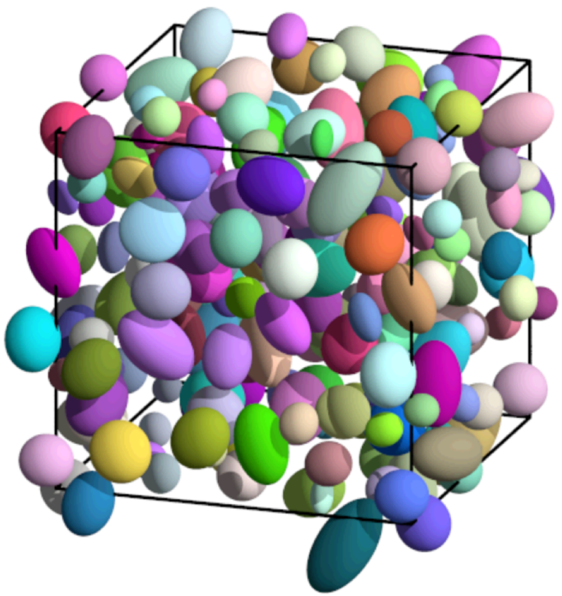
**Case B**

Case C

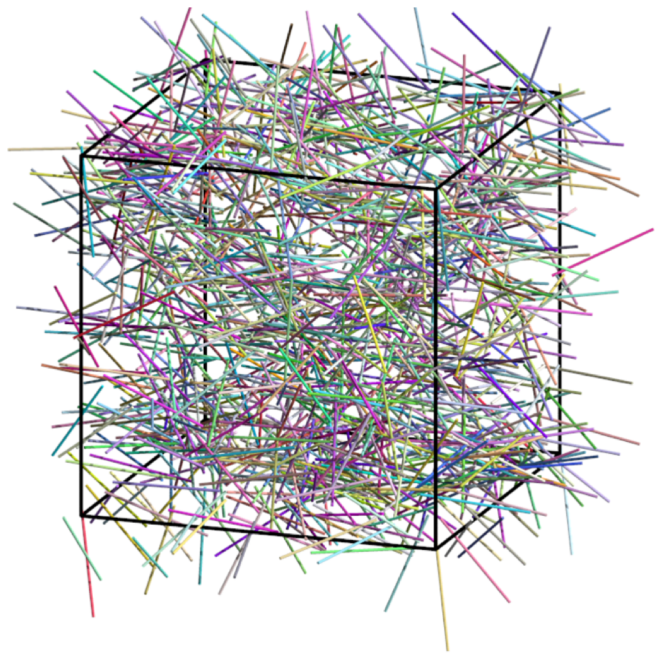
Muller (1957), Feder (1980), Wang et al. (2001), Mehrotra (2011)

# Meso-scale generation

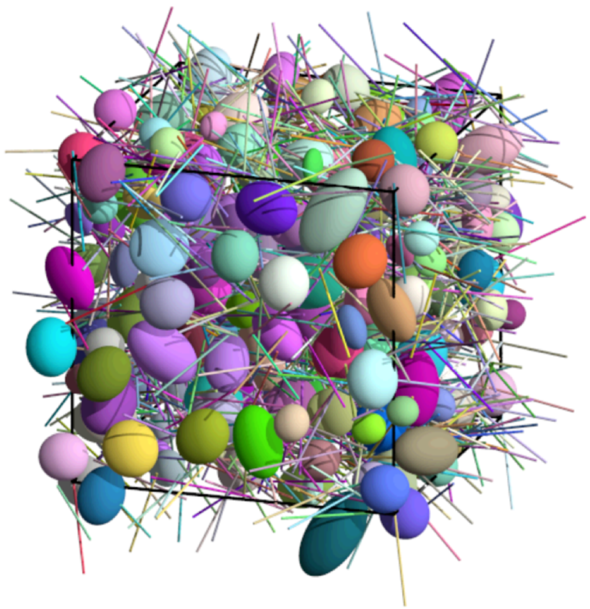
## Examples



Aggregates



Fibres

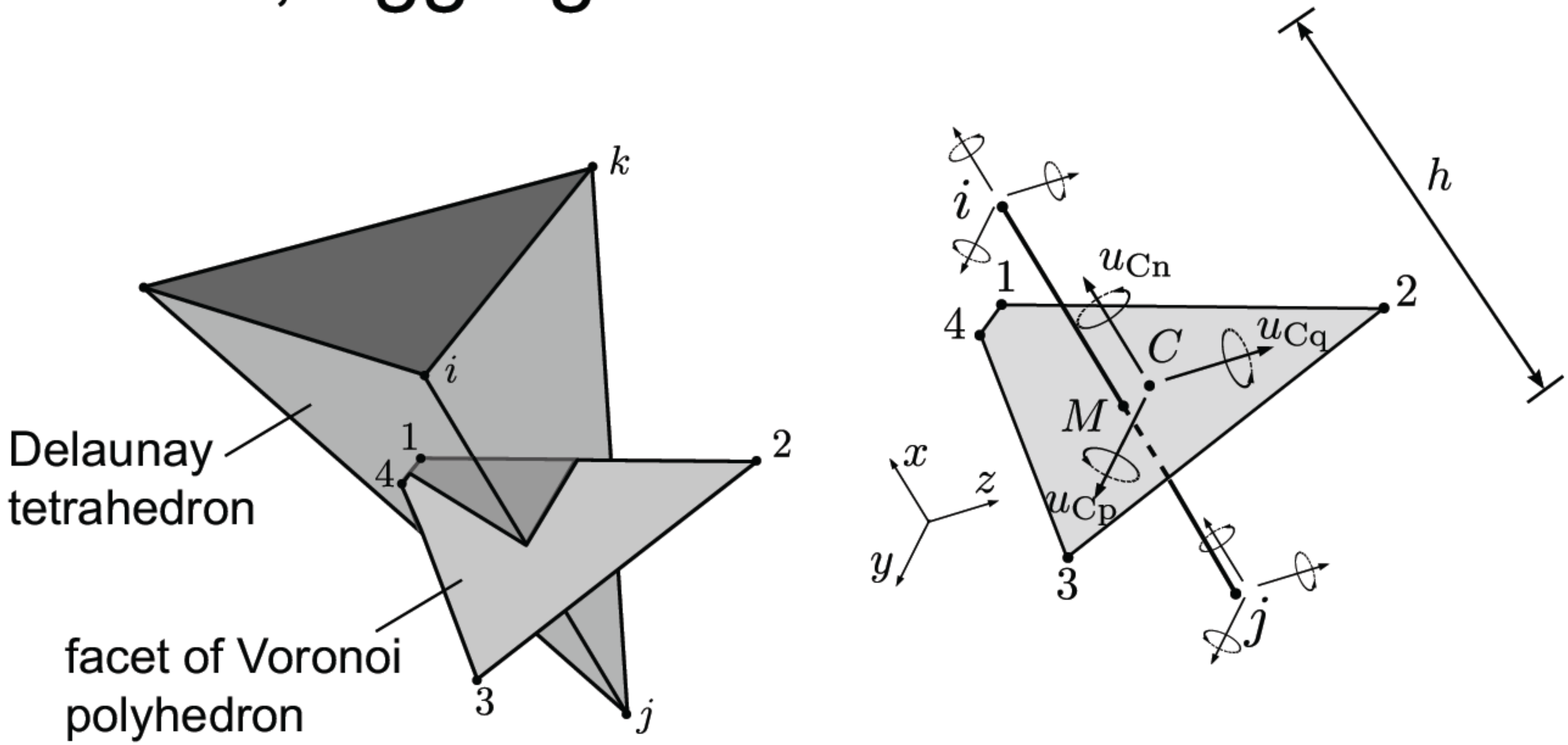


Aggregates + Fibres

# Network model

# Network model

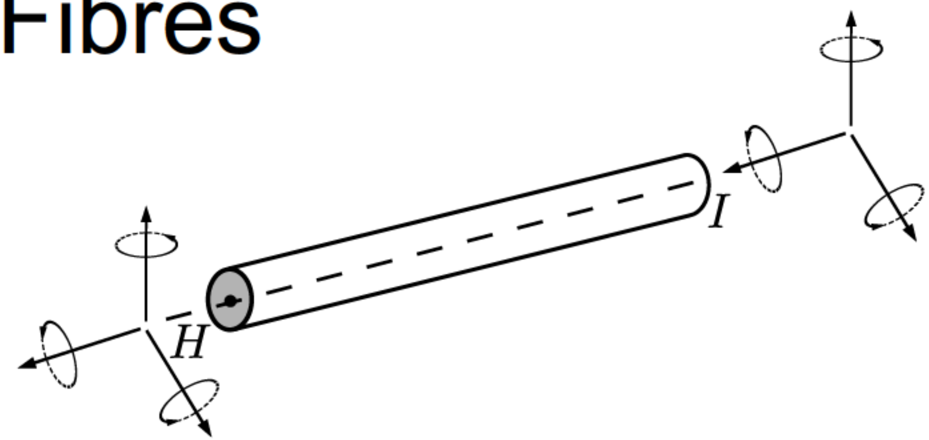
## Matrix, Aggregate and ITZ




Yip et al. (2005), Grassl and Bolander (2016)

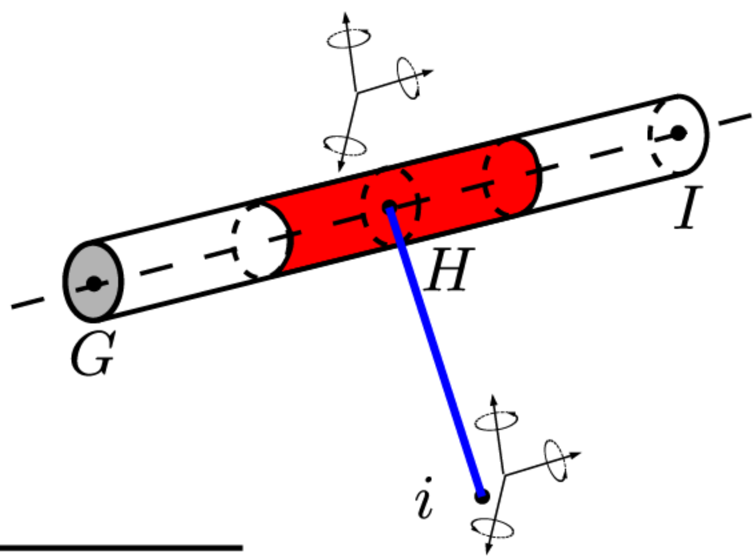
# Network model

## Fibres



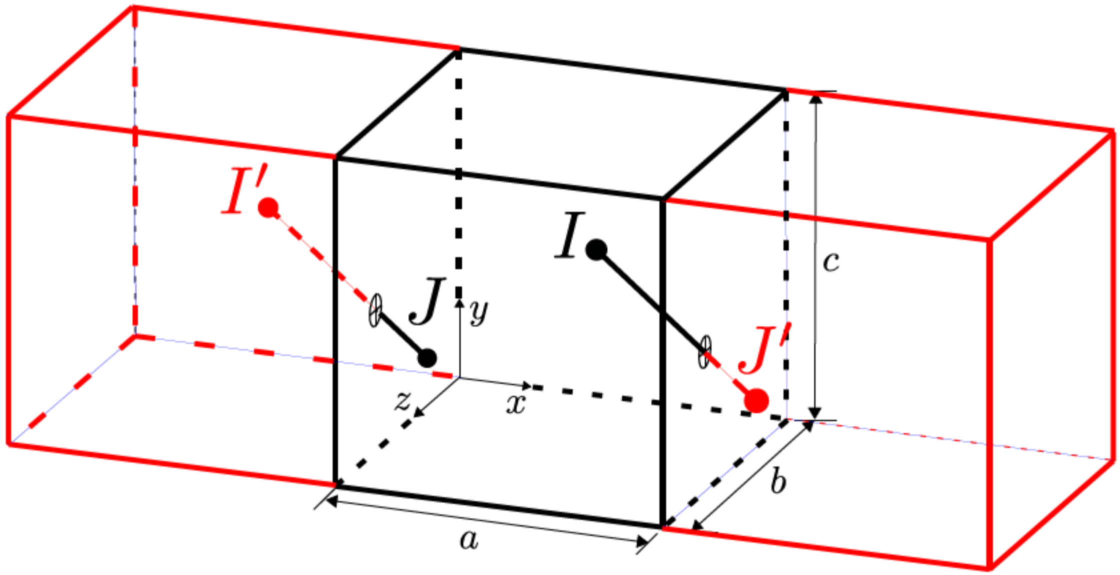

Euler-Bernoulli beams

## Links between fibres and matrix




Yip et al. (2005), Ngo and Scordelis (1967), McGuire et al. (2000)

# Periodic extension




Periodic node position

$$\mathbf{x}' = \mathbf{M}\mathbf{x}$$

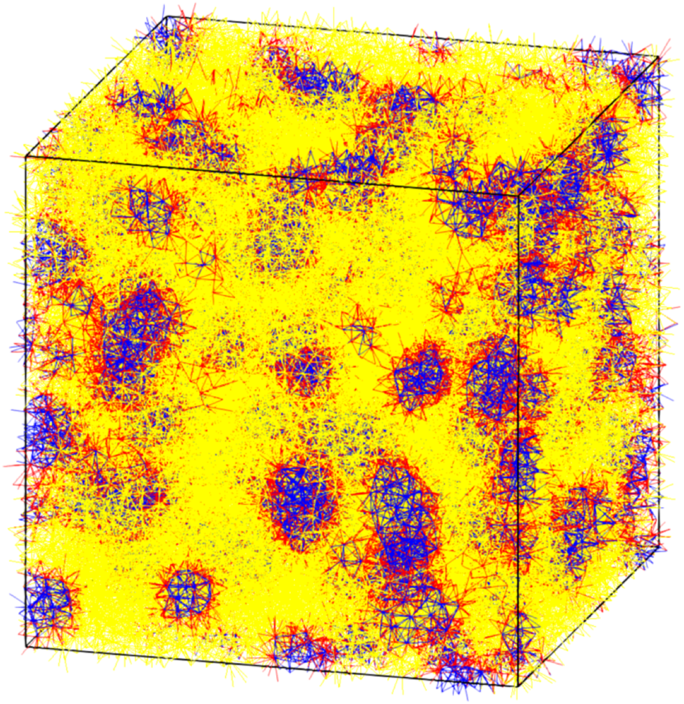
DOFs of periodic node:

$$\mathbf{u}' = \mathbf{T}\begin{pmatrix}\mathbf{u}\\ \mathbf{E}\end{pmatrix}$$

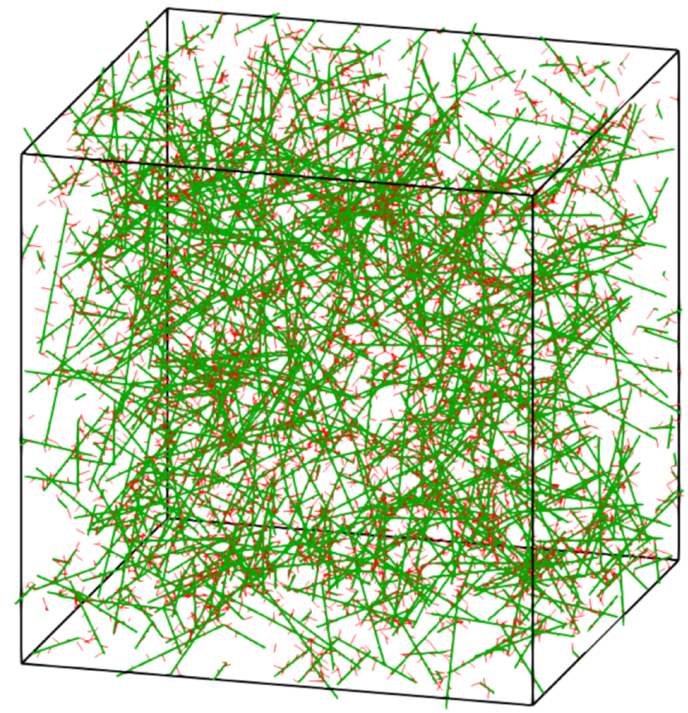
$\mathbf{E}$ Average strain applied to cell

Athanasiadis et al. (2017), Grassl and Jirásek (2010)

# Example of periodic network



Network elements



Beam elements with links

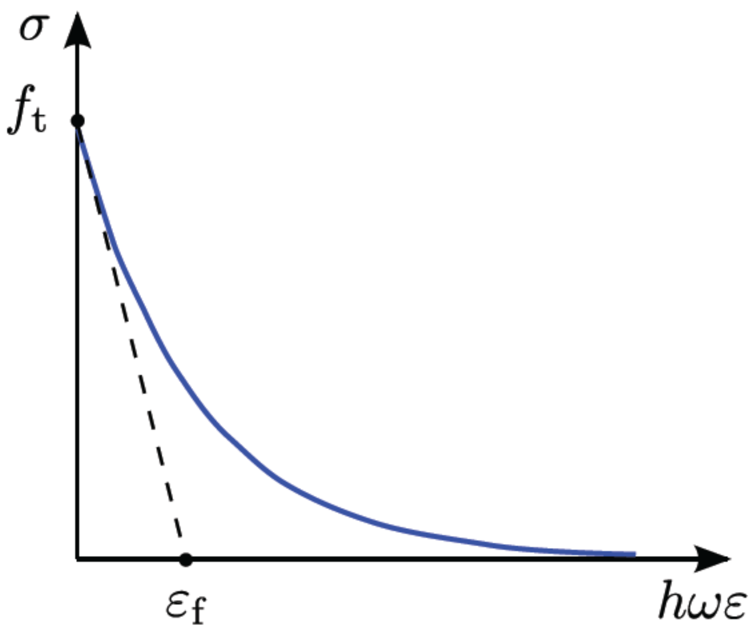
# Material models

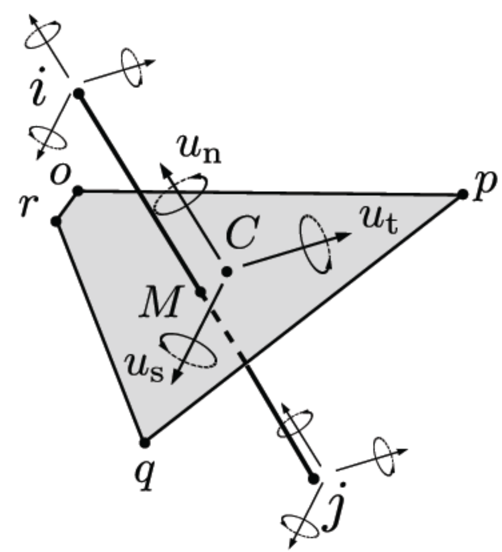
# Material models

## Matrix and ITZ: Scalar damage

$$\boldsymbol{\sigma} = (1 - \omega)\, \mathbf{D}_{\mathrm{e}} \boldsymbol{\varepsilon}$$

$$\dot{d} = \dot{\omega} \frac{1}{2} \boldsymbol{\varepsilon} : \mathbf{D}_{\mathrm{e}} : \boldsymbol{\varepsilon}$$



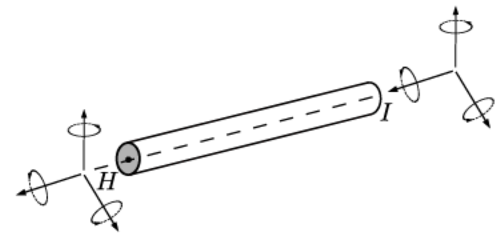

## Aggregates: Elastic

$$\boldsymbol{\sigma} = \mathbf{D}_{\mathrm{e}} \boldsymbol{\varepsilon}$$

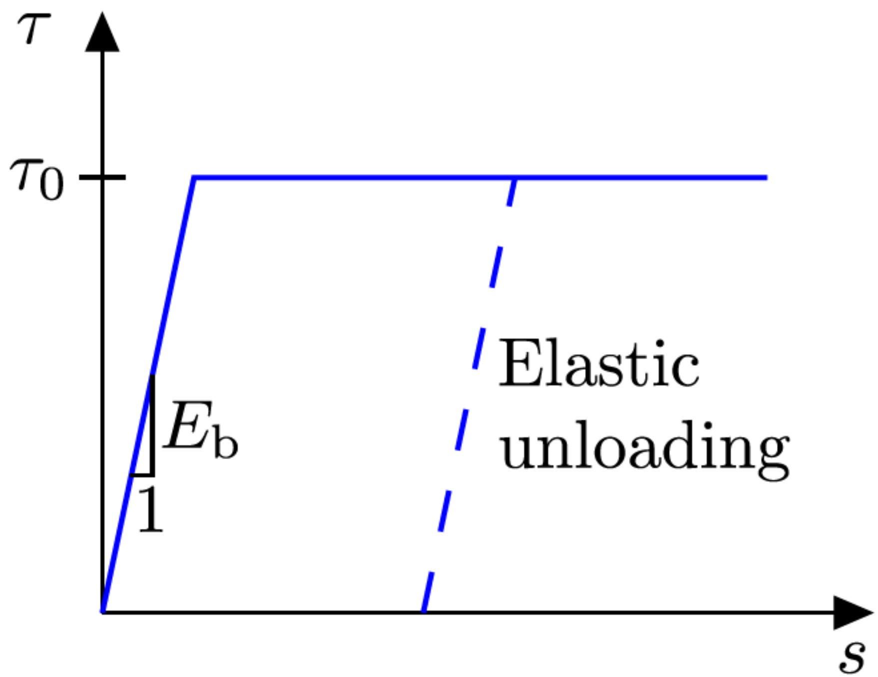
# Material models

Fibres: elastic

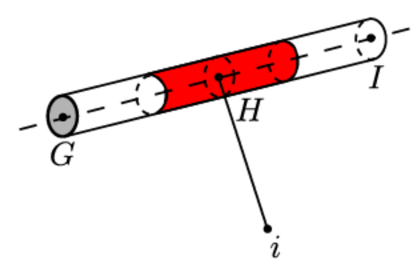
$$\mathbf{F} = \mathbf{K}_\mathrm{e}\left(E_\mathrm{f}, d_\mathrm{f}\right)\mathbf{u}$$




Links: Elasto-plastic in tangential direction

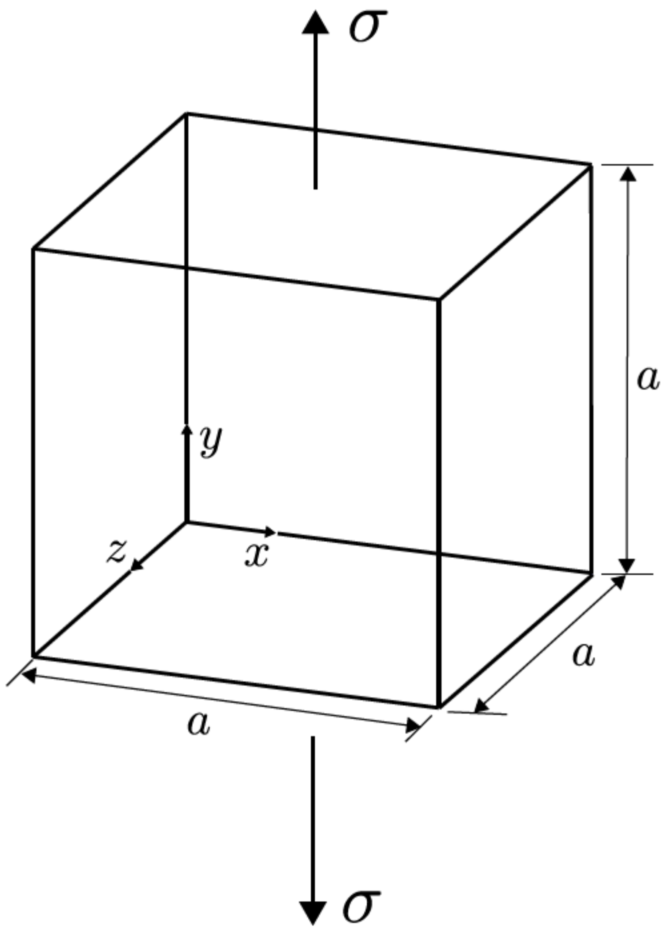



$$\dot{d} = \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_\mathrm{p}\right) : \mathbf{D}_\mathrm{e} : \dot{\boldsymbol{\varepsilon}}_\mathrm{p}$$



# Analyses

# Analyses: Direct tension




Average strain and stress control:

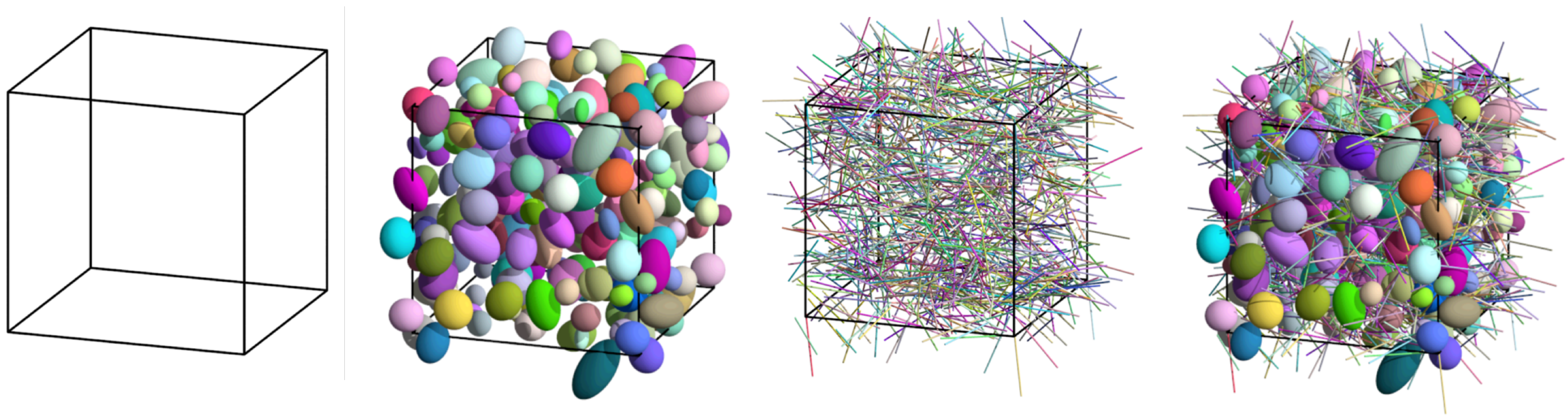
$$E_y > 0$$

$$S_x = S_z = S_{yx} = S_{zx} = S_{yz} = 0$$

# Analyses: Direct tension

## Four groups of analyses

Plain | Aggregates | Fibres | Aggregates +Fibres



10 analyses with random meso-structures and networks for each group

# Analyses: Input

## Meso-scale generation

Aggregates: $\rho_\mathrm{a} = 80\ \%$ $d_\mathrm{min} = 8$ mm $d_\mathrm{max} = 16$ mm

Fibres: $d_\mathrm{f} = 0.75$ mm $\rho_\mathrm{f} = 1\ \%$ $l_\mathrm{f} = 3$ cm

Discretisation: $a = 10$ cm $l_\mathrm{min} = 3$ mm

## Material parameters

Matrix: $f_\mathrm{t} = 3$ MPa $E = 30$ GPa $G_\mathrm{F} = 120$ J/m$^2$

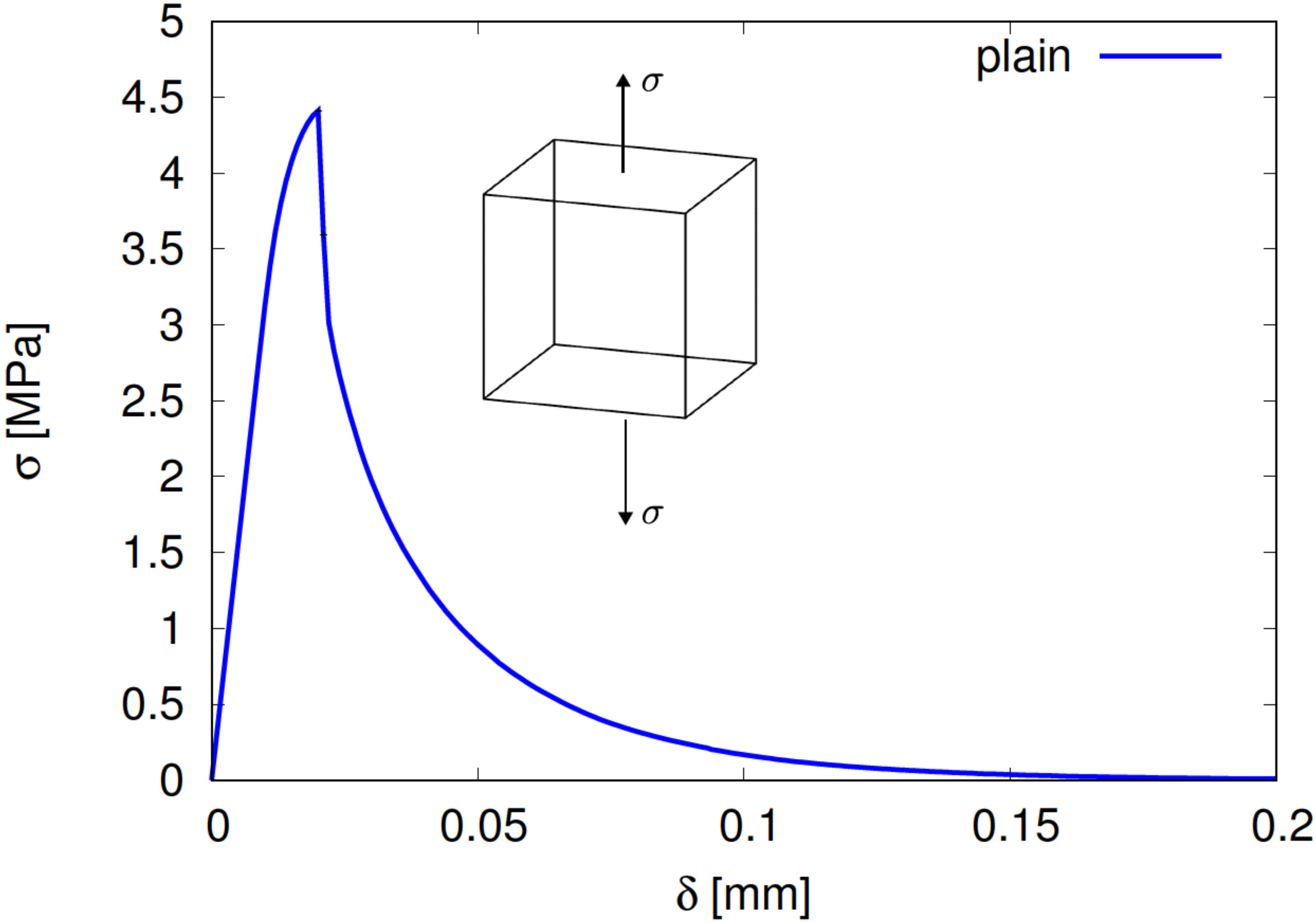
Aggregate: $E = 90$ GPa

ITZ: $E = 45$ GPa $f_\mathrm{t} = 1.5$ MPa $G_\mathrm{F} = 60$ J/m$^2$

Fibres+Links: $E_\mathrm{f} = 200$ GPa $\tau = 3$ MPa $E_\mathrm{b} = 3000$ GPa

# Results

# Results: Stress-Displacement



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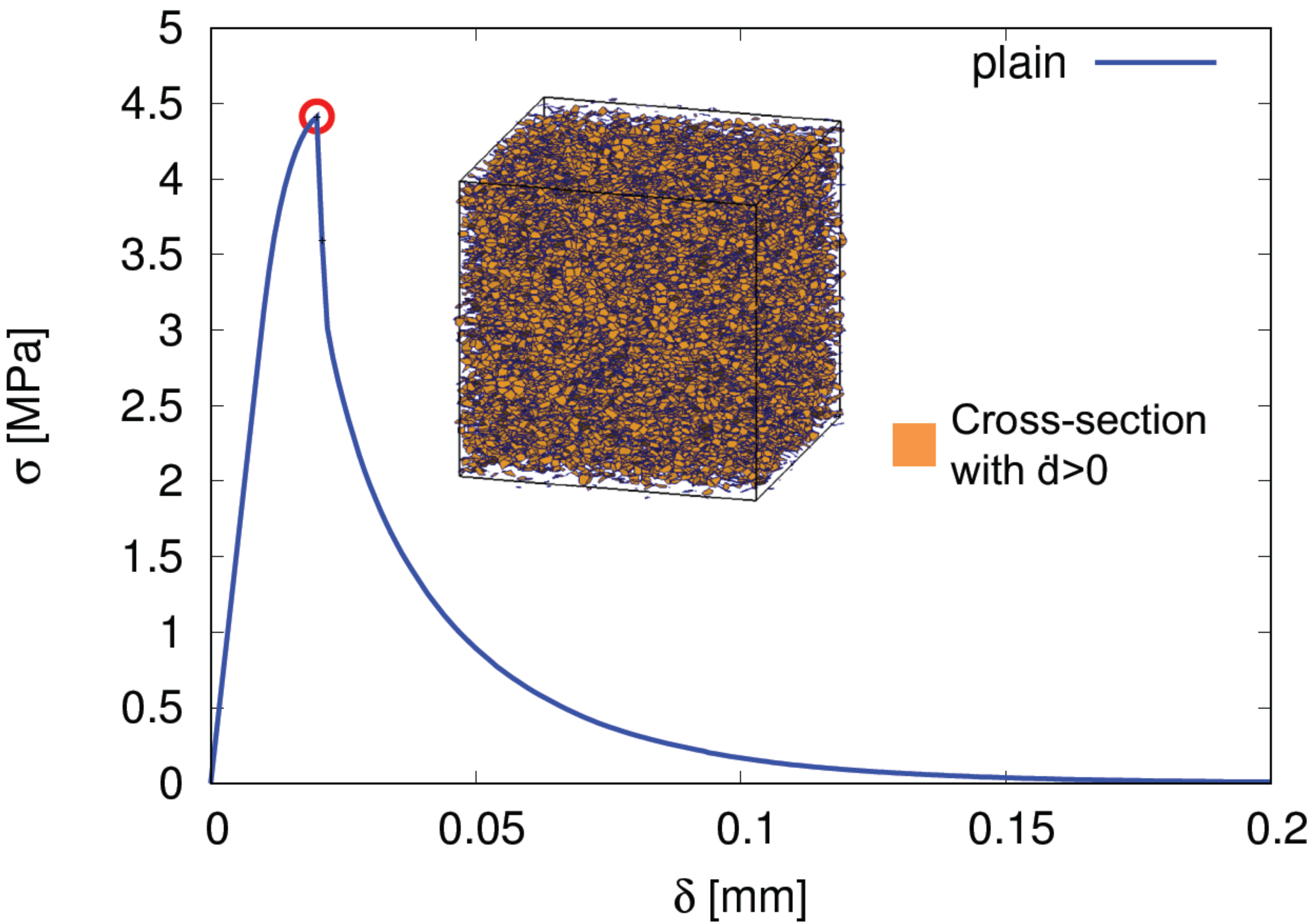

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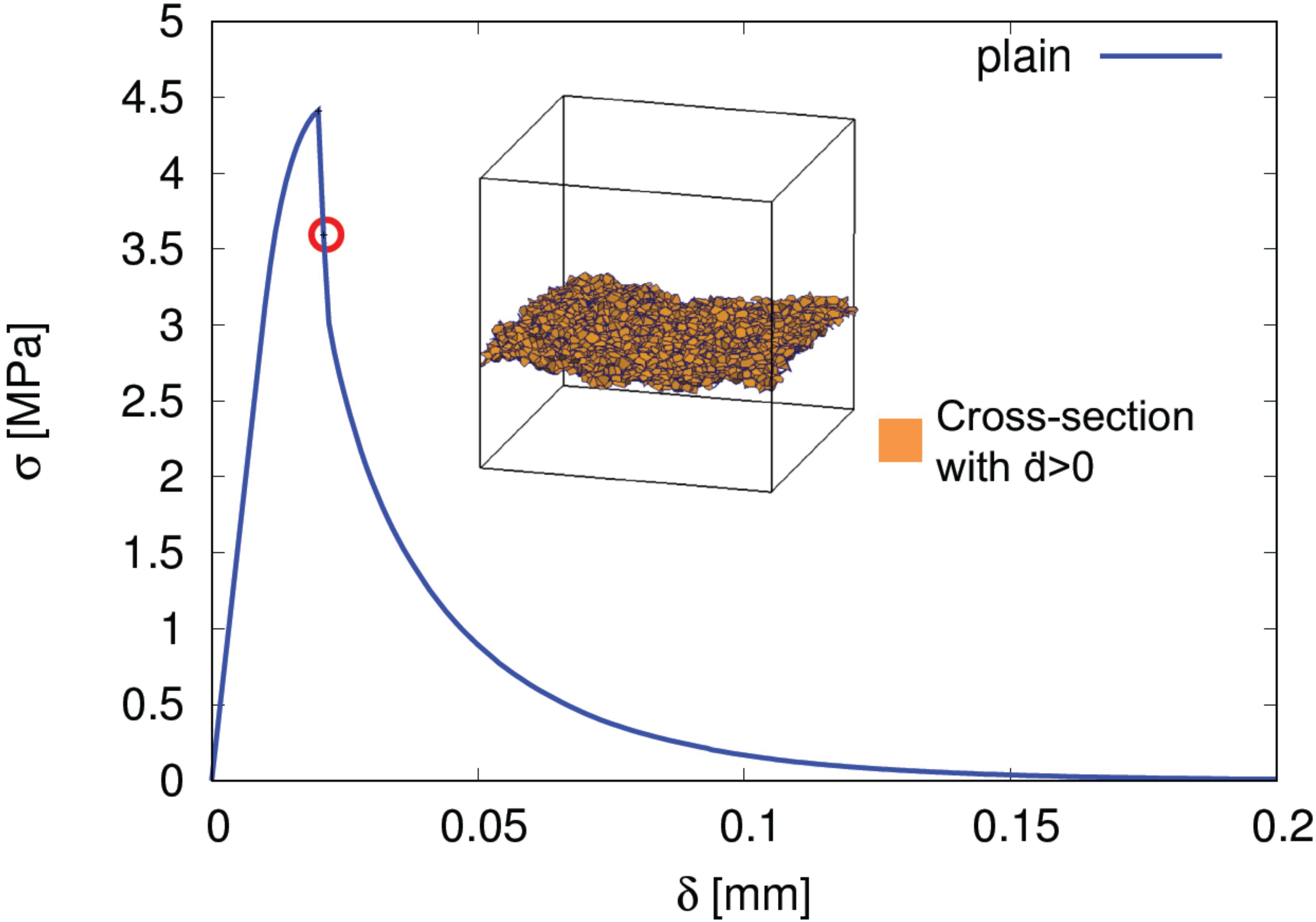

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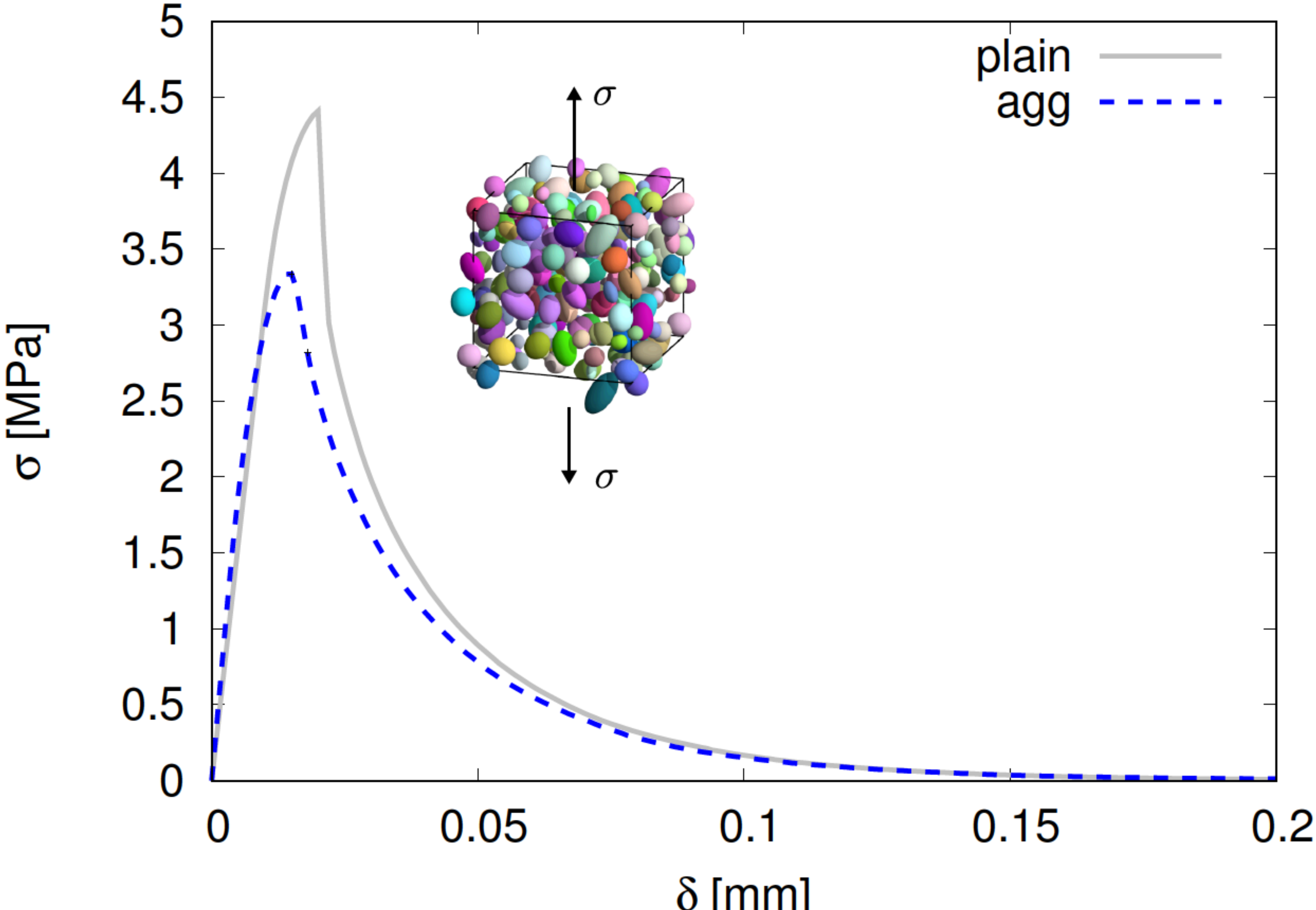

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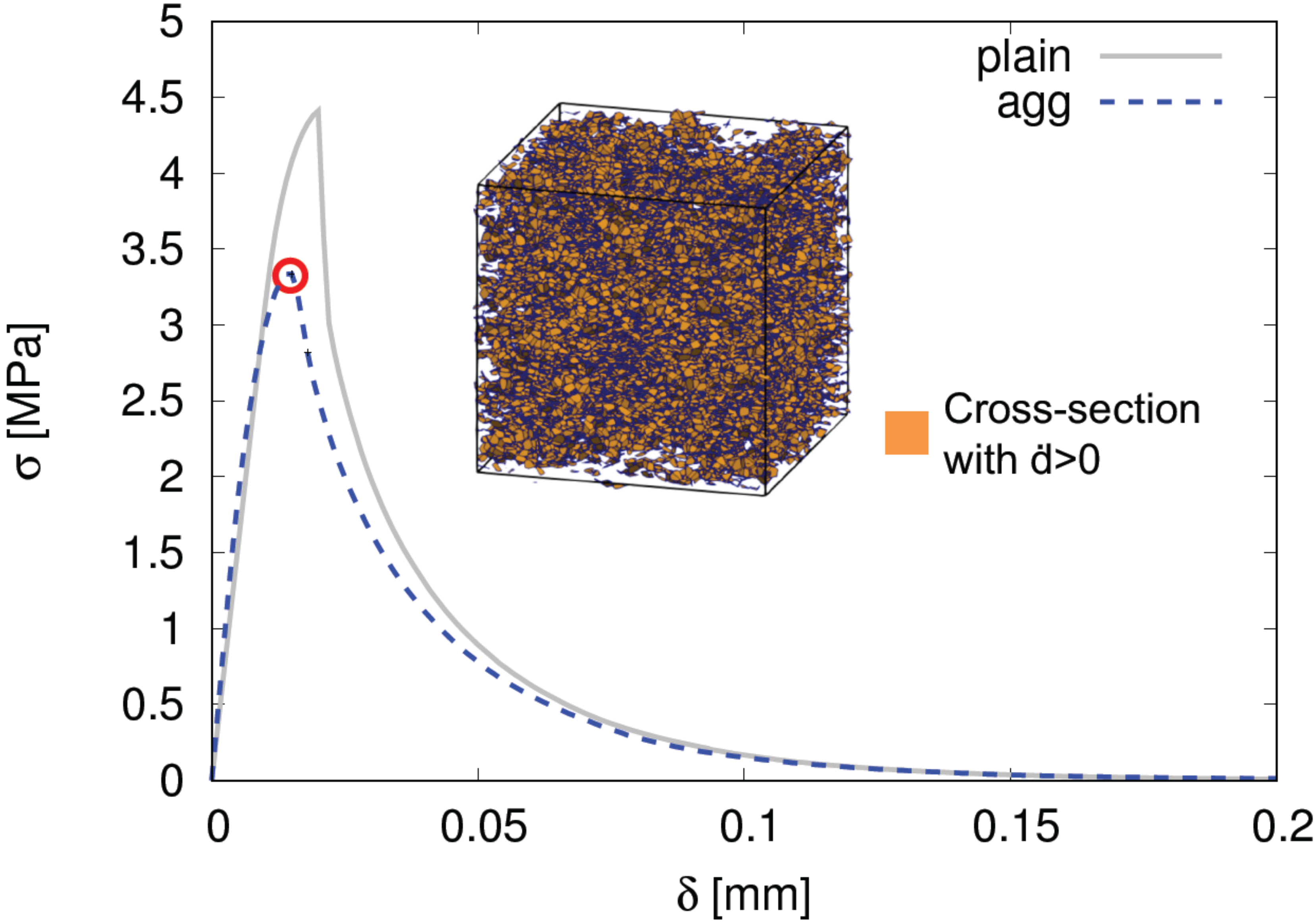

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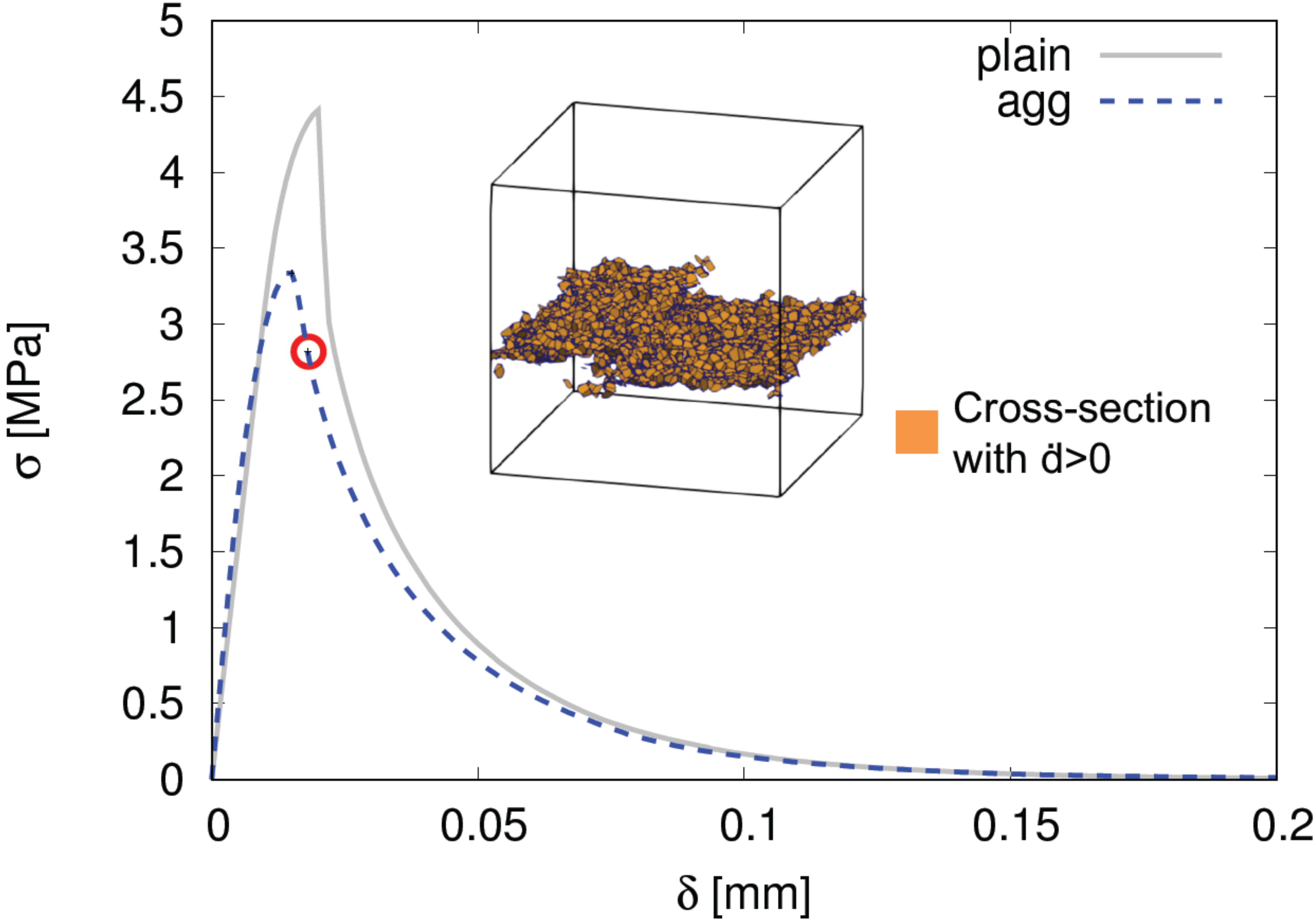

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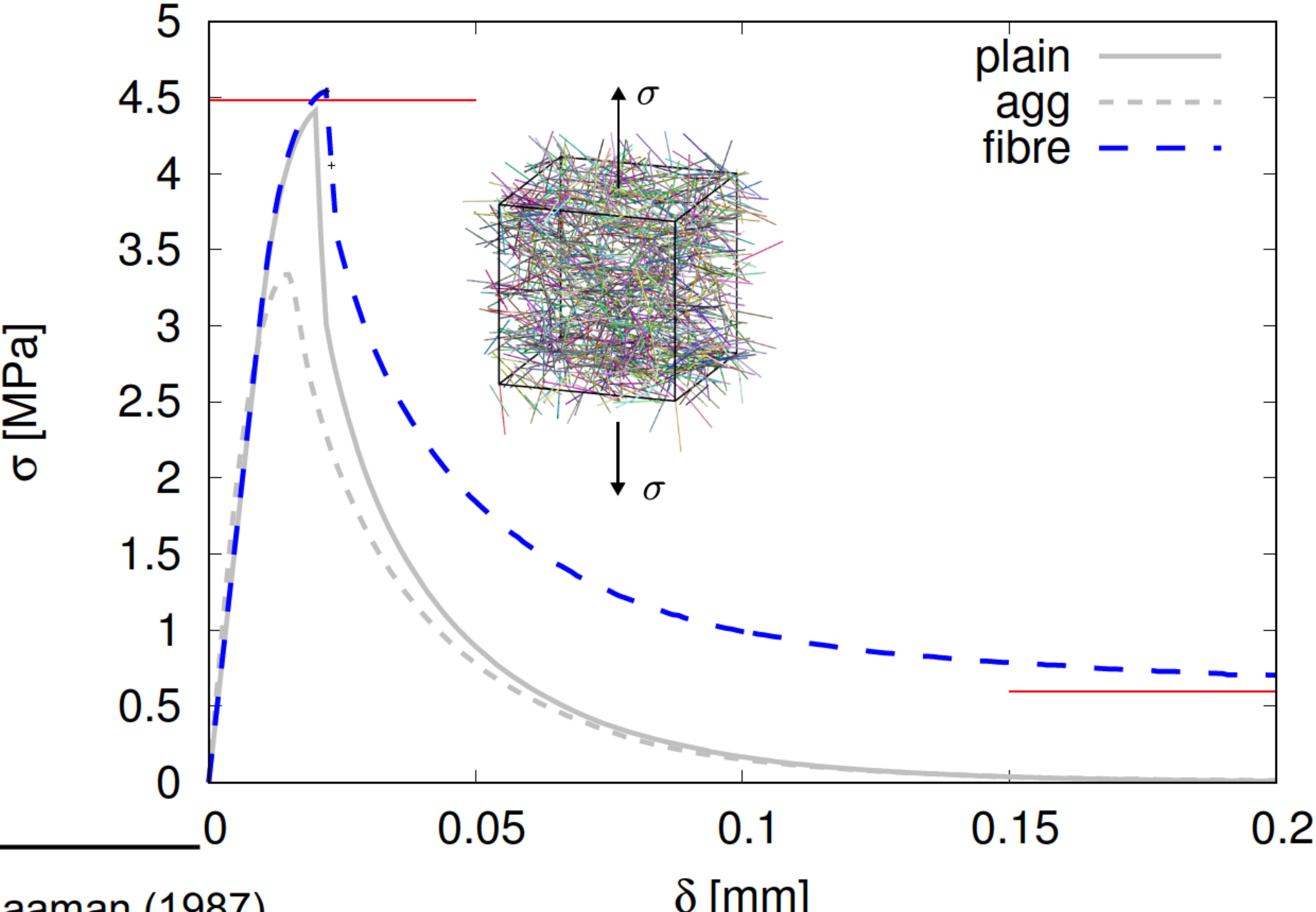
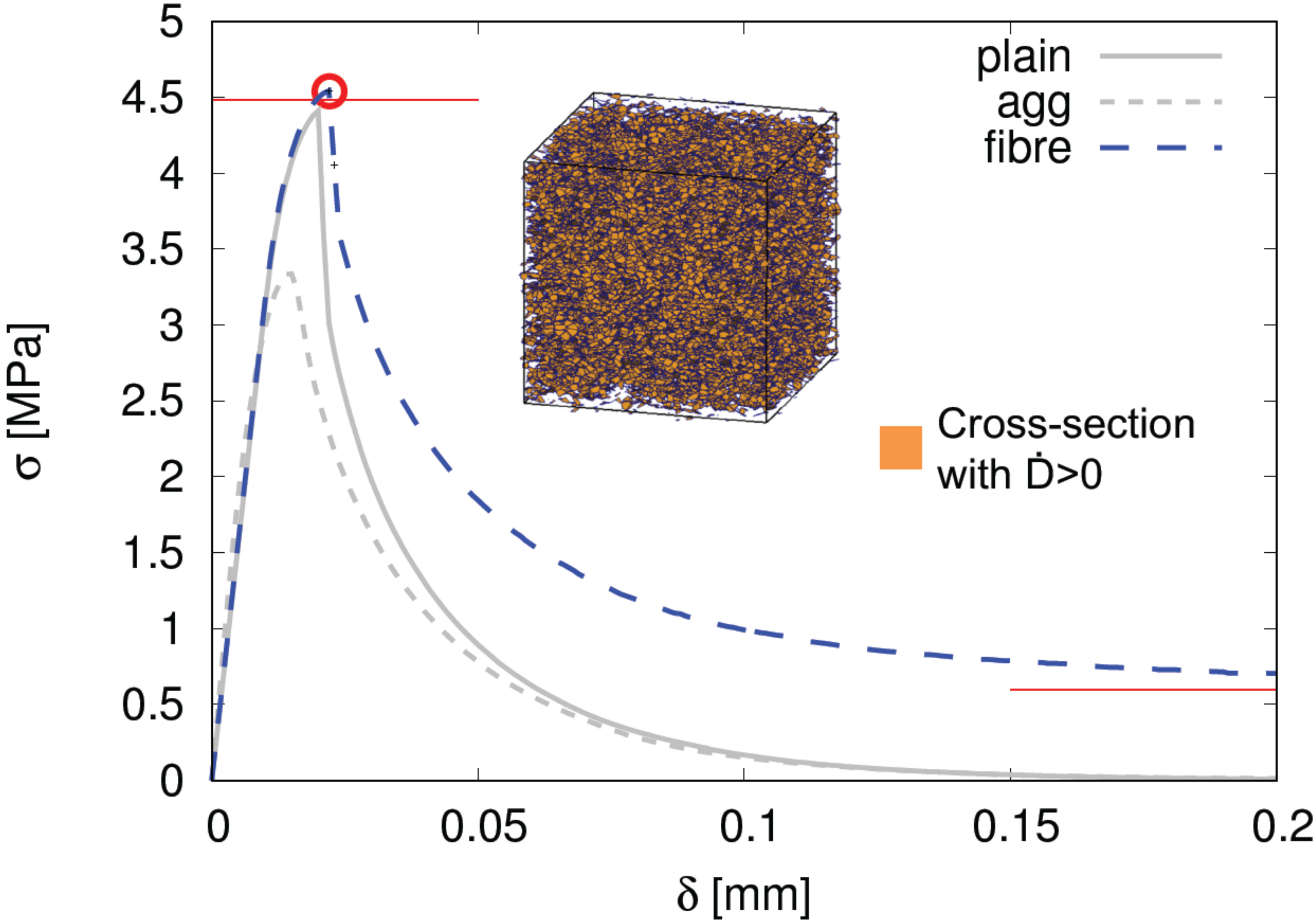



Naaman (1987)

# Results: Stress-Displacement



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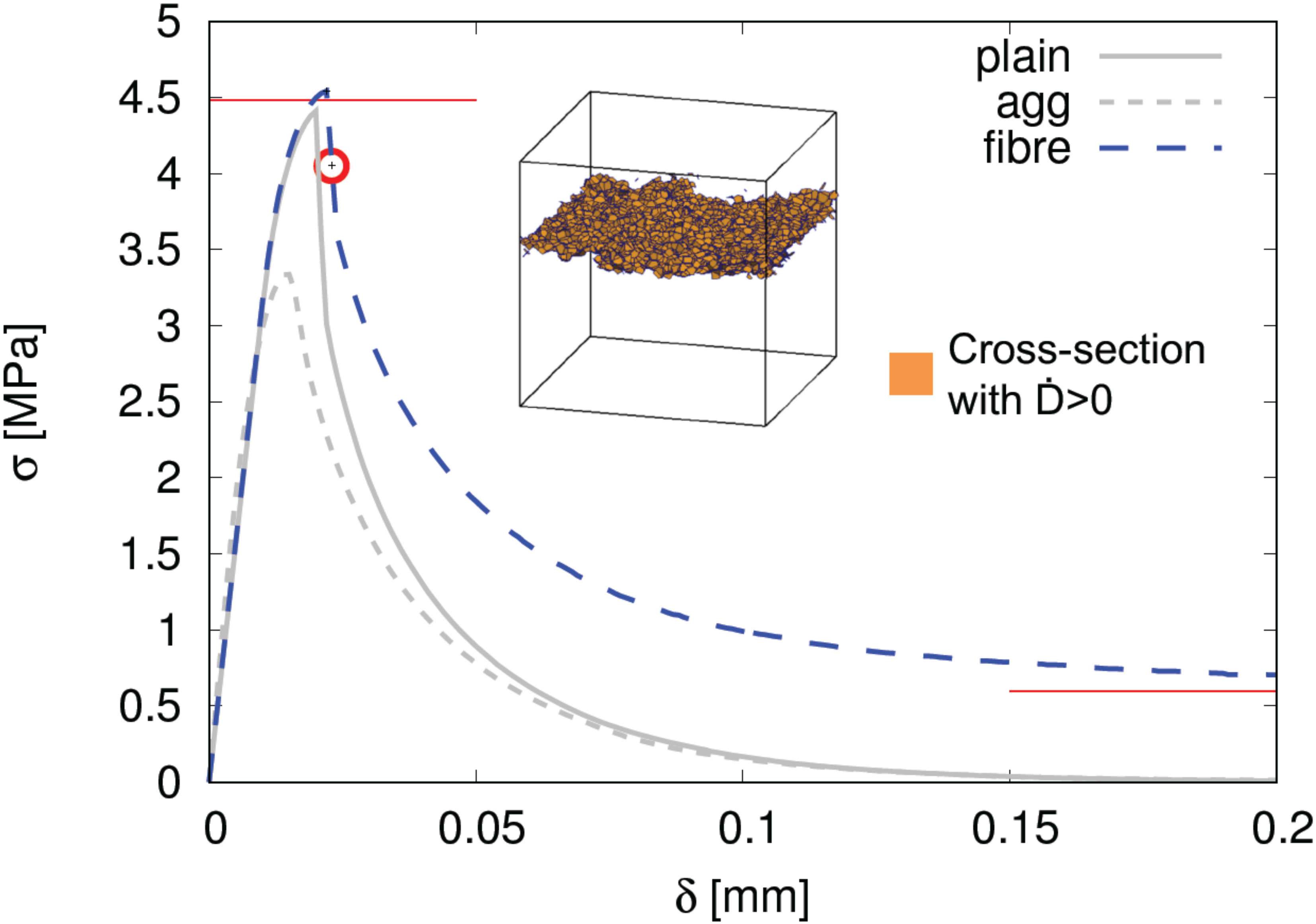

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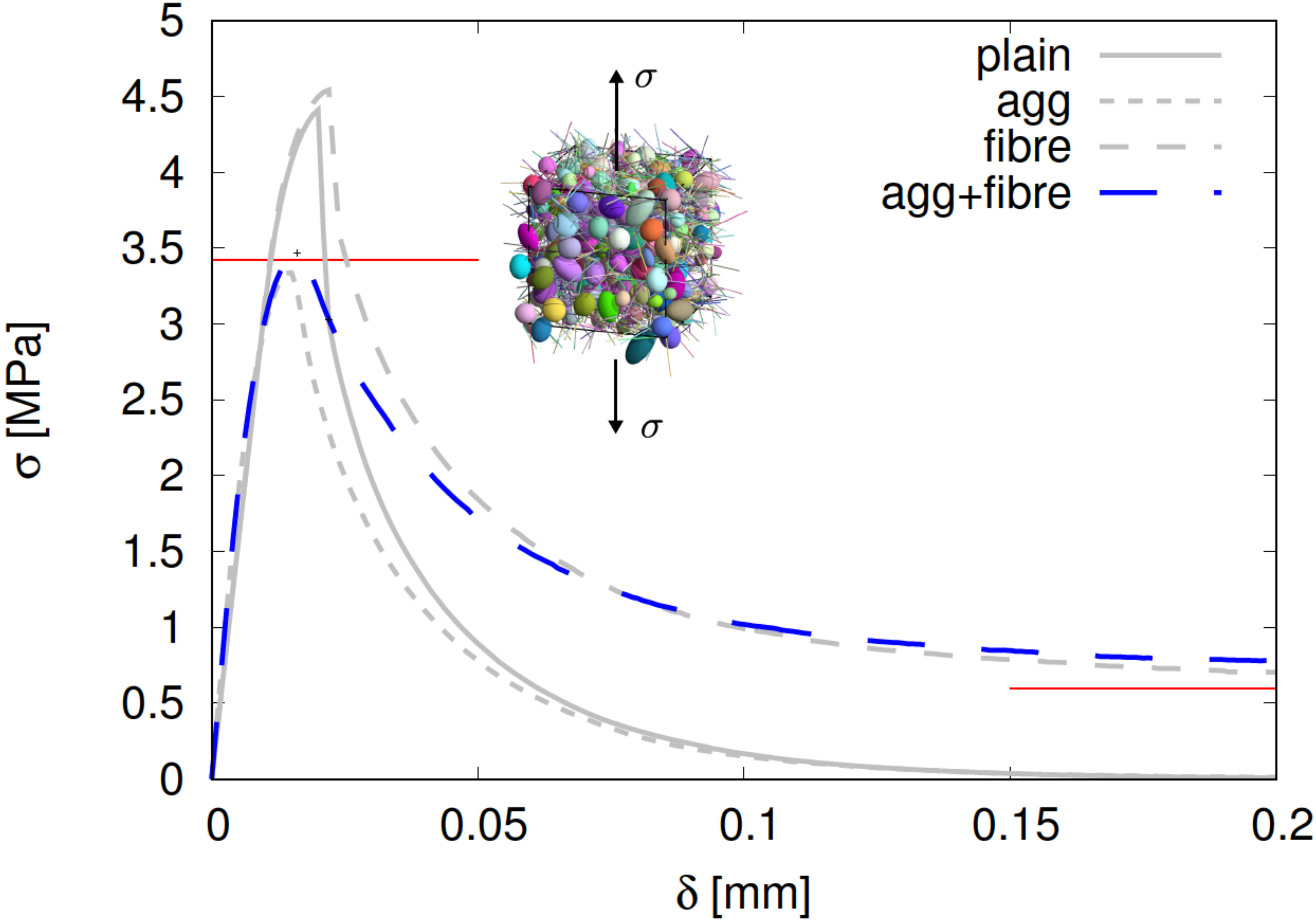

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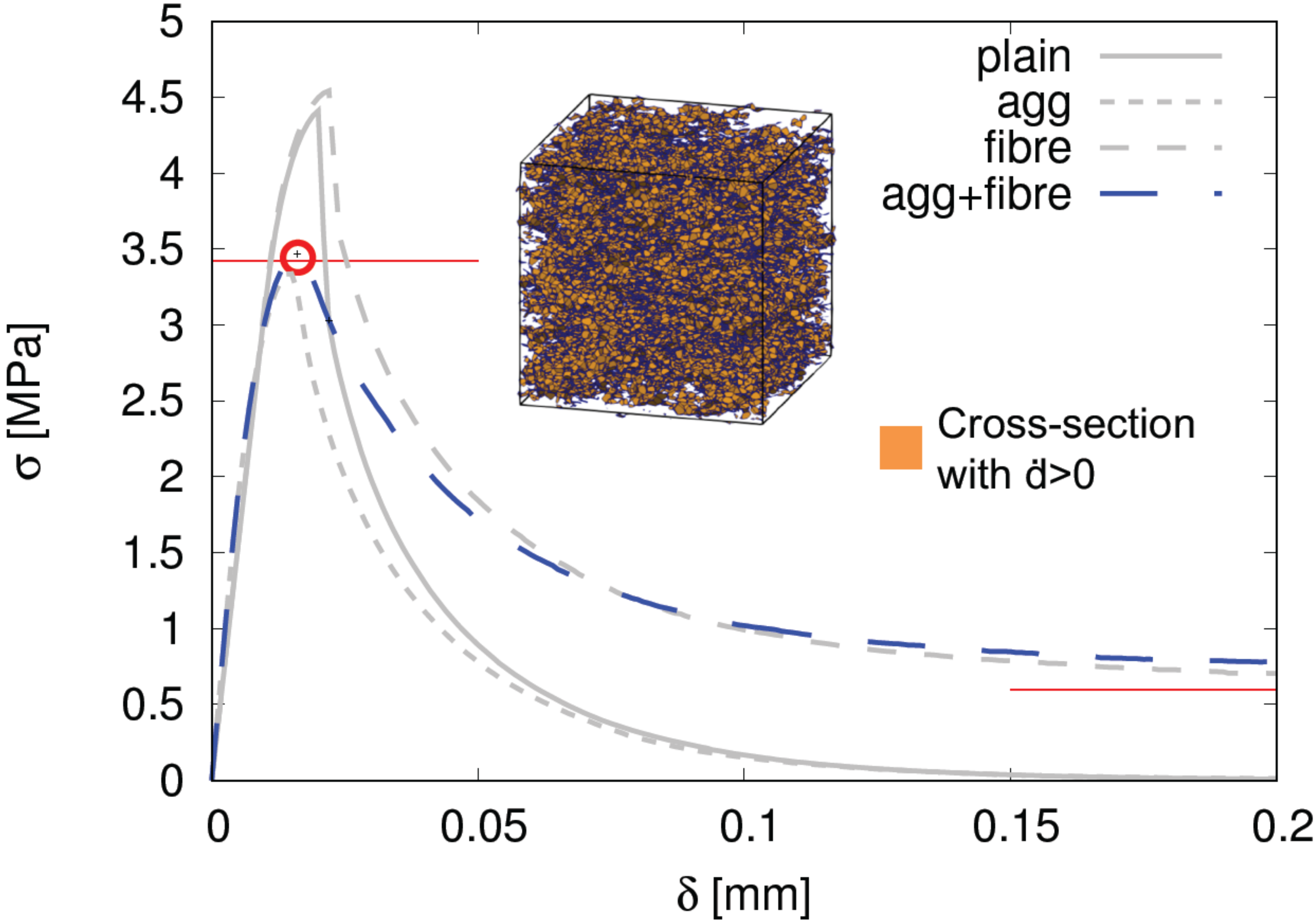

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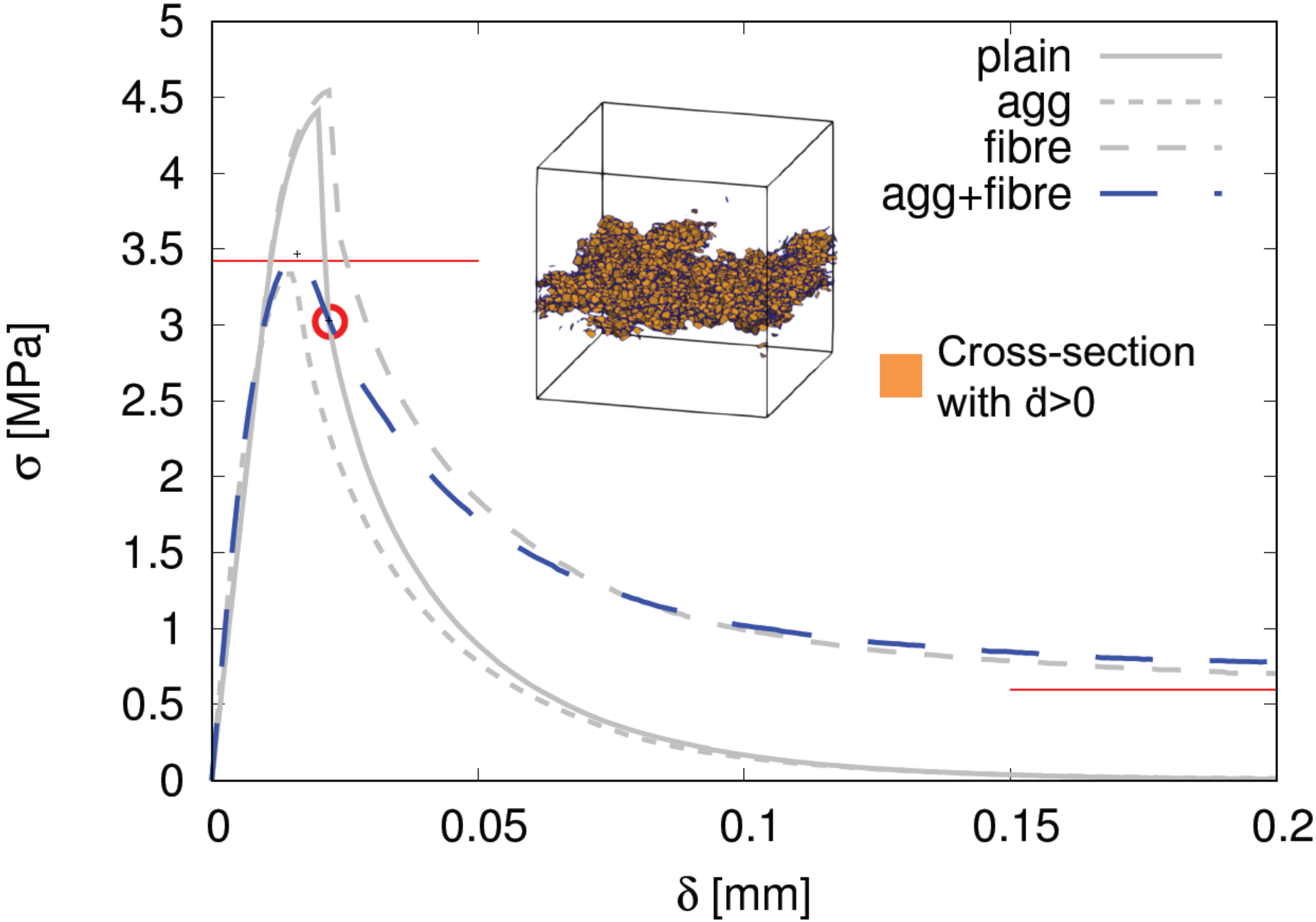
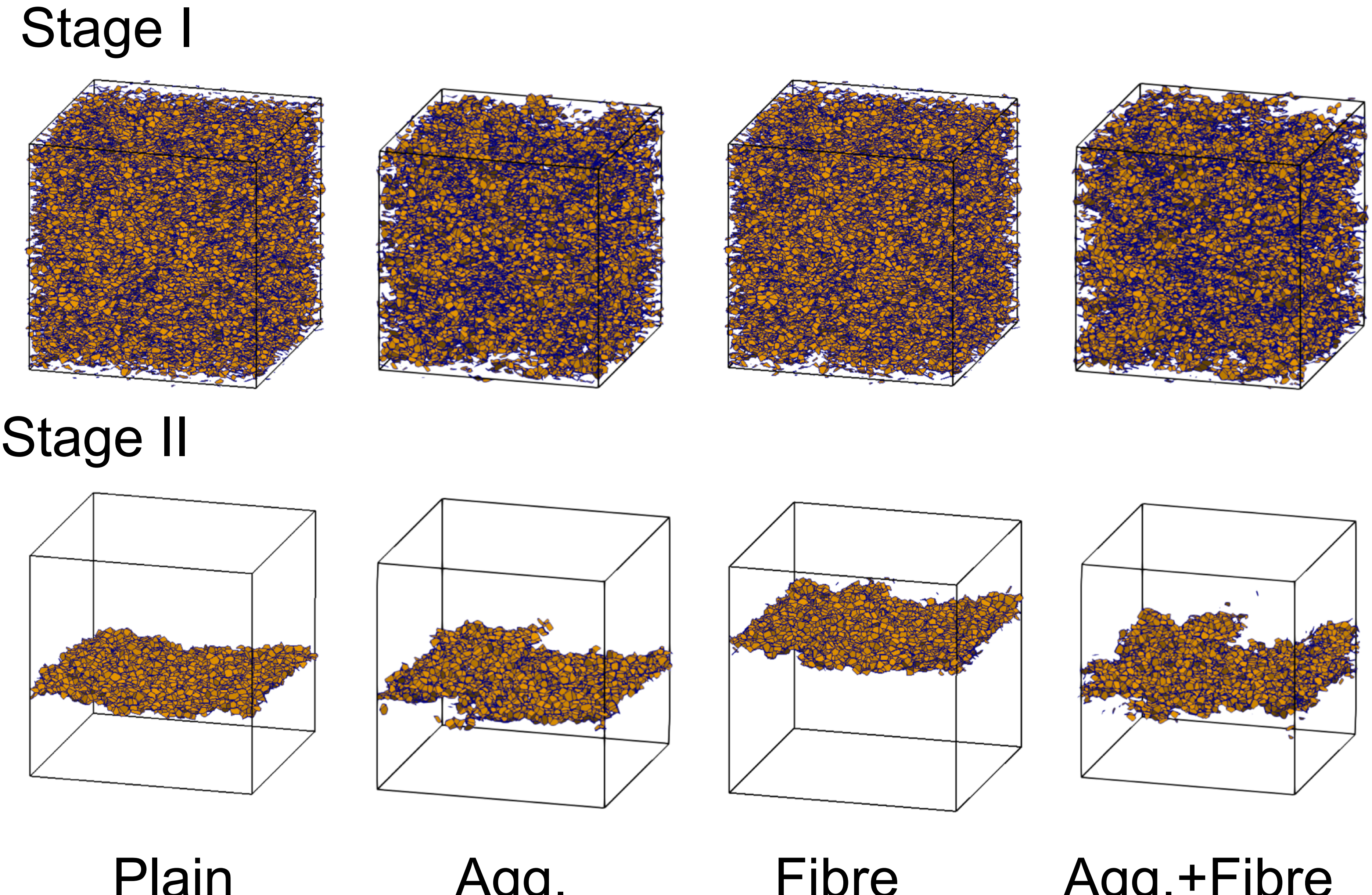


# Crack patterns in matrix and ITZ



# Roughness evaluation

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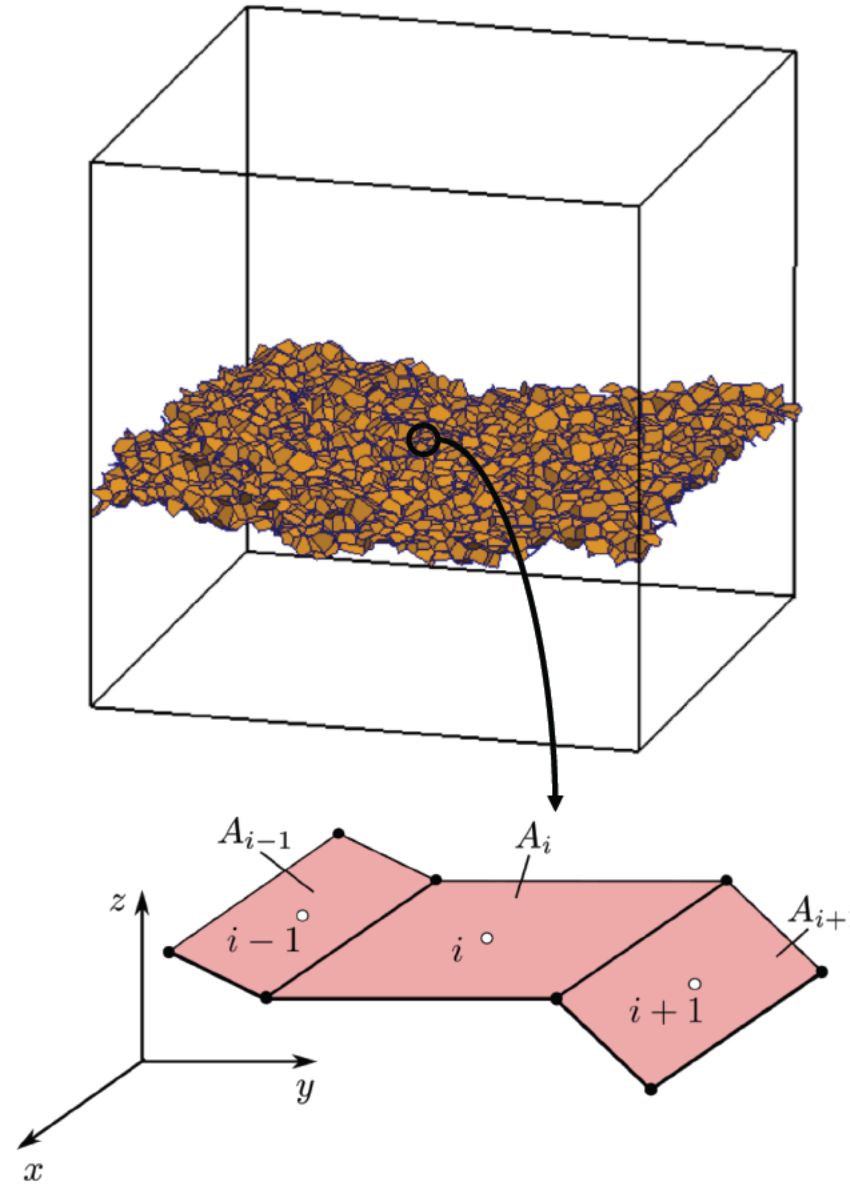
$$\bar{z} = \sum_{i=1}^{N} w_i z_i$$

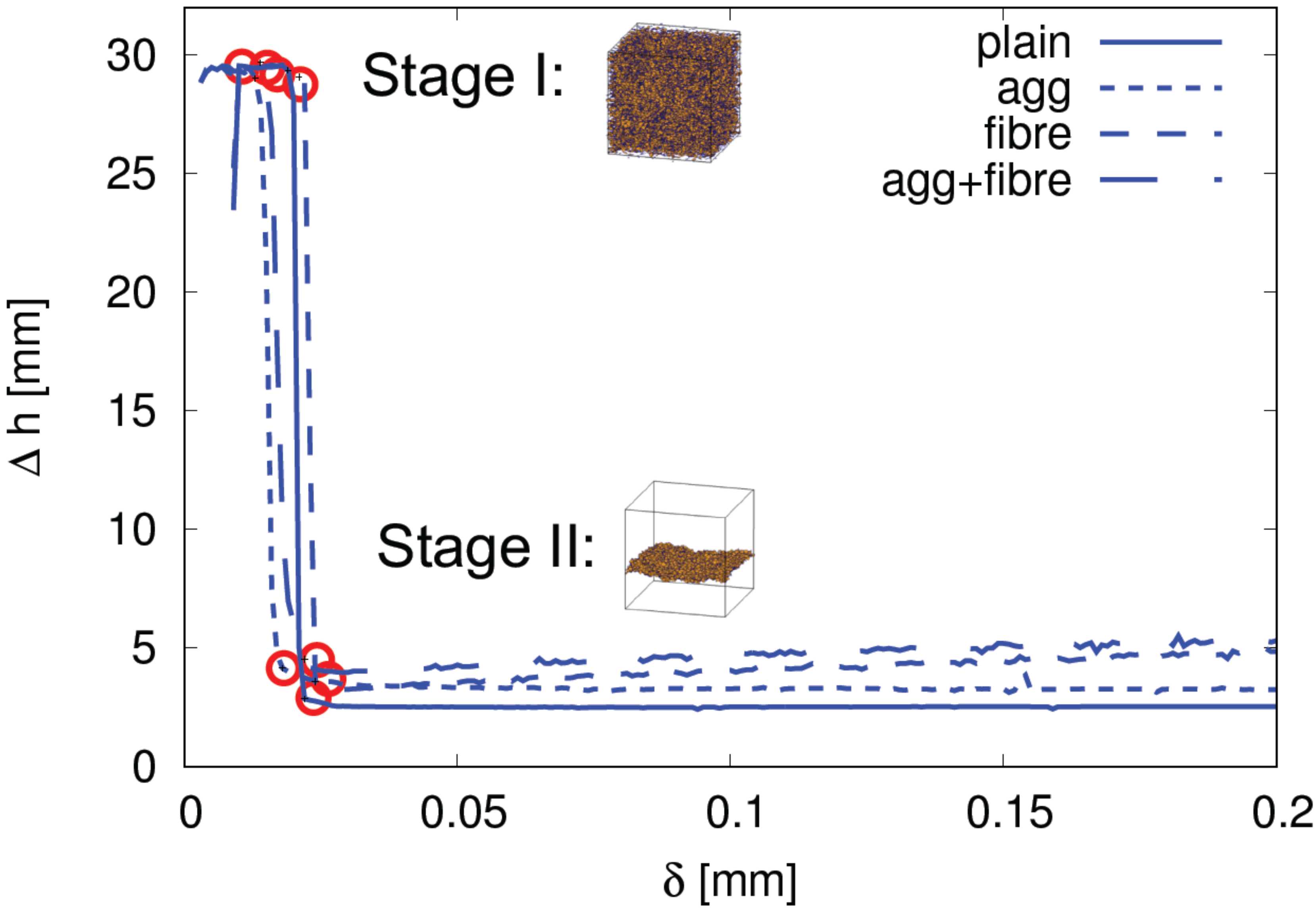
$$w_i = \frac{A_i \Delta d_i}{\sum_{k=1}^{N} A_k \Delta d_k}$$

$$\Delta h = \sqrt{\sum_{i=1}^{N} w_i (z_i - \bar{z})^2}$$




Xenos et al. (2015)

# Results: Roughness



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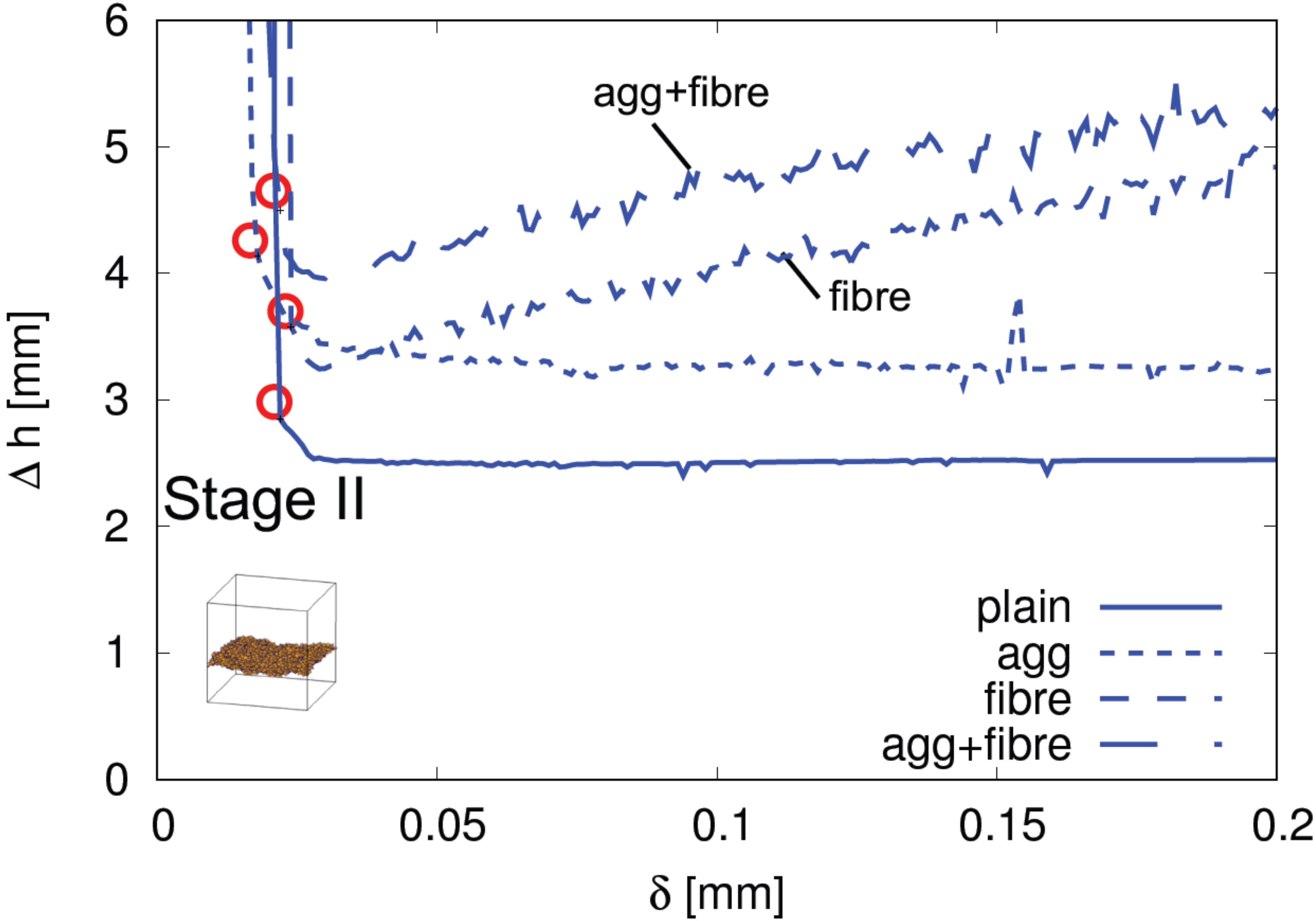

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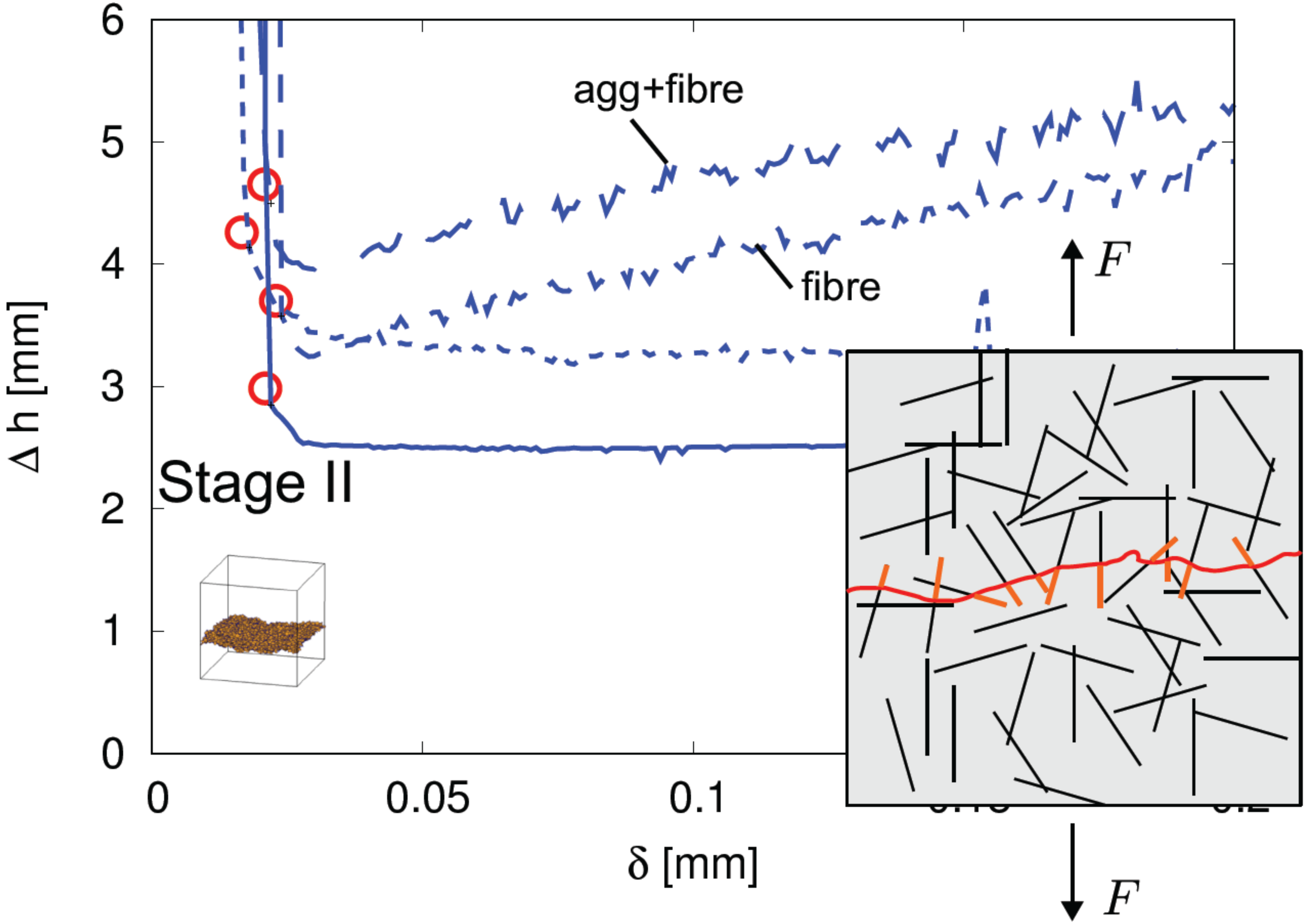


The end

# Backup slides

# Analytical expressions

At peak:

$$\sigma_{\mathrm{cc}} = \sigma_{\mathrm{mu}}\left(1 - \rho_{\mathrm{f}}\right) + \alpha_1\alpha_2\tau_0\rho_{\mathrm{f}}l_{\mathrm{f}}/d_{\mathrm{f}}$$

After cracking:

$$\sigma_{\mathrm{pc}} = 4\lambda_1\lambda_2\tau_0\rho_{\mathrm{f}}l_{\mathrm{f}}/d_{\mathrm{f}}$$

$\sigma_{\mathrm{mu}}$ Matrix strength

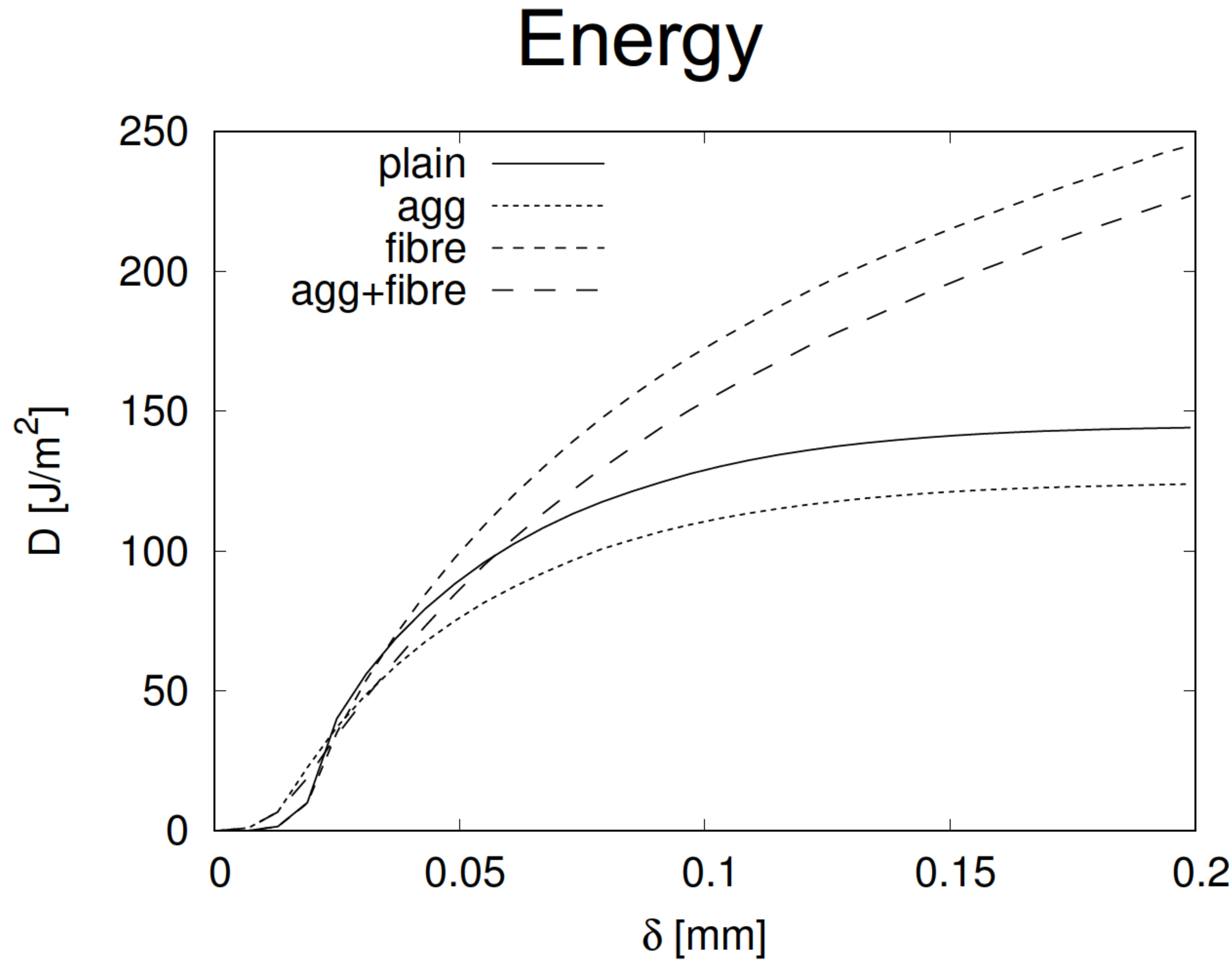
$\alpha_1$ Fibre orientation

$\alpha_2$ Fraction of bond strength mobilised

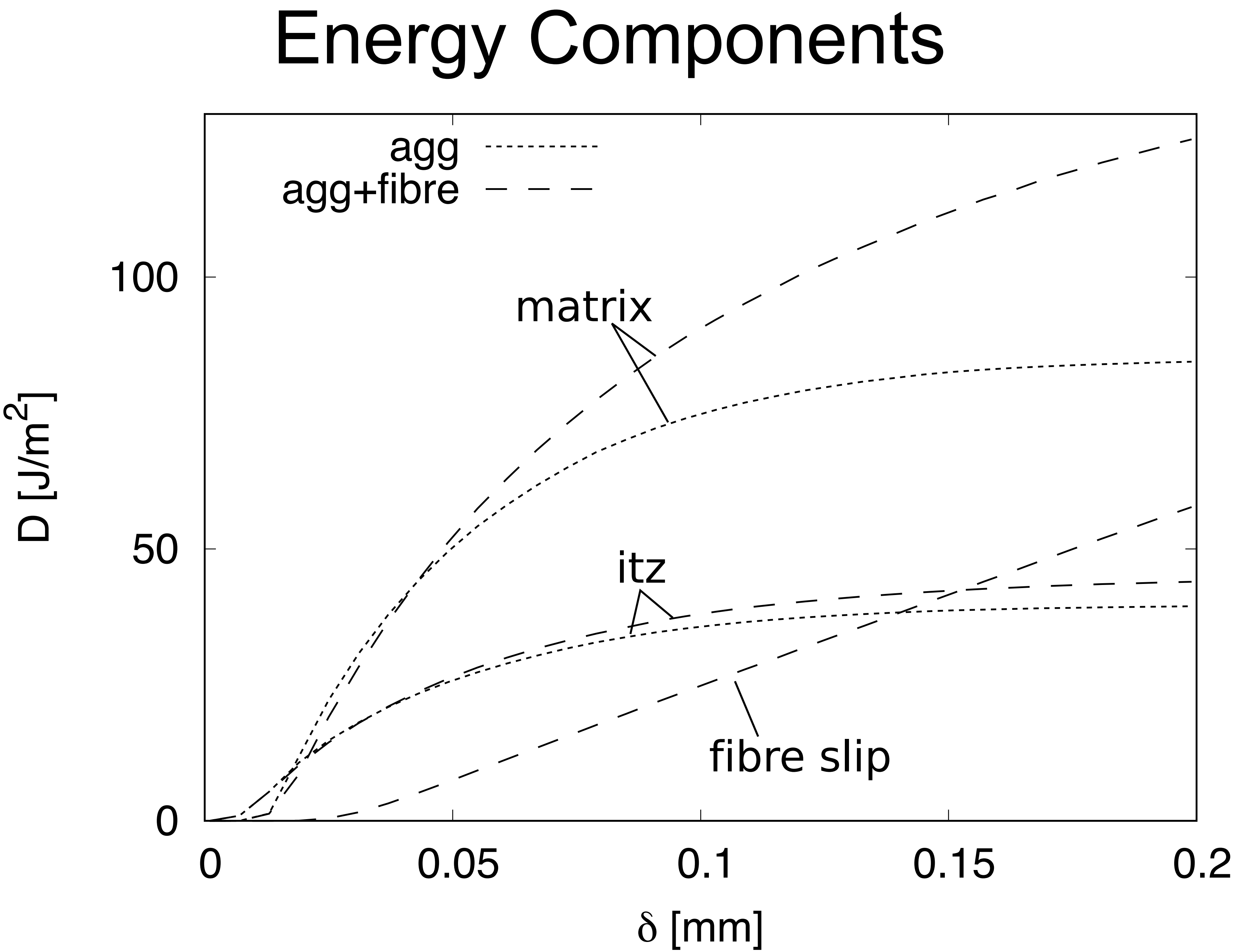
$\lambda_1$ Average pullout length

$\lambda_2$ Postcracking orientation efficiency

Naaman (1987)



Energy
250
200
150
100
50
0
plain
agg
fibre
agg+fibre
D [J/m2]
0
0.05
0.1
0.15
0.2
δ [mm]



Energy Components
agg
agg+fibre
matrix
itz
fibre slip
D [J/m2]
δ [mm]
0
50
100
0
0.05
0.1
0.15
0.2