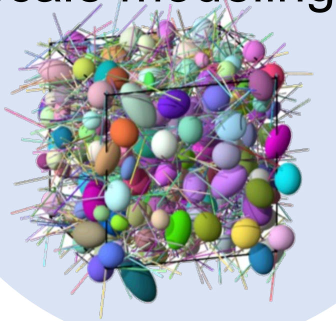
University of Glasgow

Exploring the mechanics of corrosion induced cracking in reinforced concrete

Peter Grassl

Concrete Mechanics for Performance Based Design

Meso/micro scale modeling



Improving understanding

Constitutive modelling

damage

$$\boldsymbol{\sigma} = (1-\omega)\mathbf{D}_{\mathrm{e}} : \boldsymbol{\varepsilon}$$

plasticity

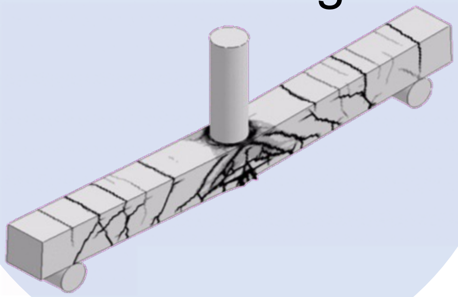
$$\boldsymbol{\sigma} = \mathbf{D}_{\mathrm{e}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{p}})$$

damage-plasticity

$$\boldsymbol{\sigma} = (1-\omega_{\mathrm{t}})\bar{\boldsymbol{\sigma}}_{\mathrm{t}} + (1-\omega_{\mathrm{c}})\bar{\boldsymbol{\sigma}}_{\mathrm{c}}$$

$$\bar{\boldsymbol{\sigma}} = \mathbf{D}_{\mathrm{e}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{p}}) = \bar{\boldsymbol{\sigma}}_{\mathrm{t}} + \bar{\boldsymbol{\sigma}}_{\mathrm{c}}$$

Structural modelling



http://petergrassl.com

Why Concrete?

Most used construction material in the world

7% of anthropogenic CO_2 emissions (mainly from cement production)

Use of concrete predicted to double in the middle of century

230 billion cubic metres of new construction by 2060 (City of size of Paris every week)

Dean et al. (2016)

What can we do?

Models for predicting response of concrete

New materials with improved performance

Techniques for retrofitting and repairing structures

Improving understanding of concrete behaviour

PhD students:

Xiaowei Liu: Dynamic loading

Gumaa Abdelrhim: Structural collapse

Ifiok Ekop: Strengthening with FRP

Chao Zhou: Fibre reinforced composites

Ismail Aldellaa: Corrosion induced cracking in reinforced concrete

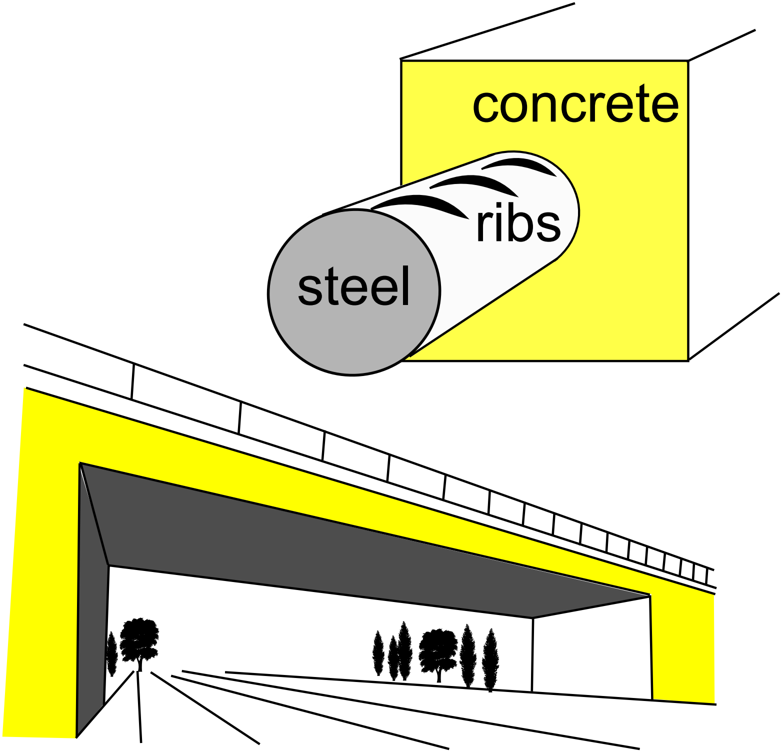
Corrosion induced cracking

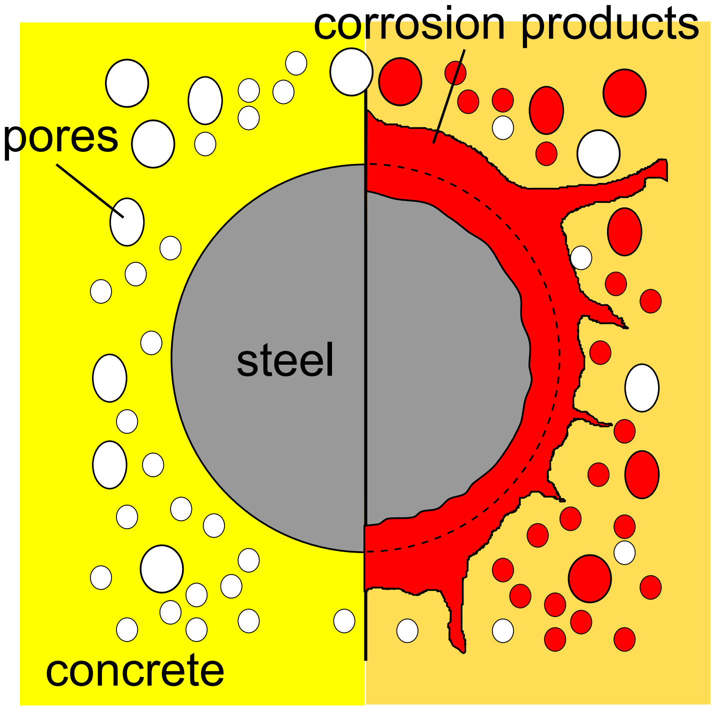
Main references:

I. Aldellaa, P. Havlásek, M. Jirásek, P. Grassl. Engineering Fracture Mechanics, April 2022, Volume 264, 108310.

I. Aldellaa and P. Grassl, 2023, In preparation.

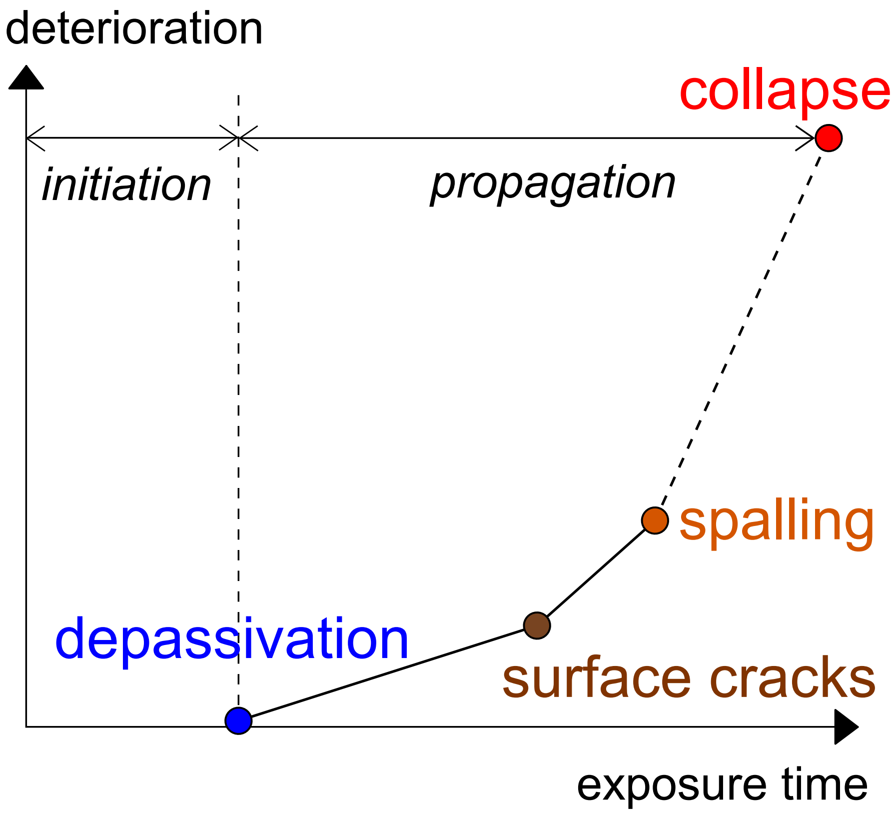
Background





Corrosion rate: $x_{\mathrm{cor}}/t \propto i_{\mathrm{cor}}$

Background



Ref: Tuutti (1982)

Main causes:

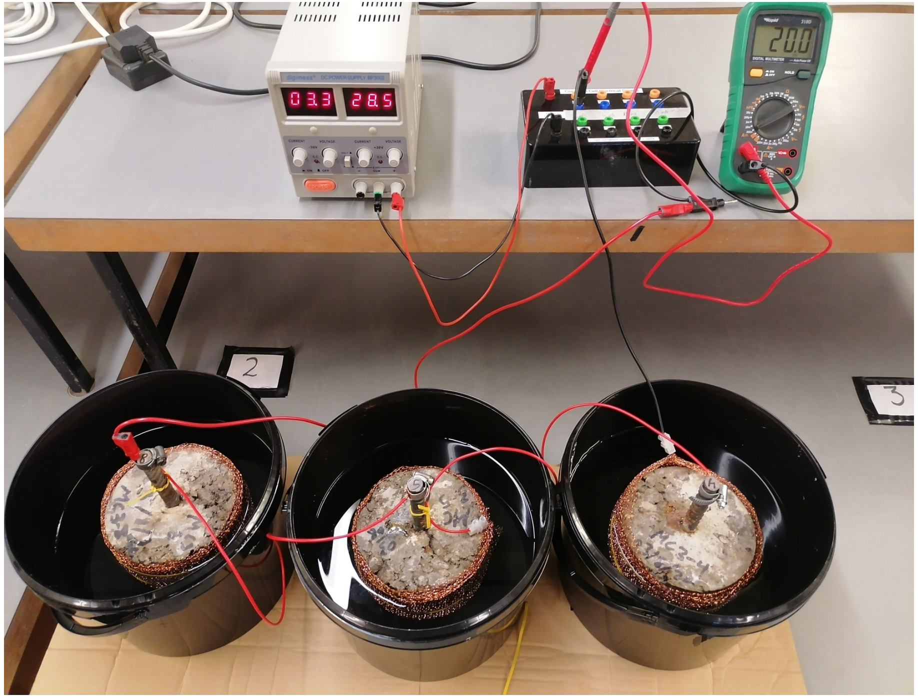
Carbonation

Ingress of chloride irons

Often $i_{\mathrm{cor}} < 10\ \mu\mathrm{A/cm}^2$

Cracking after years!

Background



Accelerated process in the lab:

$$i_{\mathrm{cor}} > 100\ \mu\mathrm{A}/\mathrm{cm}^2$$

Cracking after days!

How to link lab results to field applications?

Ref: Aldellaa and Grassl (2023)

Aim

Understand long-term effects involved in corrosion induced cracking.

Provide suggestions how to incorporate these effects in predictive modelling approaches.

Methodology

Use thick-walled cylinder models supported by lattice analyses and experiments.

Faraday's laws of electrolysis

$$\frac{x_{\mathrm{cor}}}{t} = \frac{M}{n\rho F} i_{\mathrm{cor}}$$

$$\frac{x_{\mathrm{cor}}}{t} = 0.032 i_{\mathrm{cor}}$$

$[\mu\mathrm{m/day}]$ $\quad$ $[\mu\mathrm{A/cm^2}]$

$$M = 55.9\ \mathrm{g/mol}$$

$$n = 2$$

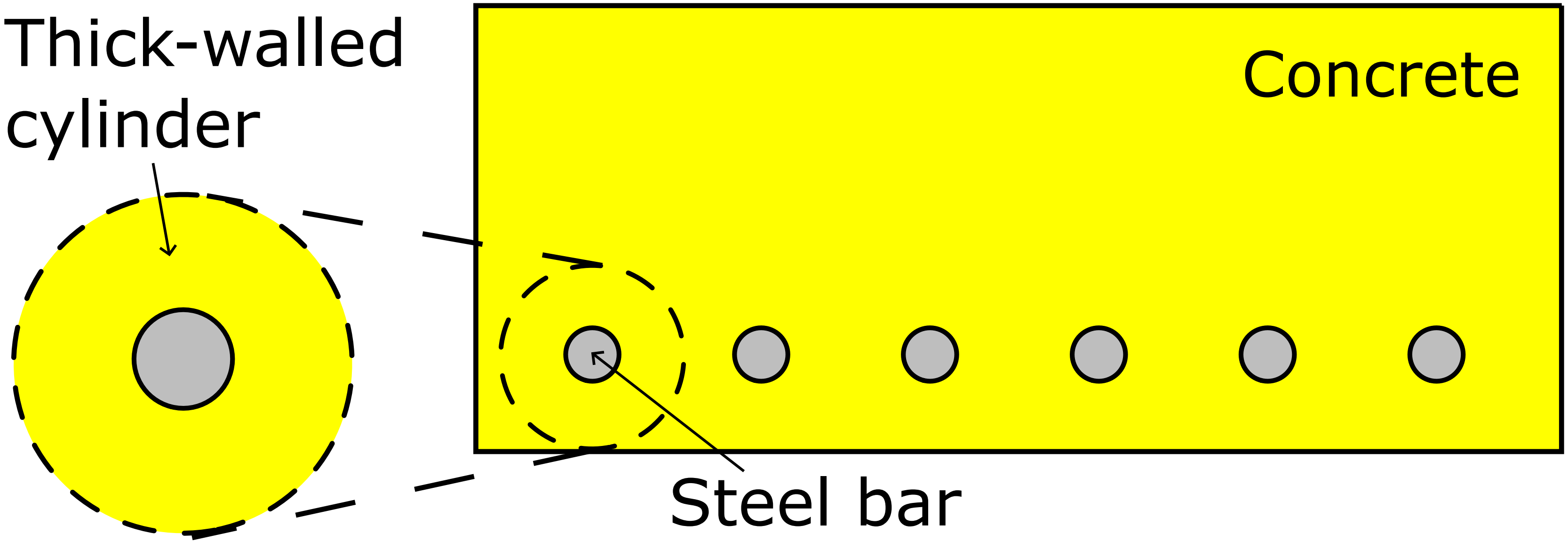
$$\rho = 7.85\ \mathrm{g/cm^3}$$

$$F = 96485\ \mathrm{C/mol}$$

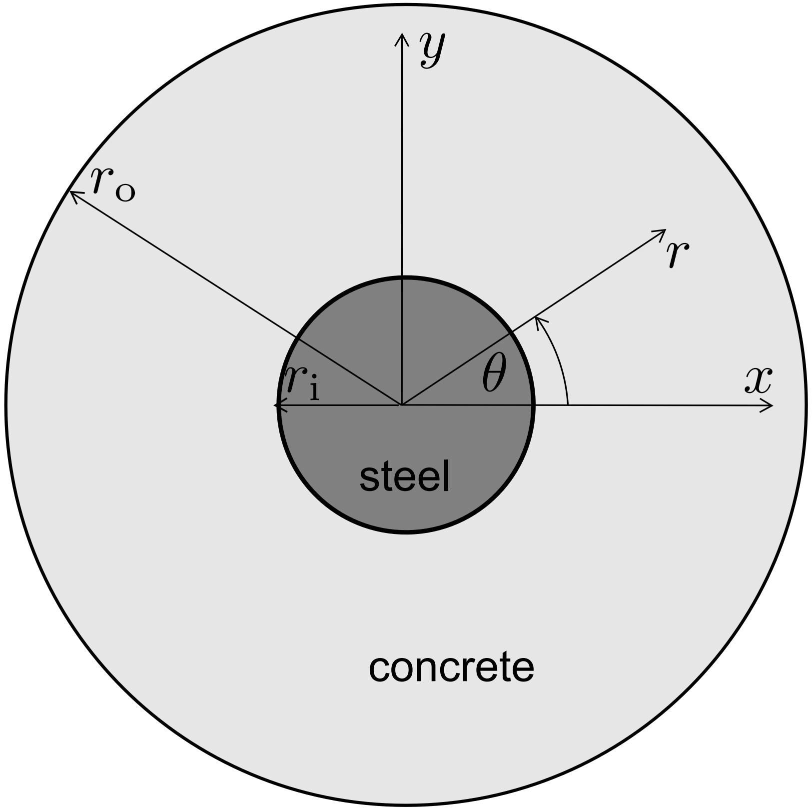
$$1\ \mathrm{C} = 1\ \mathrm{A} \times\ \mathrm{s}$$

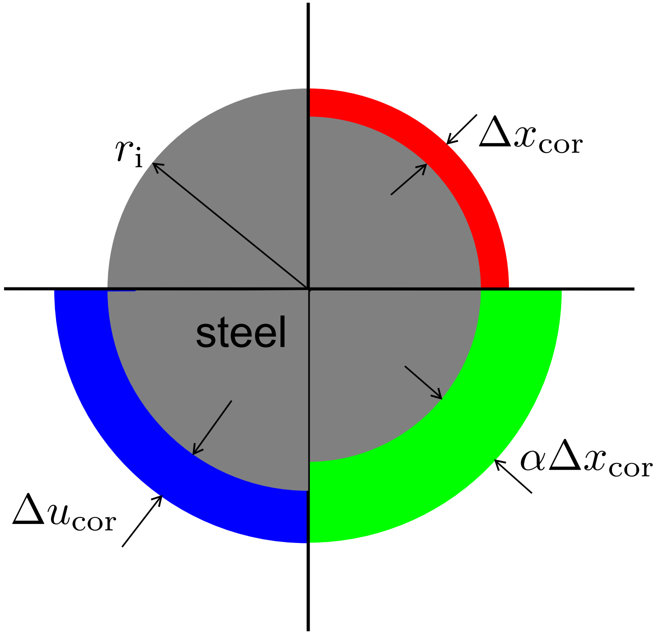
Thick-walled cylinder model

Thick-walled cylinder idealisation



Thick-walled cylinder





$$\alpha = 2$$

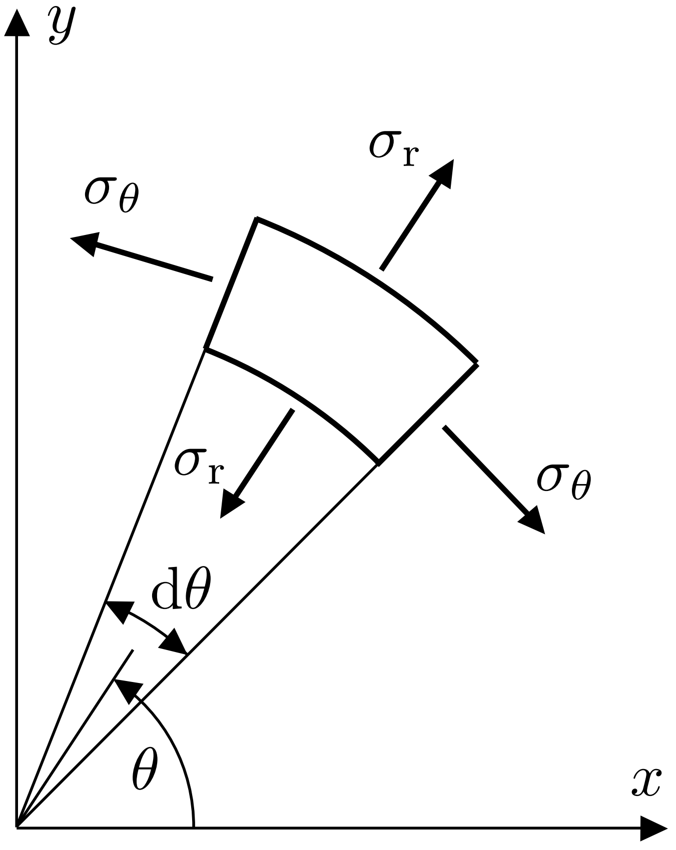
Thick-walled cylinder mechanics

Equilibrium

$$\frac{\mathrm{d}\sigma_\mathrm{r}}{\mathrm{d}r} r + \sigma_\mathrm{r} - \sigma_\theta = 0$$

Compatibility

$$\varepsilon_\mathrm{r} = \frac{\mathrm{d}u}{\mathrm{d}r} \quad \text{and} \quad \varepsilon_\theta = \frac{u}{r}$$

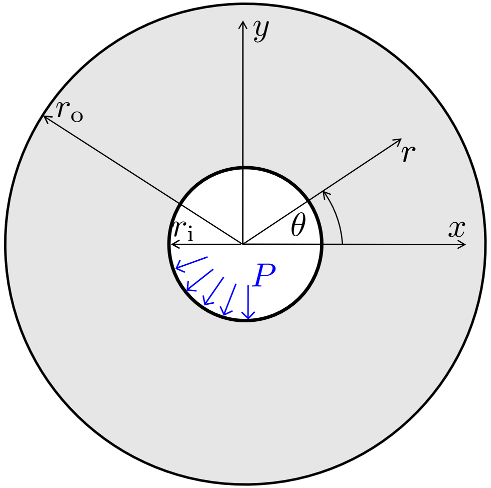


Version 1: Elastic thick-walled cylinder + plastic limit for pressure

Elastic thick-walled cylinder

Constitutive model

$$\begin{Bmatrix} \varepsilon_\mathrm{r} \\ \varepsilon_\theta \end{Bmatrix} = \frac{1}{E} \begin{pmatrix} 1 & -\nu \\ -\nu & 1 \end{pmatrix} \begin{Bmatrix} \sigma_\mathrm{r} \\ \sigma_\theta \end{Bmatrix}$$



ODE of radial displacement

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}u}{\mathrm{d}r} - \frac{u}{r^2} = 0$$

Boundary conditions

$$\sigma_\mathrm{r} = -P \text{ at } r = r_\mathrm{i}$$

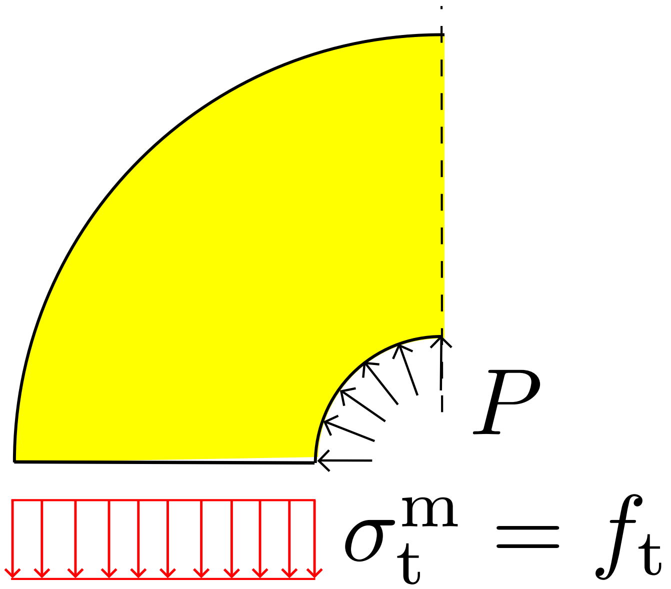
$$\sigma_\mathrm{r} = 0 \text{ at } r = r_\mathrm{o}$$

Elastic thick-walled cylinder mechanics

$$u(r) = \frac{P}{E} \frac{\left(r^2 + r_\mathrm{o}^2\right) r_\mathrm{i}^2}{\left(r_\mathrm{o}^2 - r_\mathrm{i}^2\right) r}$$

$$u_\mathrm{i} = \frac{P}{E} \frac{\left(r_\mathrm{i}^2 + r_\mathrm{o}^2\right) r_\mathrm{i}}{r_\mathrm{o}^2 - r_\mathrm{i}^2}$$

with $u_\mathrm{i} = u(r_\mathrm{i})$



Plastic limit: $P^\mathrm{crit} = f_\mathrm{t} \frac{r_\mathrm{o} - r_\mathrm{i}}{r_\mathrm{i}}$

Combine solution for radial displacement with plastic limit

$$u_{\mathrm{i}}^{\mathrm{crit}} = \frac{f_{\mathrm{t}}}{EC}\frac{r_{\mathrm{o}} - r_{\mathrm{i}}}{r_{\mathrm{i}}} \qquad C = \frac{r_{\mathrm{o}}^2 - r_{\mathrm{i}}^2}{\left((1-\nu)r_{\mathrm{i}}^2 + (1+\nu)r_{\mathrm{o}}^2\right) r_{\mathrm{i}}}$$

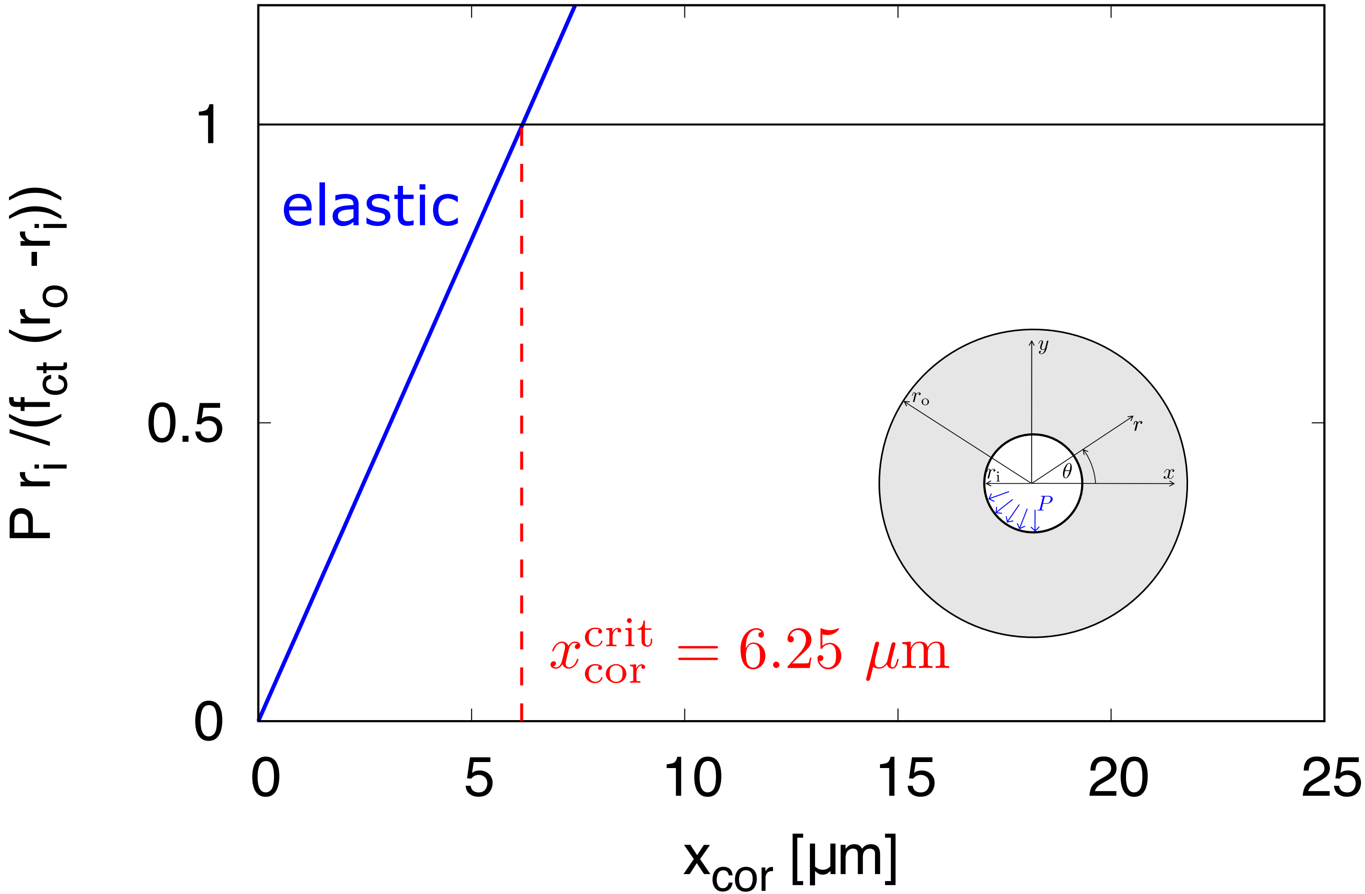
Link to corrosion penetration

$$x_{\mathrm{cor}}^{\mathrm{crit}} = \frac{u_{\mathrm{i}}^{\mathrm{crit}}}{(\alpha - 1)} = \frac{f_{\mathrm{t}}}{(\alpha - 1)\,EC}\frac{r_{\mathrm{o}} - r_{\mathrm{i}}}{r_{\mathrm{i}}}$$

and time

$$\Delta t^{\mathrm{crit}} = \frac{u_{\mathrm{i}}^{\mathrm{crit}}}{0.0315\,(\alpha - 1)\,i_{\mathrm{cor}}} = \frac{f_{\mathrm{t}}}{0.0315\,(\alpha - 1)\,i_{\mathrm{cor}}EC}\frac{r_{\mathrm{o}} - r_{\mathrm{i}}}{r_{\mathrm{i}}}$$

Elastic cylinder with plastic limit



$r_i = 8$ mm
$r_o = 58$ mm

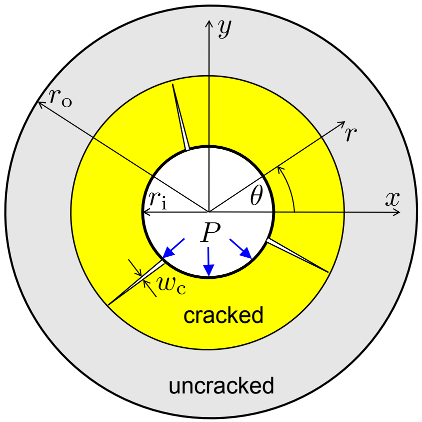
$E = 30$ GPa
$\nu = 0.2$
$f_{ct} = 3$ MPa

Version 2: Cracked cylinder

Thick-wall cylinder mechanics

Cracked constitutive model

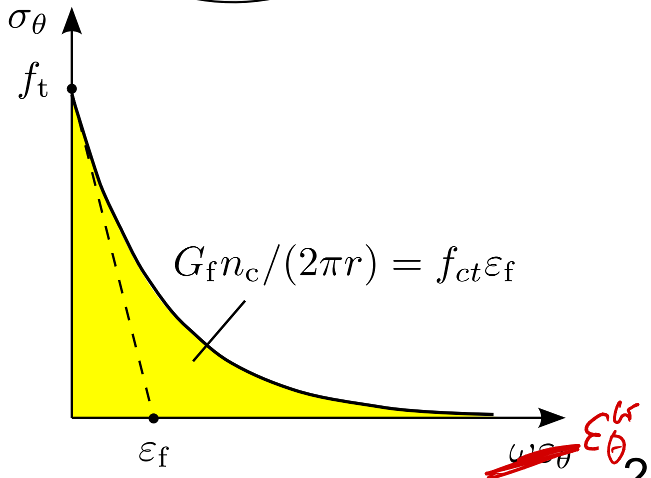
$$\begin{Bmatrix}\varepsilon_\mathrm{r}\\ \varepsilon_\theta\end{Bmatrix} = \frac{1}{E}\begin{pmatrix}1 & -\nu\\ -\nu & 1\end{pmatrix}\begin{Bmatrix}\sigma_\mathrm{r}\\ \sigma_\theta\end{Bmatrix} + \begin{Bmatrix}0\\ \varepsilon_\theta^\mathrm{cr}\end{Bmatrix}$$



Cohesive law

$$\sigma_\theta = f\left(\varepsilon_\theta^\mathrm{cr}, r\right) \equiv f_\mathrm{t}\exp\left(-\frac{\varepsilon_\theta^\mathrm{cr}}{\varepsilon_\mathrm{f}}\right)$$

$$\varepsilon_\mathrm{f} = n_\mathrm{c}G_\mathrm{f}/(f_\mathrm{t}2\pi r)$$



Set stresses in ODE

$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}u}{\mathrm{d}r} - \frac{1}{r^2}u + \frac{1}{r}\left(1-\nu\right)\varepsilon_\theta^{\mathrm{cr}} - \nu\frac{\mathrm{d}\varepsilon_\theta^{\mathrm{cr}}}{\mathrm{d}r} = 0$$

Use $\frac{\mathrm{d}\sigma_\theta}{\mathrm{d}r} = \frac{\mathrm{d}f\left(\varepsilon_\theta^{\mathrm{cr}}, r\right)}{\mathrm{d}r}$ to solve for $\frac{\mathrm{d}\varepsilon_\theta^{\mathrm{cr}}}{\mathrm{d}r}$

$$\frac{\mathrm{d}\varepsilon_\theta^{\mathrm{cr}}}{\mathrm{d}r} = \frac{1}{A}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\frac{1}{r} - \frac{u}{r^2} + \nu\frac{\mathrm{d}^2 u}{\mathrm{d}r^2}\right)$$

with

$$A = 1 - \frac{f_{\mathrm{t}}}{E\varepsilon_{\mathrm{f}}}\left(1-\nu^2\right)\exp\left(-\frac{\varepsilon_\theta^{\mathrm{cr}}}{\varepsilon_{\mathrm{f}}}\right)$$

Nonlinear ODE

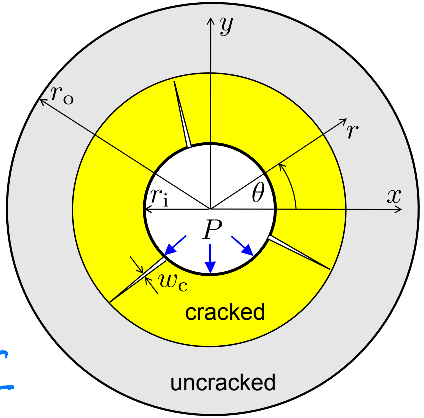
$$\frac{\mathrm{d}^2 u}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}u}{\mathrm{d}r}\frac{A-\nu}{A-\nu^2} - \frac{1}{r^2}u\frac{A-\nu}{A-\nu^2} + \frac{1}{r}\frac{A(1-\nu)}{A-\nu^2}\varepsilon_\theta^{\mathrm{cr}} = 0$$

$$A = 1 - \frac{f_\mathrm{t}}{E\varepsilon_\mathrm{f}}\left(1-\nu^2\right)\exp\left(-\frac{\varepsilon_\theta^{\mathrm{cr}}}{\varepsilon_\mathrm{f}}\right)$$

Matlab script available on github

Solve with bvp4c in MATLAB

Incremental analysis controlled by increasing u_{cor}

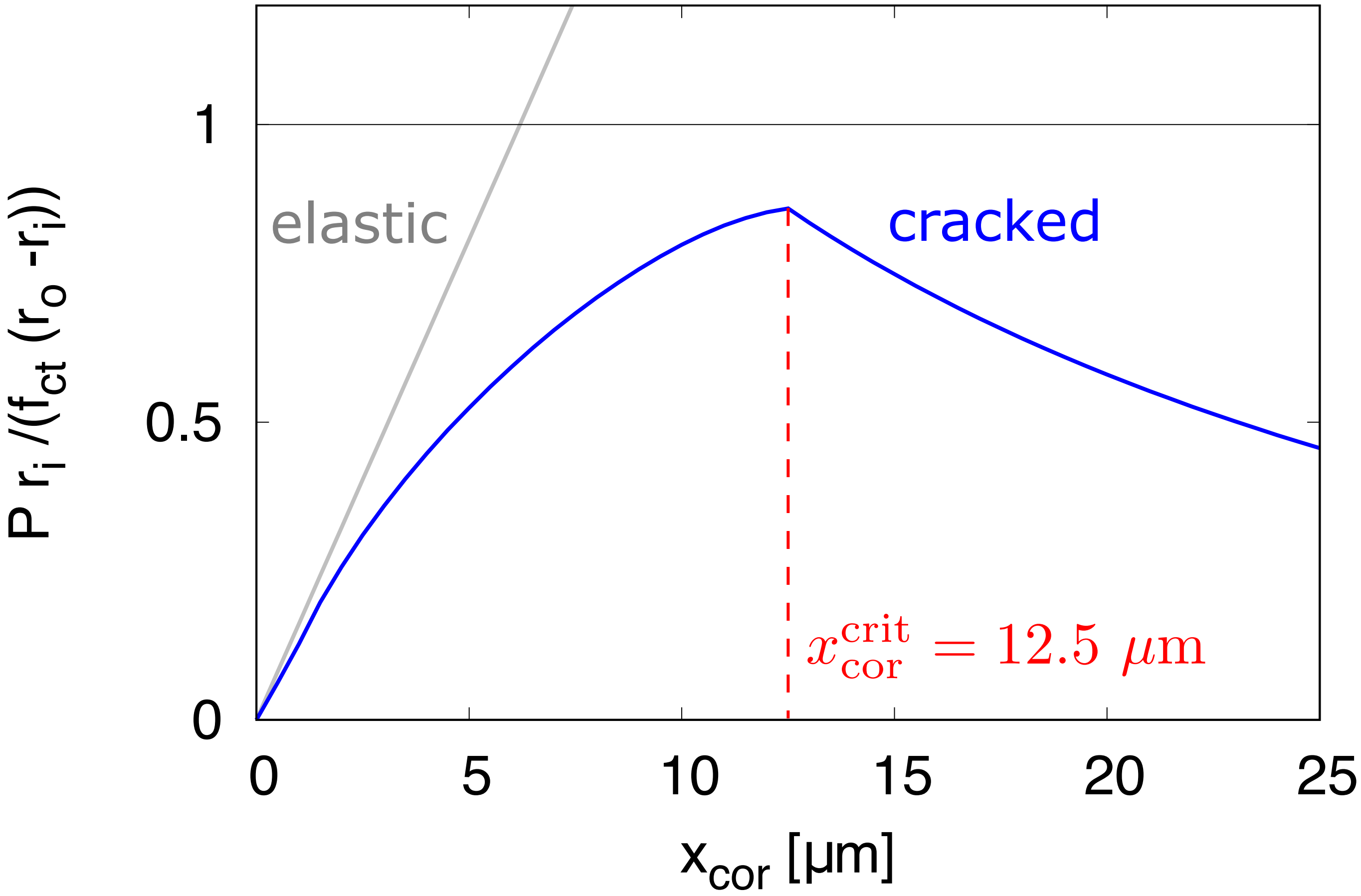


Boundary conditions

$$u(r_\mathrm{i}) = u_\mathrm{cor}$$

$$\sigma_\mathrm{r}(r_\mathrm{o}) = 0$$

Cracked cylinder



$r_{\mathrm{i}} = 8$ mm
$r_{\mathrm{o}} = 58$ mm

$E = 30$ GPa
$\nu = 0.2$
$f_{\mathrm{ct}} = 3$ MPa
$n_{\mathrm{c}} = 4$
$G_{\mathrm{F}} = 150$ N/m

Version 3: Time dependent response

Time dependent response

$$\begin{Bmatrix} \varepsilon_\mathrm{r} \\ \varepsilon_\theta \end{Bmatrix} = \frac{1}{E''} \begin{pmatrix} 1 & -\nu \\ -\nu & 1 \end{pmatrix} \begin{Bmatrix} \sigma_\mathrm{r} \\ \sigma_\theta \end{Bmatrix} + \begin{Bmatrix} 0 \\ \varepsilon_\theta^\mathrm{cr} \end{Bmatrix}$$

Modified AAEM

$$E^{''}(t, t_0) = \frac{1 - R(t, t_0) J(t_0^*, t_0)}{J(t, t_0) - J(t_0^*, t_0)}$$

t_0 start of loading

$$t_0^* = \begin{cases} 0.9 t_0 + 0.1 t & \text{if } t_0 < t < t_0 + 10 \Delta t_\mathrm{s} \\ t_0 + \Delta t_\mathrm{s} & \text{if } t_0 + 10 \Delta t_\mathrm{s} \leq t \end{cases}$$

Ref: Bažant and Jirásek (2018)

$$R(t,t_0)=\frac{1}{J(t,t_0)}\left[1+\frac{c_1(t_0)J(t,t_0)}{10J(t,t-\Delta t)}\left(\frac{J(t_m,t_0)}{J(t,t_{\mathrm{m}})}-1\right)\right]^{-10}$$

$$c_1(t_0)=0.08+0.0119\ln t_0$$

$$\Delta t = 1 \text{ day}$$

$$\Delta t_{\mathrm{s}} = 0.01 \text{ days}$$

$$J(t,t_0)=q_1+q_2 Q(t,t_0)+q_3\ln\left[1+(t-t_0)^n\right]+q_4\ln\left(\frac{t}{t_0}\right)$$

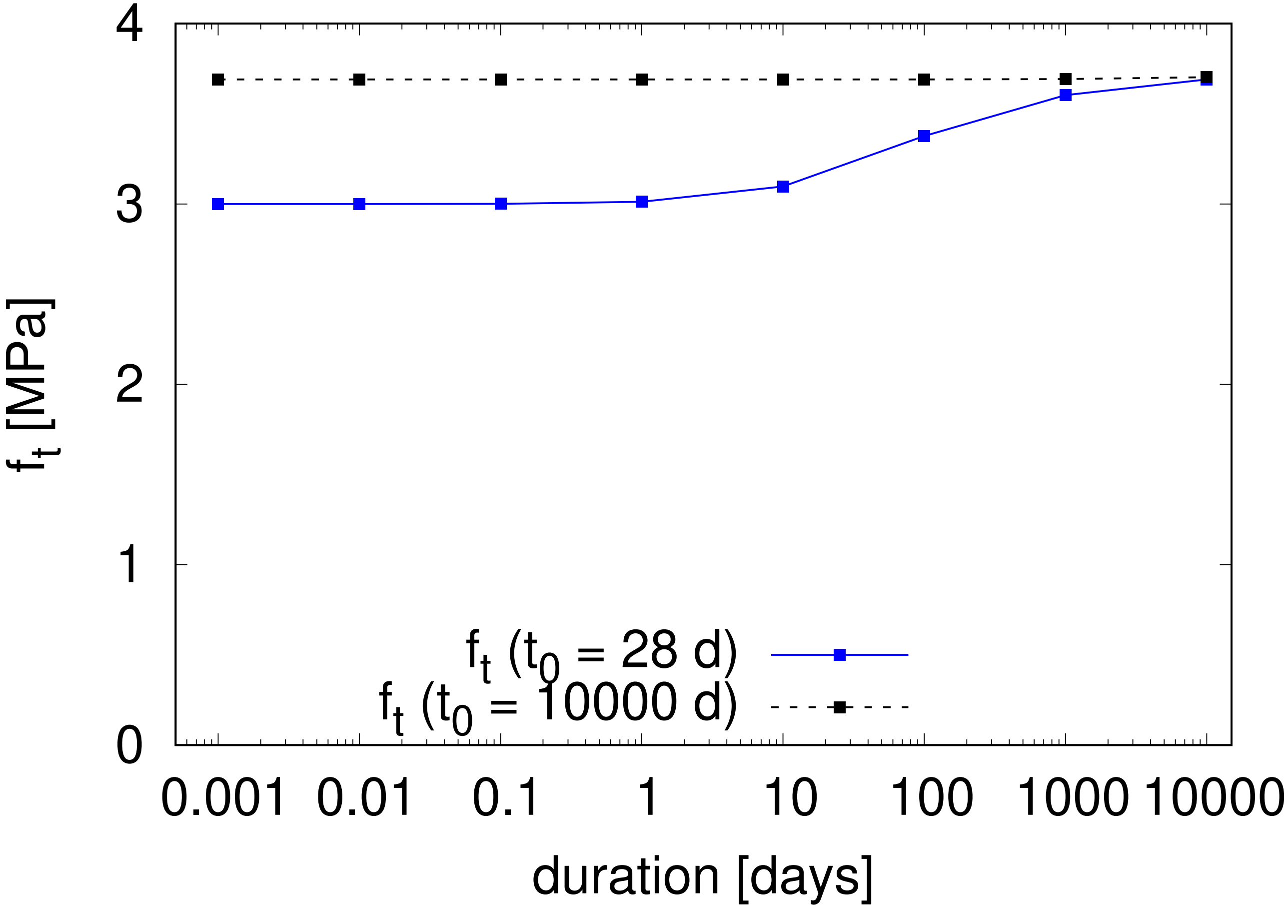
Parameters based on fc, w, c, a

Maturity according to Model Code 2010

$$f_c(t)=f_{\mathrm{c}}^{28}\exp\left(s\left[1-\sqrt{28/t}\right]\right)$$

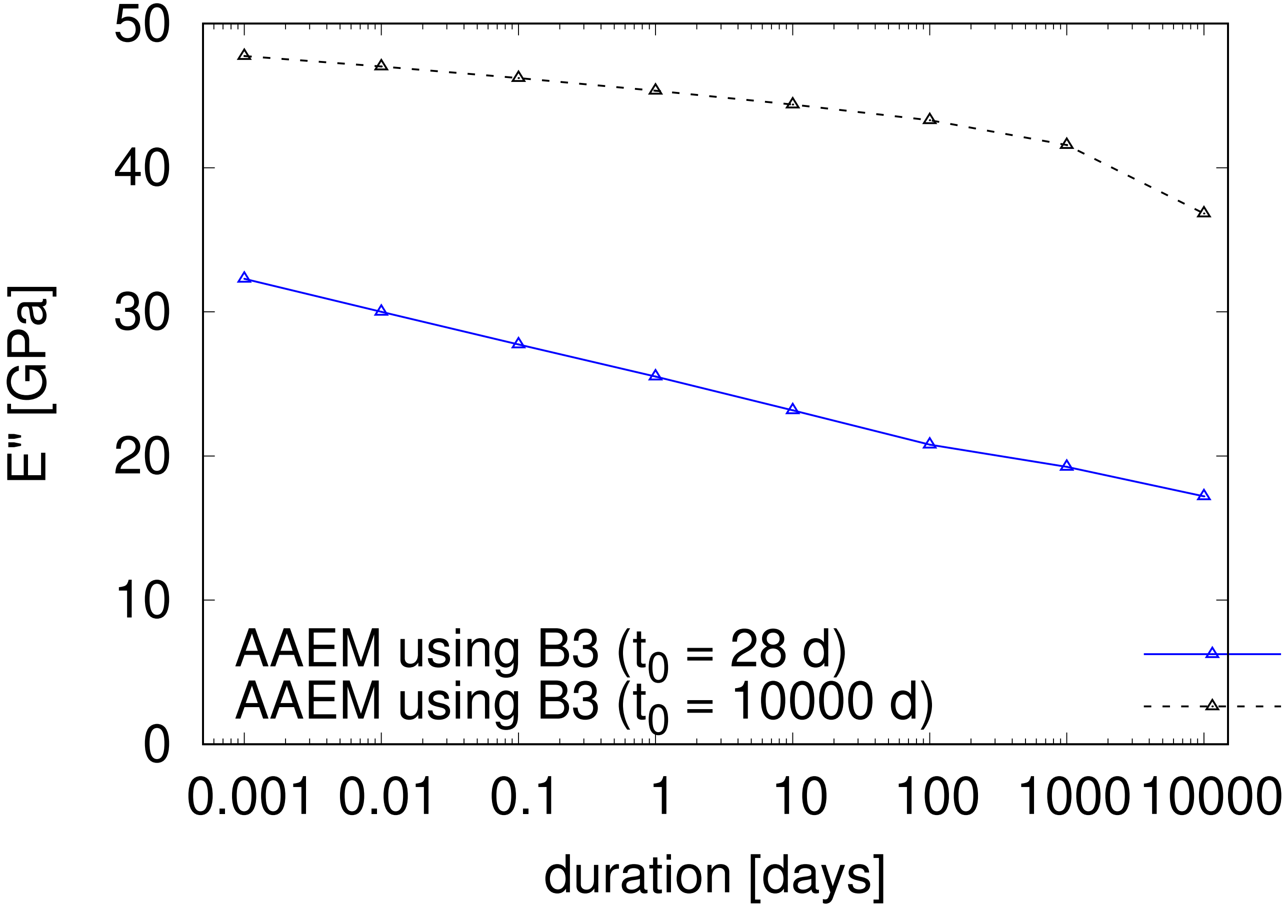
Ref: Bažant and Jirásek (2018)

Effect of maturity on tensile strength



$E_{28} = 30$ GPa
$\nu = 0.2$
$f_{\mathrm{ct}} = 3$ MPa
$f_{\mathrm{cc}} = 30$ MPa
$c = 360$ kg/m^3
$w = 180$ kg/m^3
$a = 1860$ kg/m^3

Effect of creep and maturity on effective modulus



$E_{28} = 30$ GPa

$\nu = 0.2$

$f_{ct} = 3$ MPa

$f_{cc} = 30$ MPa

$c = 360$ kg/m^3

$w = 180$ kg/m^3

$a = 1860$ kg/m^3

Scenarios for investigating effect of creep

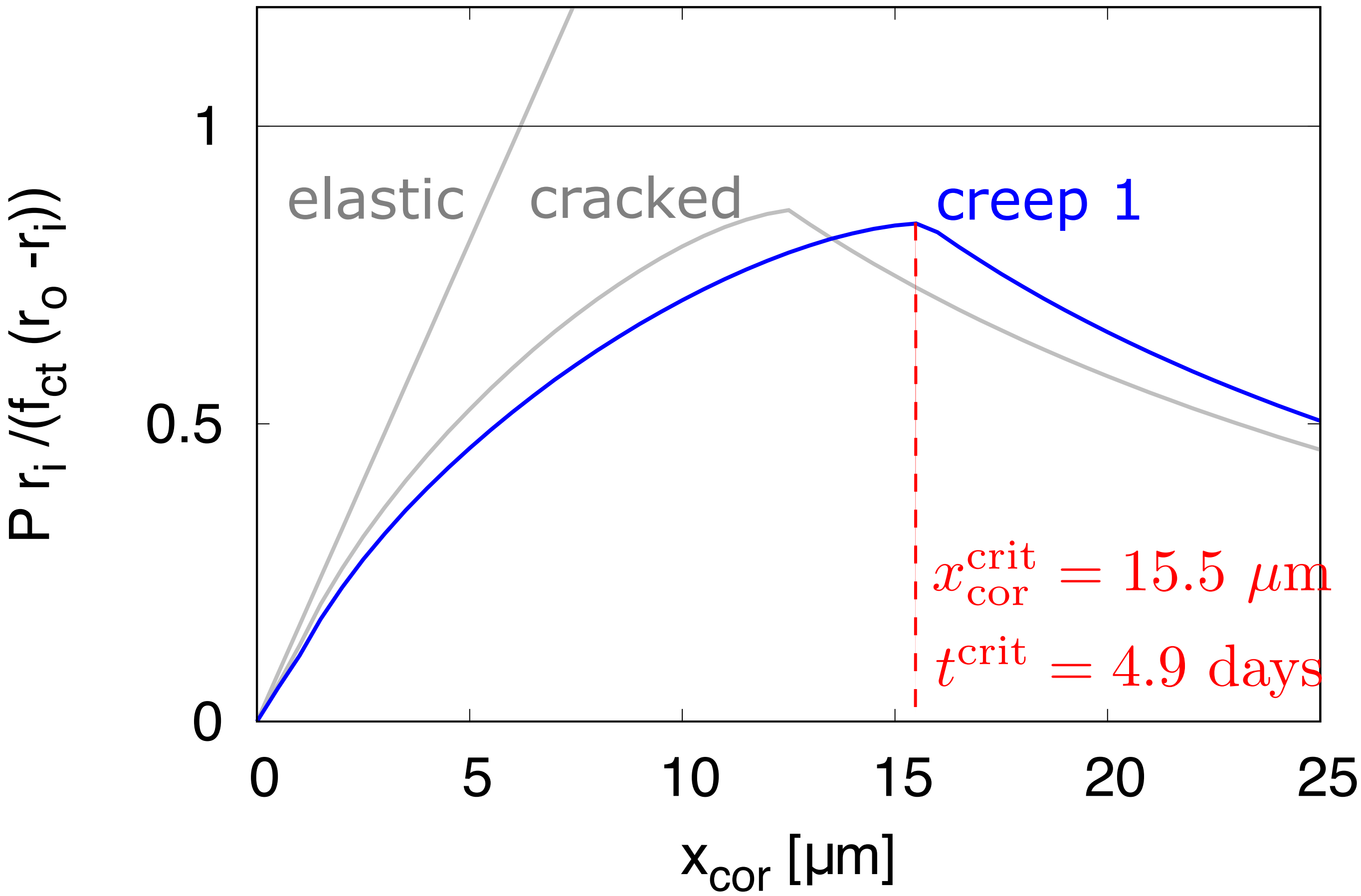
Scenario 1	Young concrete	$t_0 = 28$ days
	Accelerated corrosion	$i_{cor} = 100\ \mu A/cm^2$
Scenario 2	Young concrete	$t_0 = 28$ days
	Slow corrosion	$i_{cor} = 1\ \mu A/cm^2$
Scenario 3	Old concrete	$t_0 = 10000$ days
	Accelerated corrosion	$i_{cor} = 100\ \mu A/cm^2$
Scenario 4	Old concrete	$t_0 = 10000$ days
	Slow corrosion	$i_{cor} = 1\ \mu A/cm^2$

Lab

not realistic

In field

Scenario 1: Young concrete + accelerated corrosion

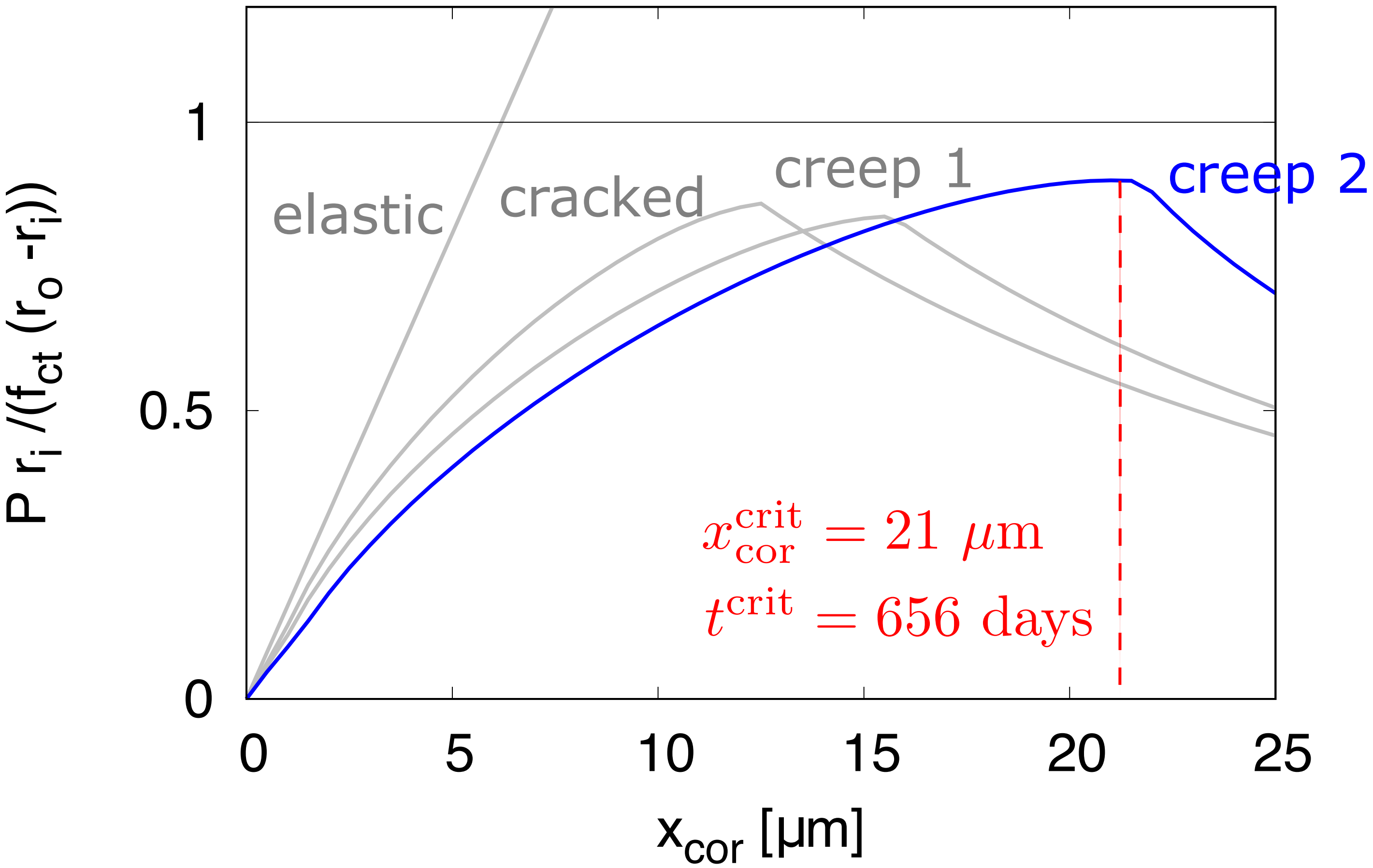


$r_i = 8$ mm
$r_o = 58$ mm

$E_{28} = 30$ GPa
$\nu = 0.2$
$f_{ct} = 3$ MPa
$f_{cc} = 30$ MPa
$c = 360\ kg/m^3$
$w = 180\ kg/m^3$
$a = 1860\ kg/m^3$

$t_o = 28$ days
$i_{cor} = 100\ \mu A/cm^2$

Scenario 2: Young concrete + slow corrosion

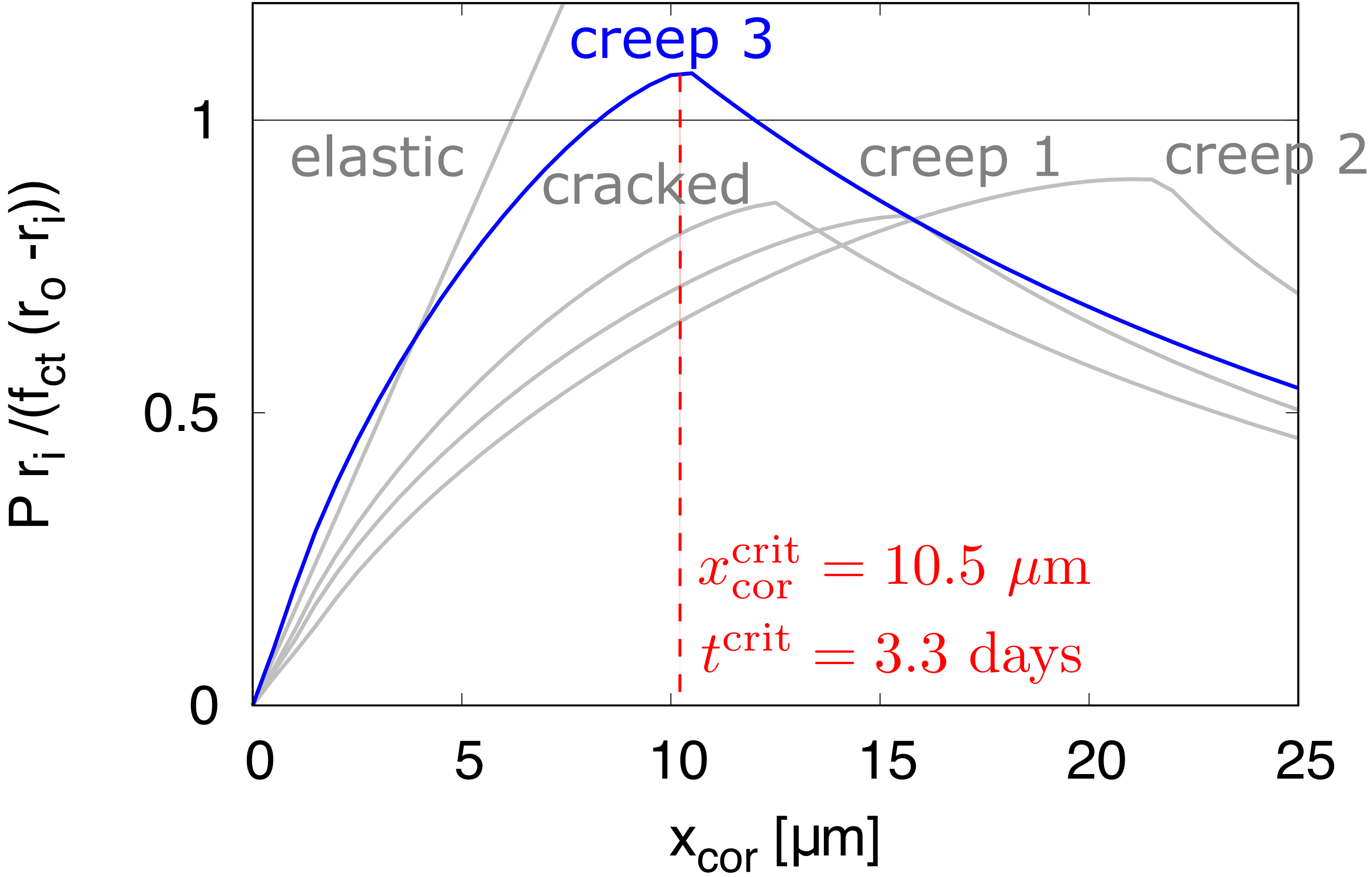


$r_i = 8$ mm
$r_o = 58$ mm

$E_{28} = 30$ GPa
$\nu = 0.2$
$f_{ct} = 3$ MPa
$f_{cc} = 30$ MPa
$c = 360\ kg/m^3$
$w = 180\ kg/m^3$
$a = 1860\ kg/m^3$

$t_o = 28$ days
$i_{cor} = 1\ \mu A/cm^2$

Scenario 3: Old concrete + accelerated corrosion

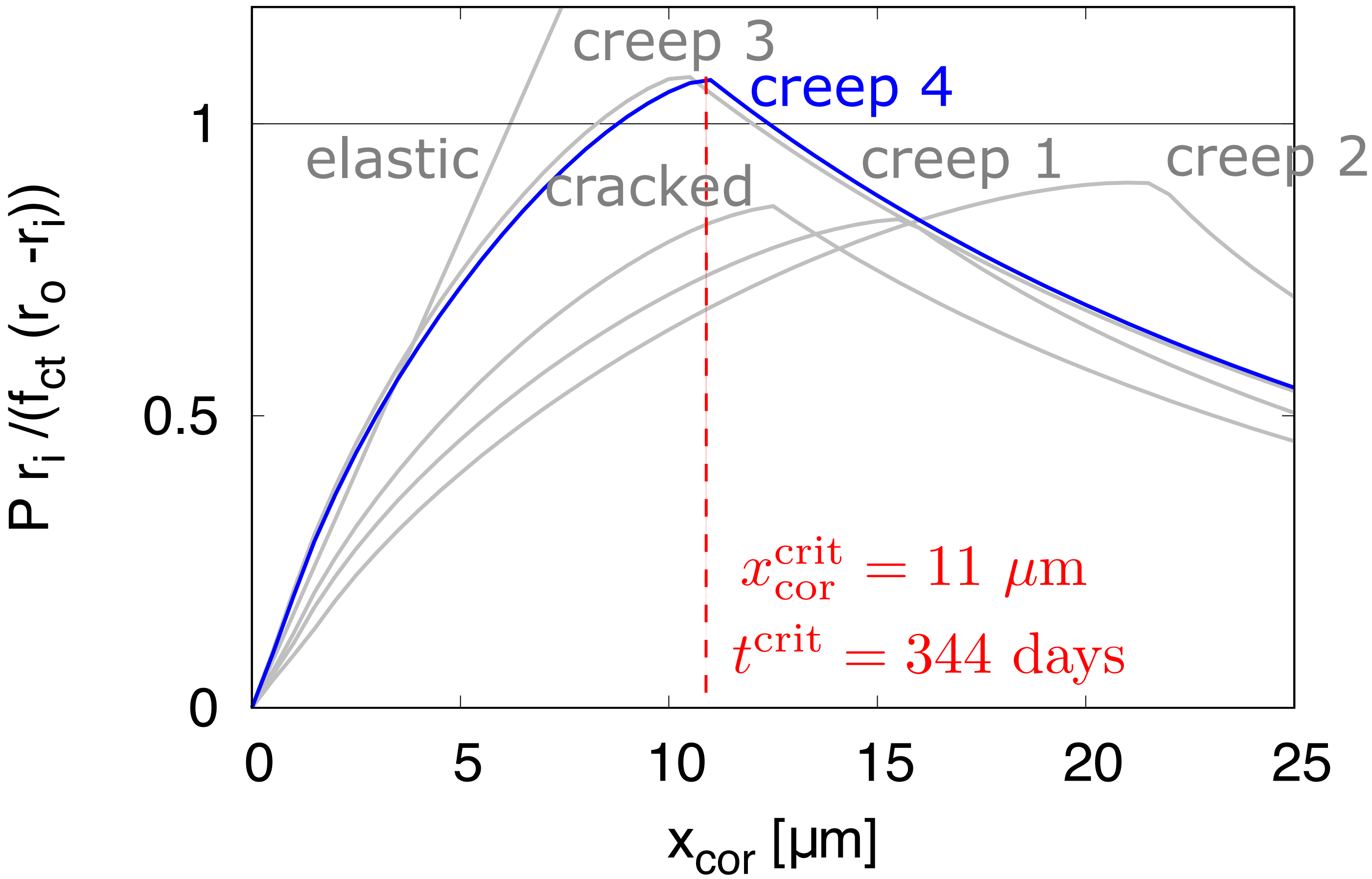


$r_i = 8$ mm
$r_o = 58$ mm

$E_{28} = 30$ GPa
$\nu = 0.2$
$f_{ct} = 3$ MPa
$f_{cc} = 30$ MPa
$c = 360\ kg/m^3$
$w = 180\ kg/m^3$
$a = 1860\ kg/m^3$

$t_o = 10000$ days
$i_{cor} = 100\ \mu A/cm^2$

Scenario 3: Old concrete + slow corrosion



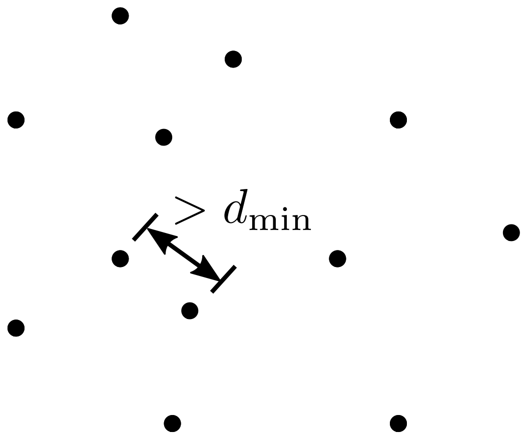
$r_i = 8$ mm
$r_o = 58$ mm

$E_{28} = 30$ GPa
$\nu = 0.2$
$f_{ct} = 3$ MPa
$f_{cc} = 30$ MPa
$c = 360\ kg/m^3$
$w = 180\ kg/m^3$
$a = 1860\ kg/m^3$

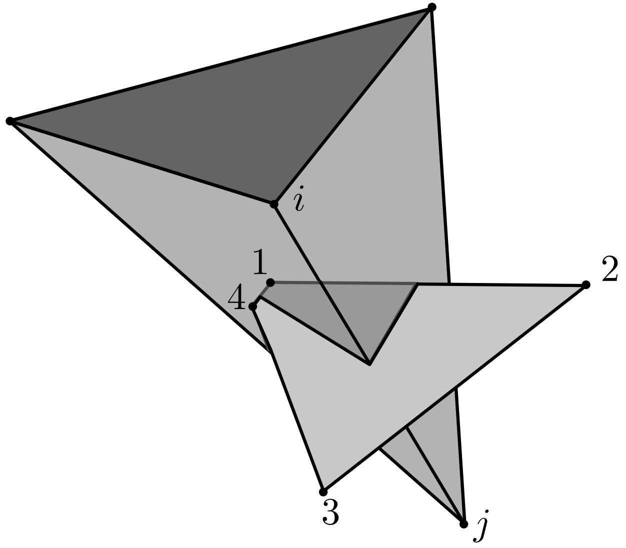
$t_o = 10000$ days
$i_{cor} = 1\ \mu A/cm^2$

Lattice model and experiments

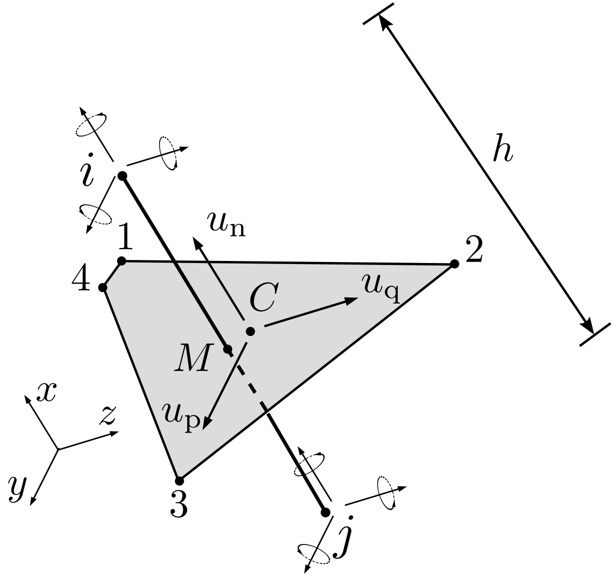
Lattice discretisation



Constrained random points



Tessellations

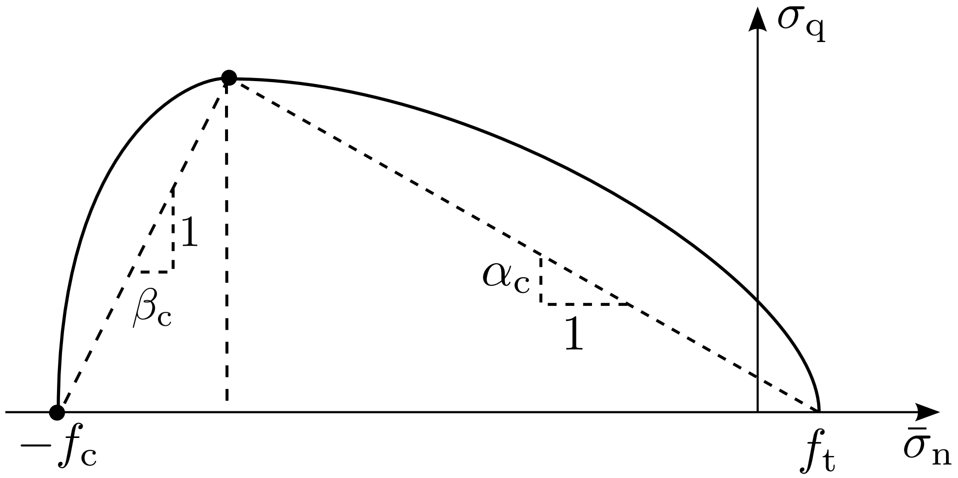


Lattice element

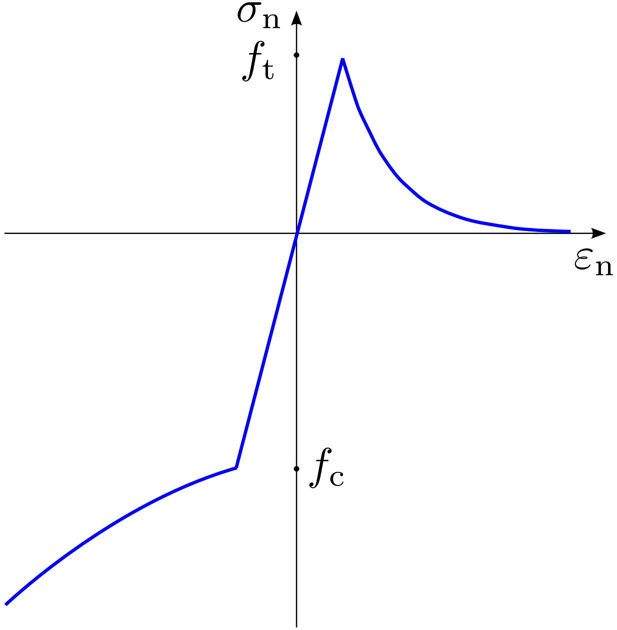
Ref: Grassl and Bolander (2016), Yip et al. (2005), Schlangen and van Mier (1992), Kawai (1977)

Lattice constitutive model

$$\boldsymbol{\sigma} = (1-\omega)\,\mathbf{D}_{\mathrm{e}}\,(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{p}}) = (1-\omega)\bar{\boldsymbol{\sigma}}$$



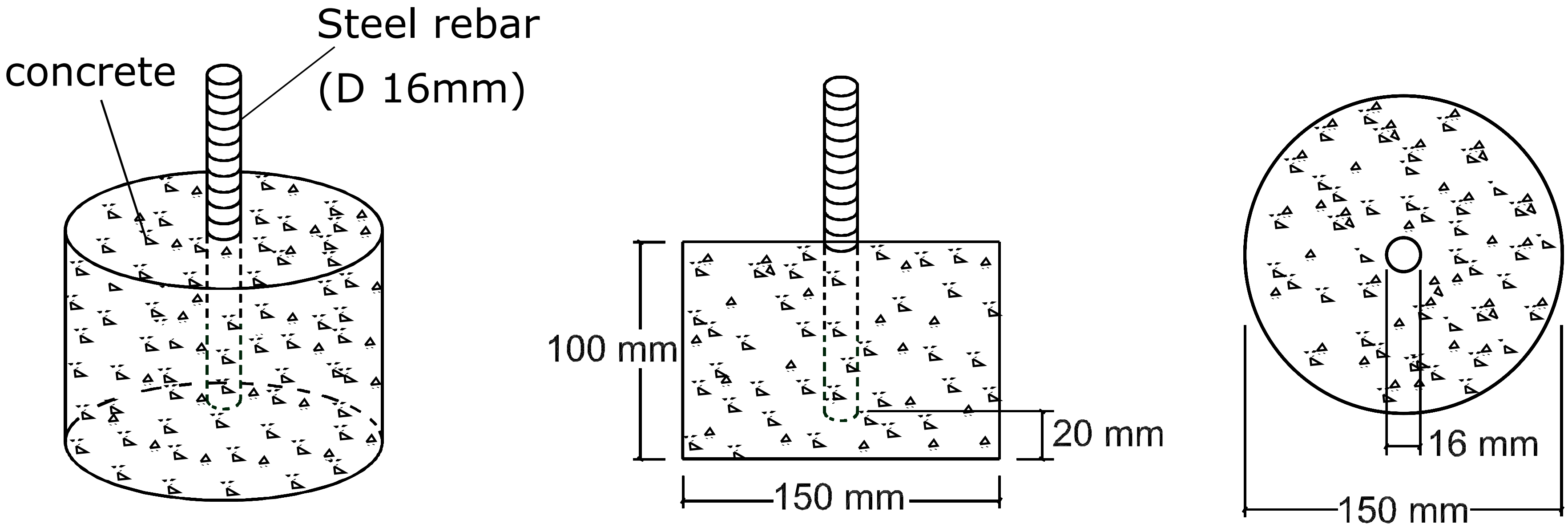
Yield surface



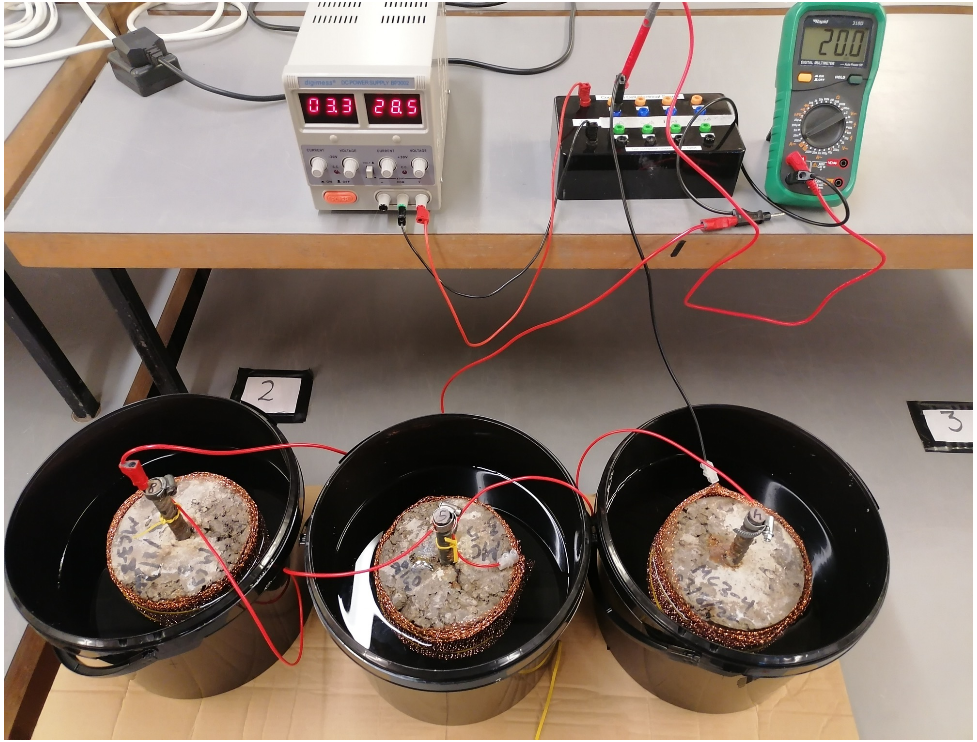
Stress-strain

Ref: Grassl and Bolander (2016), Grassl (2009)

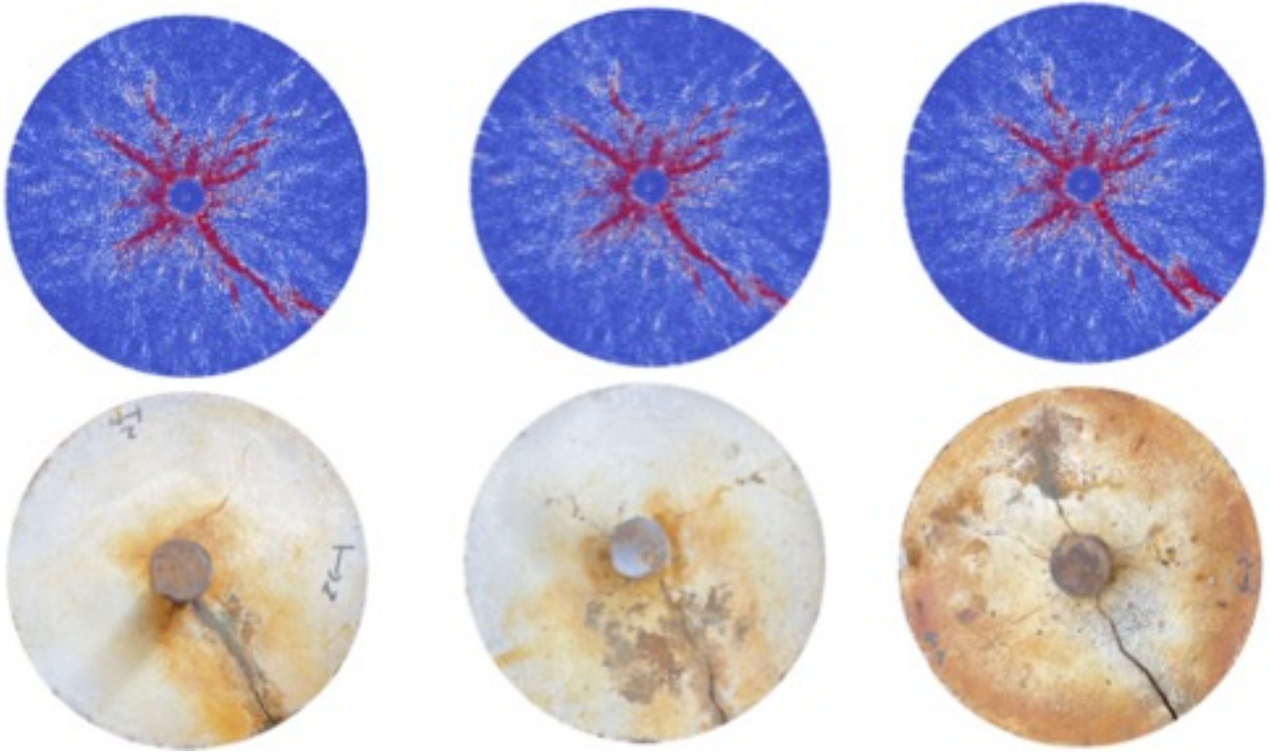
Experiments - Geometry



Experiments - Setup

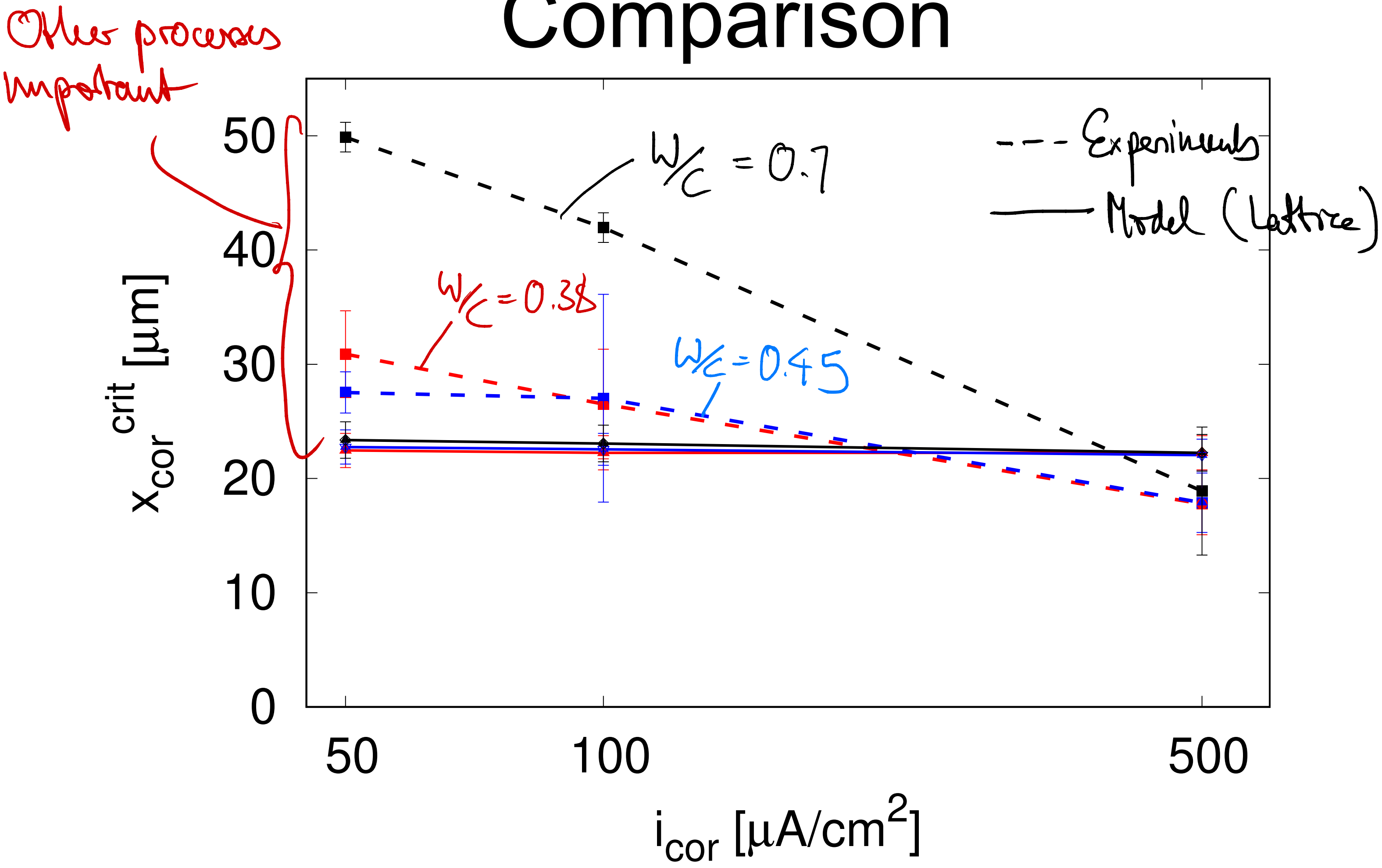


Crack patterns



w/c = 0.7 | w/c = 0.45 | w/c = 0.38

Comparison


Other processes important
W/C = 0.7
W/C = 0.38
W/C = 0.45
Experiments
Model (Lattice)
50
40
30
20
10
0
50
100
500
x_{cor}^{crit} [μm]
i_{cor} [μA/cm^2]

Conclusions

Inclusion of cracking significantly affects critical corrosion penetration.

Creep has moderate effect on accelerated corrosion for young concrete.

Very small effect of creep on slow corrosion for old concrete.

Experiments indicate that cracking, creep and maturity alone is not enough to predict corrosion induced cracking. Transport of corrosion products required?

University of Glasgow

Thank you for listening

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