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THREE-DIMENSIONAL LATTICE MODEL FOR COUPLING OF FRACTURE AND FLOW

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Abstract. In this paper, a three-dimensional modelling approach to combine fracture and inviscid flow is proposed. Lattices of structural elements for the mechanical behaviour (elasticity and fracture) and conduit elements for the flow are combined. The spatial arrangement and the properties of the lattice elements is based on irregular Voronoi and Delaunay tessellations of the domain, respectively. The transport properties of the conduit elements are chosen to evolve with the crack openings of the neighbouring structural elements. The new lattice approach is applied to stationary potential flow.

1 INTRODUCTION

Fracture increases the permeability of cementitious materials, which is known to accelerate deterioration of these materials when exposed to aggressive environments. The coupling of fracture and flow is therefore important for modelling the durability of cementitious materials. However, modelling of these coupled processes with continuum mechanics presents several challenges, including the discrete representation of crack formation and opening. Lattice models are an attractive alternative in that they describe well the discontinuities that arise from fracture processes in concrete materials[1,2]. In addition, lattice approaches to modelling flow have been validated through comparisons with theory and finite element solutions[3]. Furthermore, two-dimensional lattice approaches have been developed to couple the individual lattices representing mechanical behaviour (i.e. elasticity and fracture) and flow[4,5]. These models have been used to analyse stationary and nonstationary flow fields and it was shown that basic theoretical solutions could be accurately reproduced. Furthermore, the coupling of flow and fracture was analysed in

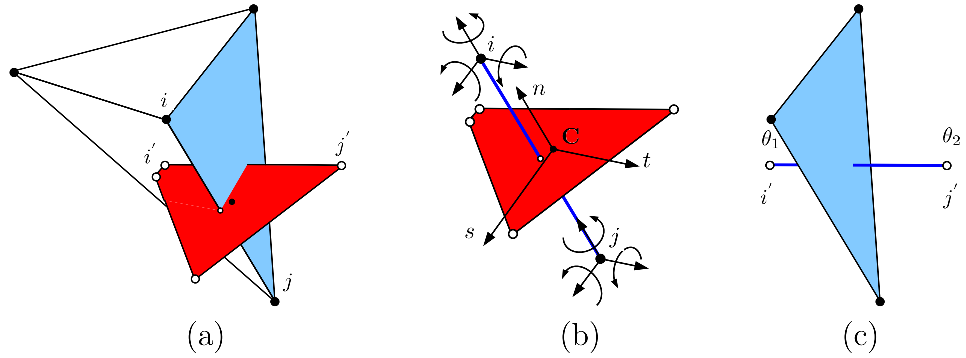

Figure 1: Lattice model: (a) Delaunay tetrahedra and Voronoi polyhedra facet. (b) Structural lattice element. (c) Flow lattice element.

comparison with several benchmark problems and the sensitivity of the numerical results with respect to the size of the lattice elements was studied[5]. The present paper describes basic concepts and preliminary steps toward extending this model to three dimensions.

2 COMBINED MODEL FOR MECHANICAL LOADING AND FLOW

The present three-dimensional numerical model for the coupling of potential flow and fracture is based on lattices of one-dimensional structural and conduit elements. The spatial arrangement of the lattice elements and their cross-sectional properties are based on Delaunay and Voronoi tessellations of the domain to be modelled. For a three-dimensional domain, the Voronoi tesselation results in polyhedra with Voronoi facets, edges and vertices. On the other hand, the Delaunay tessellation consists of tetrahedra with Delaunay facets, edges and vertices. In Fig. 1a, a Delaunay tetrahedron and a facet of a Voronoi polyhedron are shown.

For the mechanical lattice, the lattice elements are placed on the edges of the Delaunay tetrahedra. The geometry of the mid cross-section of the lattice elements is determined by the corresponding facet of the Voronoi polyhedra (Fig. 1b). Each node has six degrees of freedom, that is three translations and three rotations, which determine the displacement jump at the centroid C of the element's mid cross-section. The displacement jumps are transformed into strains by the element length. An isotropic damage model is used to relate the strains to stresses. The evolution of damage is controlled by a stress-crack opening curve, so that the mechanical response is independent of the length of the lattice elements used.

The flow lattice consists of one-dimensional conduit elements, for which the locations and cross-sectional properties are obtained from the Voronoi and Delaunay tessellation. The conduit elements are placed on the edges of the Voronoi polyhedra. Furthermore, the cross-sectional areas of the conduit elements are set equal to the facet areas of the corresponding Delaunay tetrahedra (Fig. 1c). The increase of the diffusivity due to cracking is

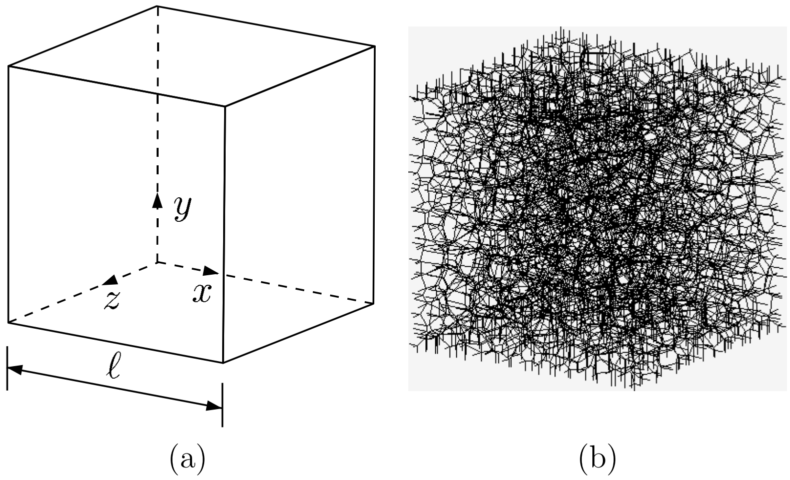

Figure 2: Uniaxial flow: (a) Geometry of the cubic domain. (b) Irregular lattice of conduit elements.

described by the crack-openings of the three structural lattice elements, which are placed on the boundary of the mid cross-section of the conduit element.

3 RESULTS

As a step toward extending the coupled flow-fracture analyses to three dimensions, the new flow lattice is used to analyse uniaxial stationary flow in a cubic domain shown in Fig. 2a. The domain is discretised by an irregular lattice of conduit elements, which are placed on the edges of the Voronoi polyhedra as described in the previous section (Fig. 2b). The cross-sectional areas of the conduit elements are chosen in two ways. In the first approach, cross-sectional areas are determined by the dual Delaunay tessellation. In the second approach, a constant cross-sectional area is used for all conduit elements. The nodes on the left and right hand sides ($x = 0$ and $x = \ell$) of the model domain of unit length and diffusivity are subjected to constant potentials of $\theta = 0$ and $\theta = 1$, respectively. For the other four faces, the boundary flux is assumed to be zero. The exact solution for this problem is $\theta = x$. The flow along the x-direction for $y = z = \ell/2$ for the two approaches is shown in Fig. 3. The accuracy of the modelling approach is assessed by comparing the L_2 error norm to the exact solution. The error for a constant cross-sectional area is $e_{\mathrm{r}} = 0.0042$, whereas the error for cross-sectional areas obtained from the Delaunay tessellation is $e_{\mathrm{r}} = 4.72 \times 10^{-17}$.

Consequently, the lattice with conduit elements placed on the edges of Voronoi tessellation and cross-sections obtained from the dual Delaunay tessellation results in an accurate description of the stationary flow field. The results of the present study complement results obtained from a dual lattice approach, in which the conduit elements are placed on the edges of the Delaunay tessellation and the cross-sections are the facets of the Voronoi tessellation[3].

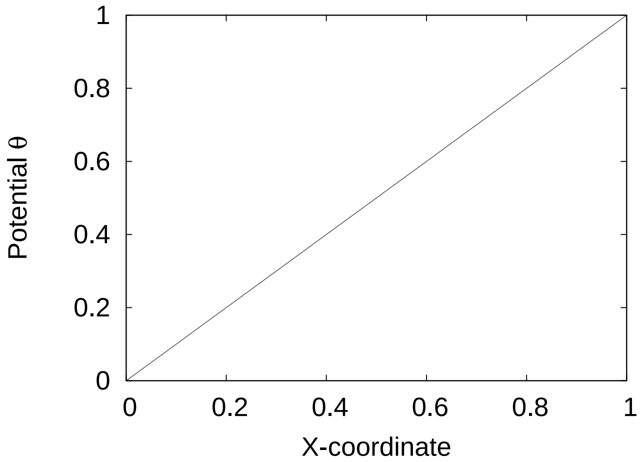

Figure 3: Uniaxial flow: Distribution of the potential along the x-axis for $y = z = \ell/2$.

4 CONCLUSIONS

The new coupled lattice approach, which is based on an irregular Delaunay and Voronoi tessellation of the domain, allows for precise three-dimensional representation of stationary potential flow. Based on previous two-dimensional results, the approach provides an objective (mesh size independent) means for simulating the influence of cracking on flow. Ongoing work involves the extension of this capability to three-dimensional analyses, along with investigations of non-stationary flow through fractured media.

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